Entanglement swapping with artificial atoms

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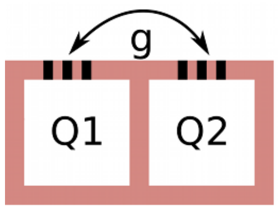
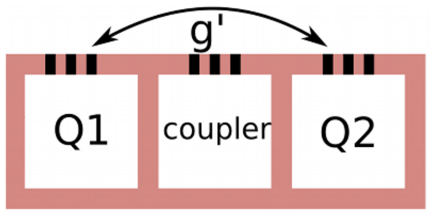
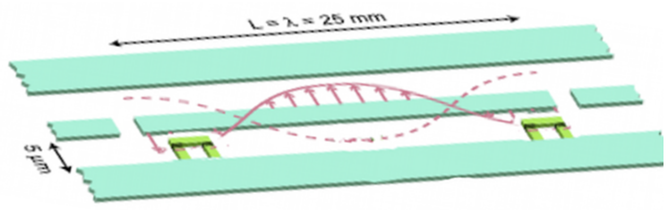
Motivation

- Qubits, the building blocks of a future quantum computer, become useful when quantum communication between them (entanglement) can be established
- Entanglement plays a crucial role in quantum information processing e.g. teleportation protocol, quantum cryptography
- Entanglement of solid states qubits usually is achieved when direct or indirect interaction between the qubits exists.







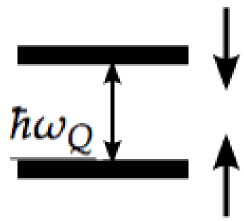
- We discuss a method of entangling *non-interacting* (distant) artificial atoms using the entanglement swapping scheme.

Artificial atoms

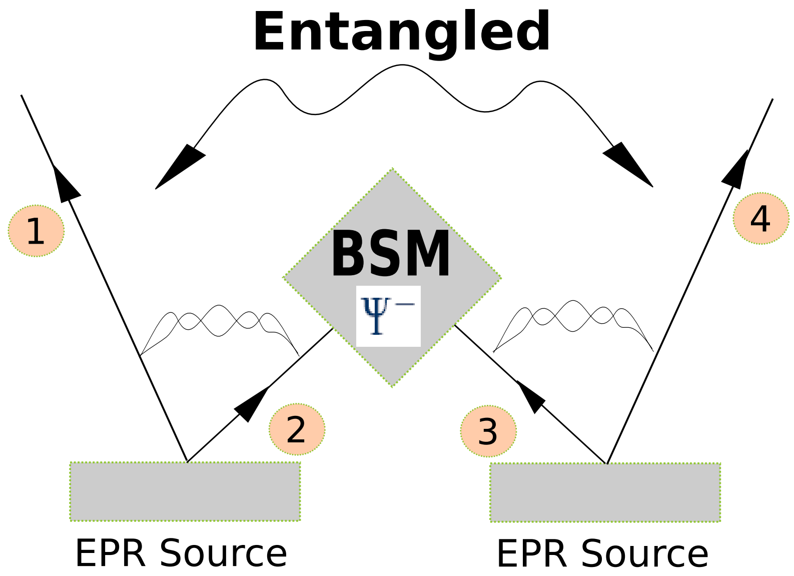
Artificial atoms, are made up of more than one atom, but are like single atoms in one important way: when you provide the right amount (or quanta) of energy, they will give off coloured light.
Nanometer-scale *boxes* in which the electrons are forced to exist at specific, discrete energy levels

- quantum dots
- superconducting circuits (charge qubits)
  (resonance fluorescence experiment , O. Astafiev et al. Science **327**, 840 (2010))
- non-superconducting quantum ring qubits

artificial atom $\rightarrow$ 2-level system $\rightarrow$ qubit



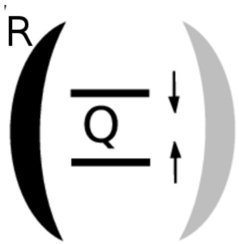
Entanglement swapping



M. Żukowski et al. PRL **71** *,4287, (1993)*

- As far: on photons (*J-W. Pan et al. PRL* **80**, *18, (1998)*), atoms (*Moehring et al. Nature* **449**, *2007*), quantum dots (*Barret et al. PRA* **71**, *2005*)

Entanglement swapping with artificial atoms



$$H_{QR} = \frac{\hbar\omega_Q}{2}\sigma_z + \hbar\omega_R\left(a^\dagger a + \frac{1}{2}\right) - \hbar g\left(a\sigma_+ + a^\dagger\sigma_-\right)$$

