Rotonlike instability and pattern formation in spinor Bose-Einstein condensates

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Toruń, 2 September 2010

Outline

- Spinor condensates
- Roton excitations and instability
- Emergence of regular patterns in BECs
- Related experimental results

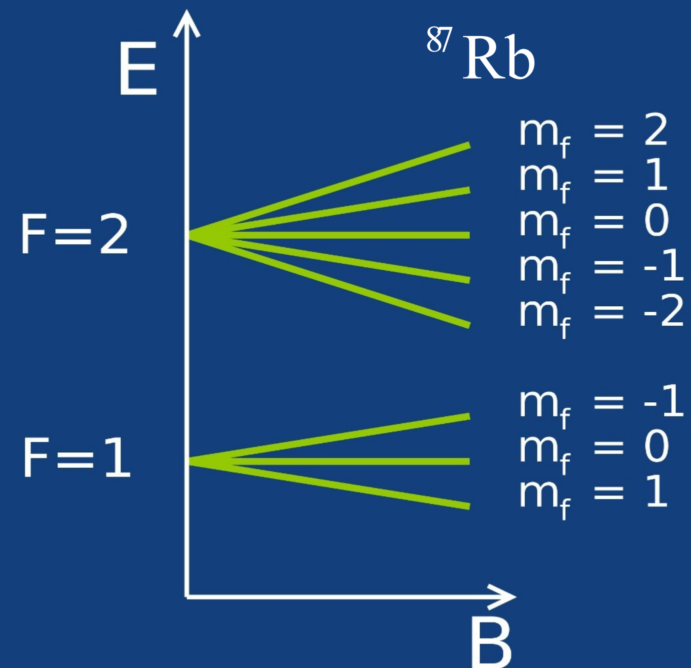
Spinor condensates

- Atoms are characterised by the electron spin J and the nuclear spin I. Their interaction is responsible for the appearance of the hyperfine structure
- The quantum number is the total spin $F = J + I$
- The projection of the spin F determines the magnetic quantum number $m_f = -F..F$. The magnetic field lifts the degeneracy of states with different m_f (Zeeman effect)

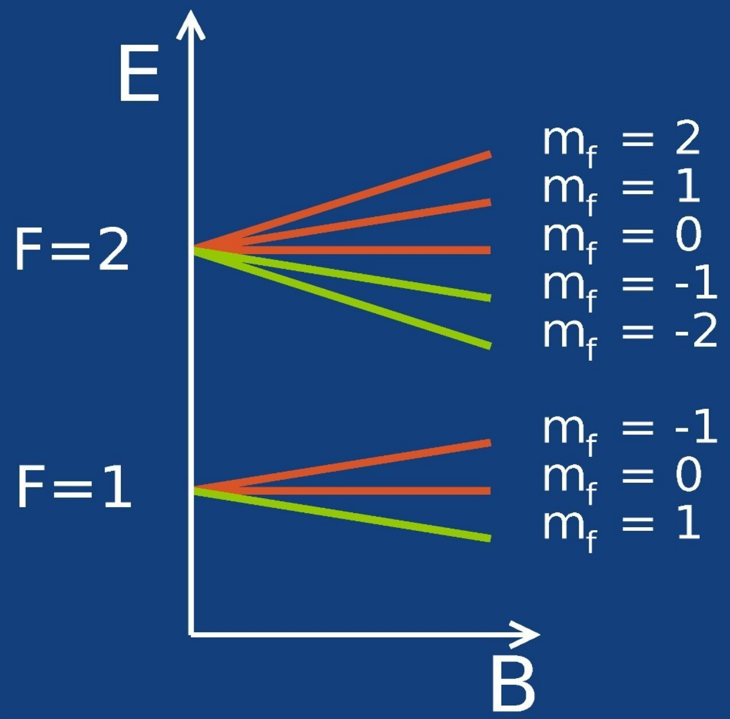
Alkali metals: $J = 1/2$

^{32}Na, ^{87}Rb	$I = 3/2$	$F = 1, 2$
^{85}Rb	$I = 5/2$	$F = 2, 3$
^{133}Cs	$I = 7/2$	$F = 3, 4$

^{52}Cr $\quad J = 3 \quad I = 0 \quad F = 3$