Resonant case ($\omega_R = \omega_Q$, number of photons $n = 0, 1$)

- One excitation in the system being coherently exchanged between the qubit and the cavity
- Strong coupling limit $g > \kappa_Q, \gamma_R$

Time evolution

The initial state: $|\psi(0)\rangle = |\downarrow 0\rangle_1 \otimes |\uparrow 1\rangle_2$ - separable

The time evolution of the system is generated by the Hamiltonian H_{QR}:

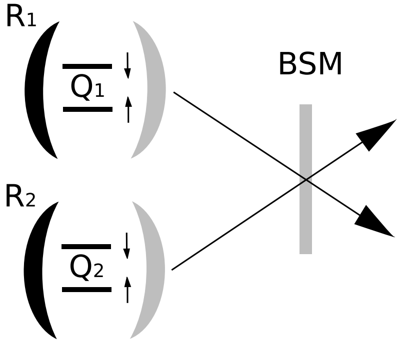
$$\begin{aligned} |\psi(t)\rangle_1 &= e^{-i\omega_{R_1} t} \left(\cos(g_1 t) |\downarrow 0\rangle - i \sin(g_1 t) |\uparrow 1\rangle \right), \\ |\psi(t)\rangle_2 &= e^{-i\omega_{R_2} t} \left(-i \sin(g_2 t) |\downarrow 0\rangle + \cos(g_2 t) |\uparrow 1\rangle \right) \end{aligned}$$

The $(QR)_i$ state $|\Psi(t)\rangle_i$ is entangled for $g_i t \neq m\frac{\pi}{2}$, m integer

$$|\psi(t)\rangle = |\psi(t)\rangle_1 \otimes |\psi(t)\rangle_2$$

Bell measurement

We perform the Bell state measurement (BSM) on photons leaving the cavity



$$|\tilde{\psi}(t')\rangle = |\psi^-\rangle_{RR}\langle\psi^-|\psi(t')\rangle$$

$$|\psi^-\rangle_R = 1/\sqrt{2}\,(|01\rangle - |10\rangle)$$

After the BSM

$$\begin{aligned}|\Psi(t')\rangle &= |\Psi^-\rangle_R \otimes |QQ(t')\rangle \\ |QQ(t')\rangle &= e^{-i(\omega_{R_1}+\omega_{R_2})t'}[\cos(g_1 t')\cos(g_2 t')|\downarrow\uparrow\rangle \\ &\quad + \sin(g_1 t')\sin(g_2 t')|\uparrow\downarrow\rangle]\end{aligned}$$

and the density matrix $\rho_{QQ}(t') = \frac{|QQ\rangle\langle QQ|}{Tr(|QQ\rangle\langle QQ|)}$

Entanglement measures

- Linear entropy

$$S_L = 1 - Tr\left[(\rho_1)^2\right],$$

$$\rho_1 = Tr_2\left(\rho_{1,2}\right)$$

$S_L = 0$ for disentangled , 0.5 for maximally entangled pure states

- Negativity

$$N = (0, -\sum_i \lambda_i)$$

where λ_i are negative eigenvalues in respect of partial transposition.
$N = 0$ for disentangled , 0.5 for maximally entangled states .
Both for pure and mixed states.

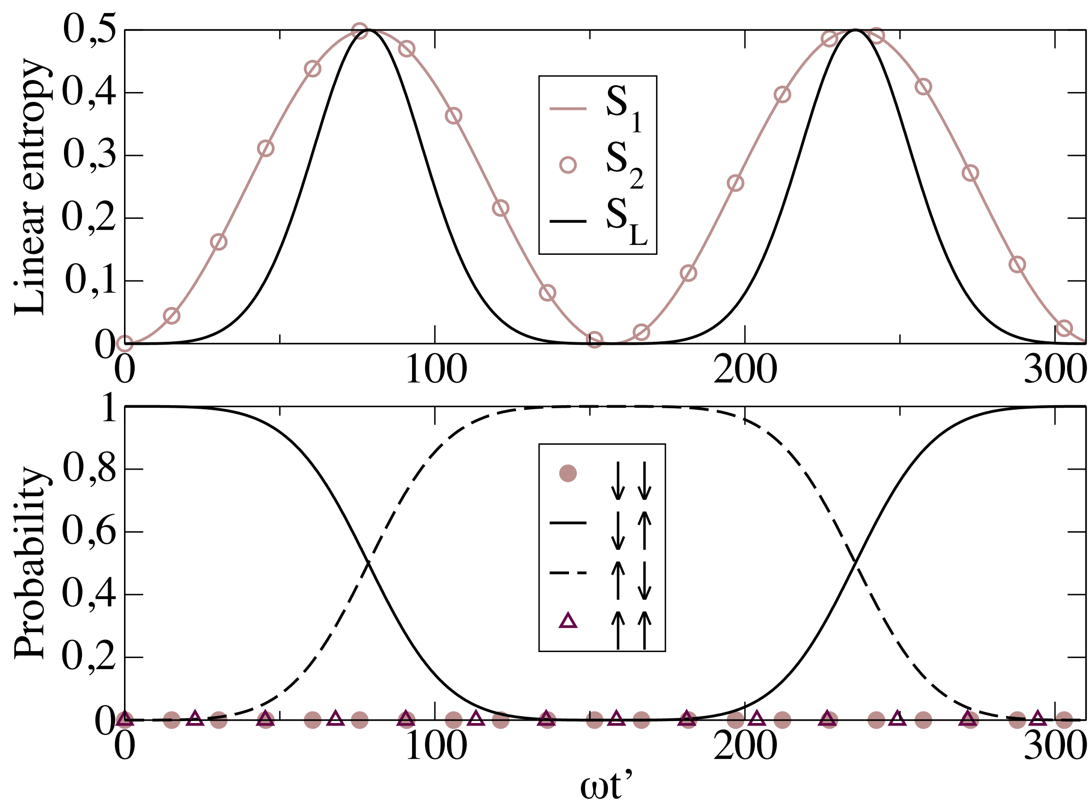
Results A $|\Psi(0)\rangle = |\downarrow 0\rangle \otimes |\uparrow 1\rangle$

$$S_L = 1 - \frac{\cos^4(g_1 t')\cos^4(g_2 t) + \sin^4(g_1 t')\sin^4(g_2 t')}{\left(Tr\left[\rho_{QQ}\right]\right)^2},$$

$$Tr\left[\rho_{QQ}\right] = \cos^2(g_1 t')\cos^2(g_2 t') + \sin^2(g_1 t')\sin^2(g_2 t')$$

$$g_1 = g_2 = 0.01\omega_R$$

$$S_L = 2P_{\downarrow\uparrow}P_{\uparrow\downarrow}$$



Kurpas et al. EPJ D **50**, *201 (2008)*

Results B $|\Psi(0)\rangle = |\downarrow 0\rangle \otimes |\downarrow 0\rangle$

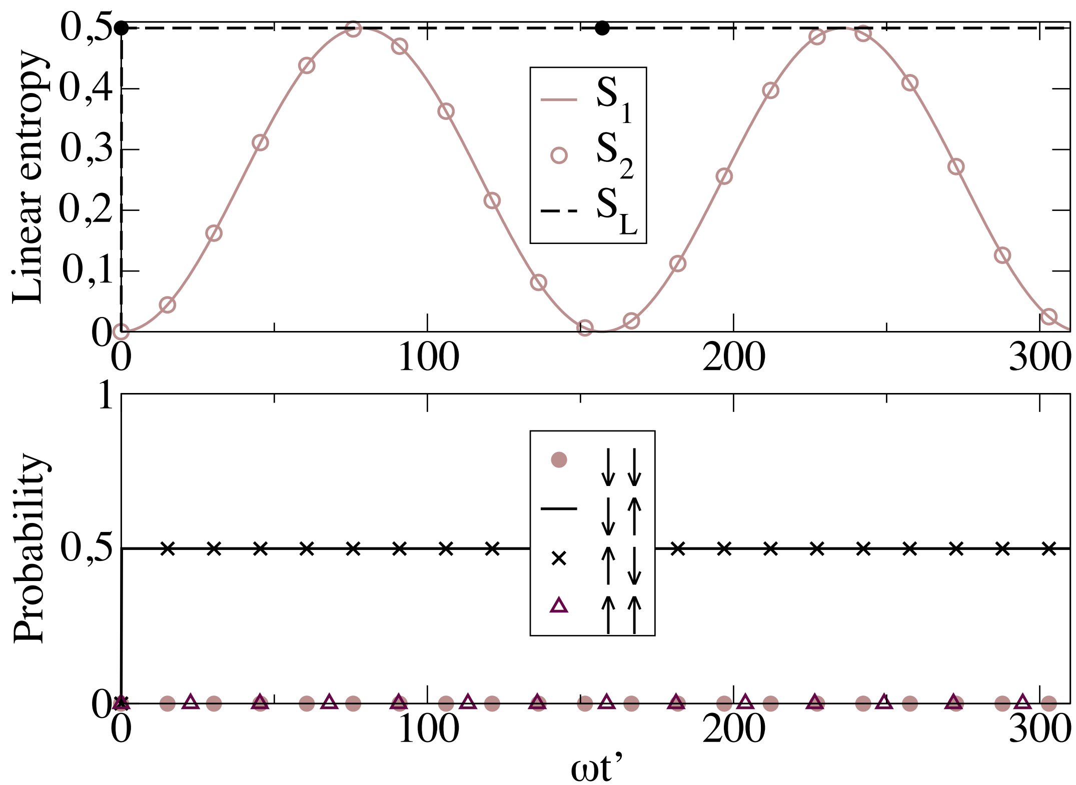
$$S_L = 1 - \frac{\sin^4(g_1 t')\cos^4(g_2 t)' + \cos^4(g_1 t')\sin^4(g_2 t')}{\left(Tr\left[\rho_{QQ}\right]\right)^2}$$

$$Tr\left[\rho_{QQ}\right] = \sin^2(g_1 t')\cos^2(g_2 t') + \cos^2(g_1 t')\sin^2(g_2 t')$$

$$g_1 = g_2 = g$$

$$g = 0.01\omega_R,\ 2gt' \neq \pi/2$$

$$S_L = \frac{1}{2}\left(\frac{\sin(2gt')}{\sin(2gt')}\right)^4 = \frac{1}{2}$$



General results

$$|\Psi^{\pm}\rangle = 1/\sqrt{2}\,(|01\rangle \pm |10\rangle)$$
$$|\Phi^{\pm}\rangle = 1/\sqrt{2}\,(|00\rangle \pm |11\rangle)$$

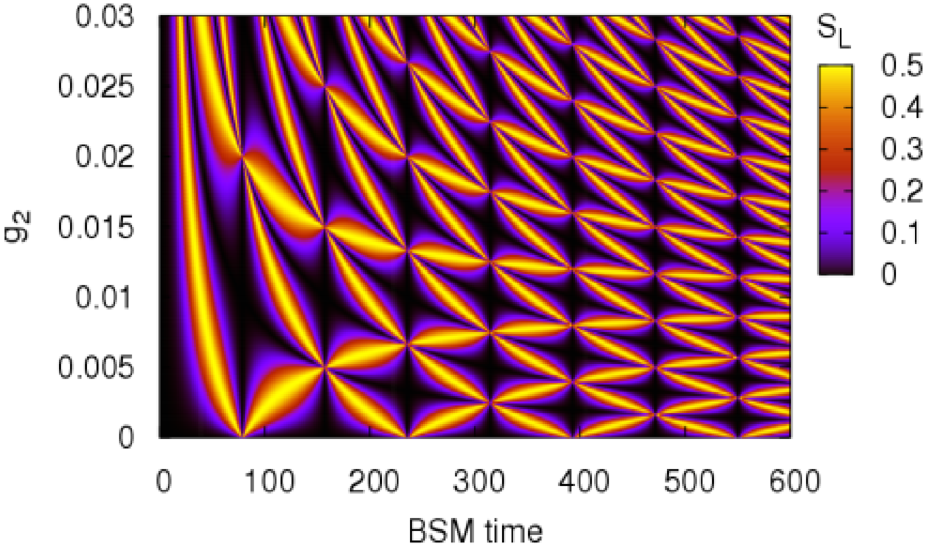
$\lvert\Psi(0)\rangle$	$S_L^{\Psi\pm}$	$S_L^{\Phi\pm}$
$\lvert\downarrow 0\rangle_1 \otimes \lvert\uparrow 1\rangle_2$ $\lvert\uparrow 1\rangle_1 \otimes \lvert\downarrow 0\rangle_2$	$2\left(\frac{\alpha\beta}{\zeta}\right)^2$	$2\left(\frac{\gamma\delta}{\eta}\right)^2$
$\lvert\downarrow 0\rangle_1 \otimes \lvert\downarrow 0\rangle_2$ $\lvert\uparrow 1\rangle_1 \otimes \lvert\uparrow 1\rangle_2$	$2\left(\frac{\gamma\delta}{\eta}\right)^2$	$2\left(\frac{\alpha\beta}{\zeta}\right)^2$

$$\alpha = \cos(g_1 t)\cos(g_2 t)\,, \quad \beta = \sin(g_1 t')\sin(g_2 t')\,,$$
$$\gamma = \cos(g_1 t')\sin(g_2 t')\,, \quad \delta = \sin(g_1 t')\cos(g_2 t')\,,$$
$$\zeta = \cos^2(g_1 t')\cos^2(g_2 t') + \sin^2(g_1 t')\sin^2(g_2 t')\,,$$
$$\eta = \sin^2(g_1 t')\cos^2(g_2 t') + \cos^2(g_1 t')\sin^2(g_2 t')\,,$$

General results

$$S_L = 2\left(\frac{\alpha\beta}{\zeta}\right)^2, \ |\Psi(0)\rangle = |\downarrow 0\rangle|\uparrow 1\rangle$$

A



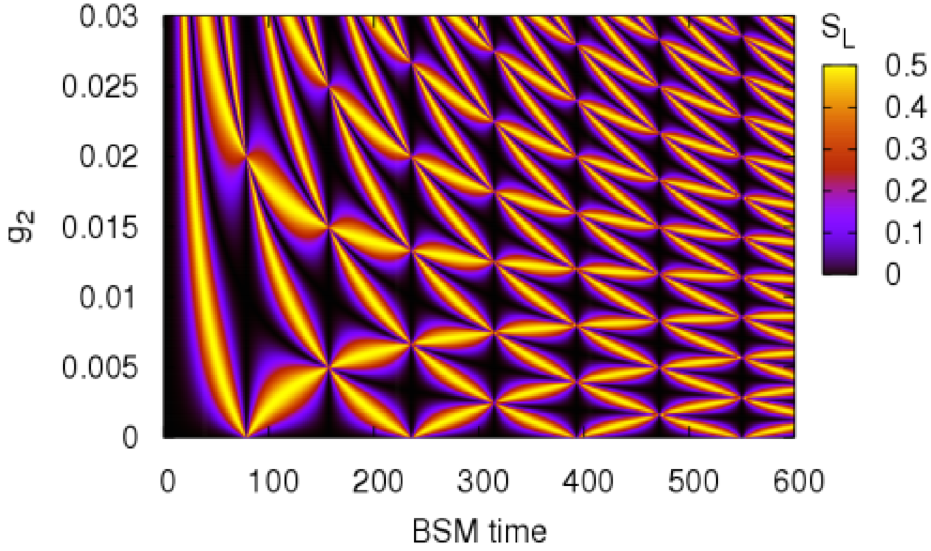
$$g_1 = 0.01\omega_R$$

Kurpas et al. *EPJ D* **50**, *201 (2008)*

General results

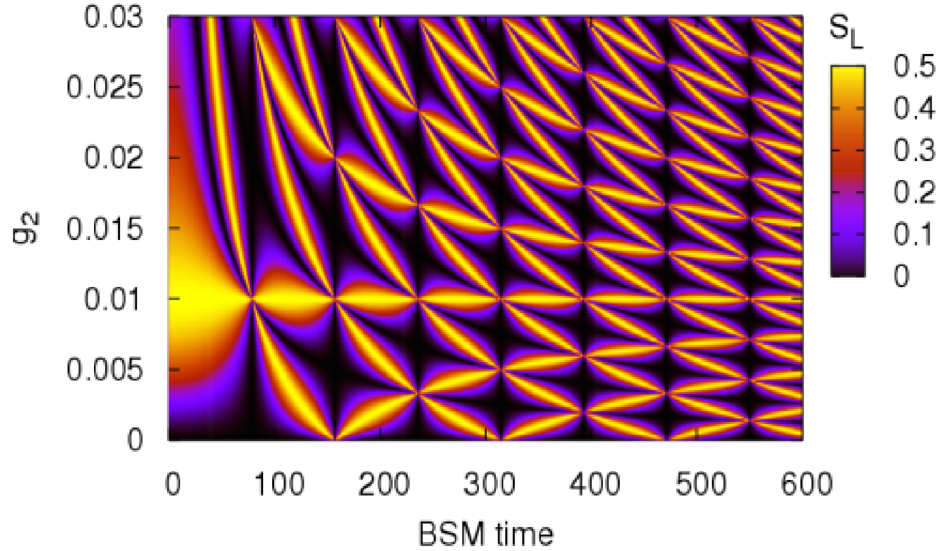
$$S_L = 2\left(\frac{\alpha\beta}{\zeta}\right)^2, \; |\Psi(0)\rangle = |\downarrow 0\rangle|\uparrow 1\rangle \qquad S_L = 2\left(\frac{\gamma\delta}{\eta}\right)^2, \; |\Psi(0)\rangle = |\downarrow 0\rangle|\downarrow 0\rangle$$

A



$g_1 = 0.01\omega_R$

B



$g_1 = 0.01\omega_R$

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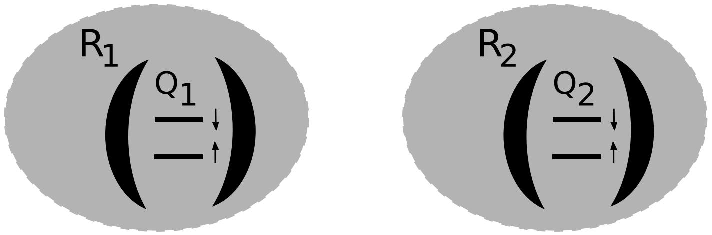
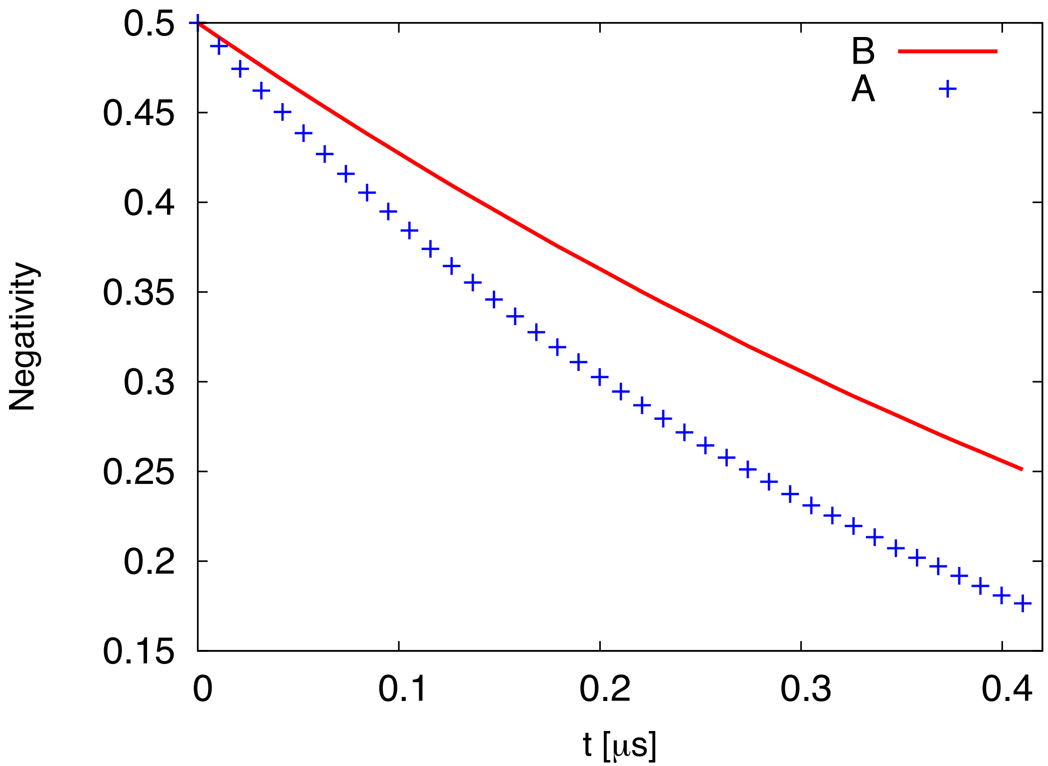
Decoherence

$$\dot{\rho}(t) = -i/\hbar[H_{sys}, \rho(t)] - \frac{1}{2}\sum_{k=\{\kappa,\gamma\}} \left(L_k^\dagger L_k \rho(t) + \rho(t) L_k^\dagger L_k - 2 L_k \rho(t) L_k^\dagger\right)$$

$$L_\kappa = \sqrt{\kappa}\sigma^-,\ L_\gamma = \sqrt{\gamma} a,\ T_Q \sim 1\mu s,\ T_R \sim 0.3\mu s$$





Conclusions

- The ability to entangle distant qubits that never interacted before
- The result of swapping is sensitive to the initial states and projectors, some configurations are favorable
- Possibility of entangling qubits in large registers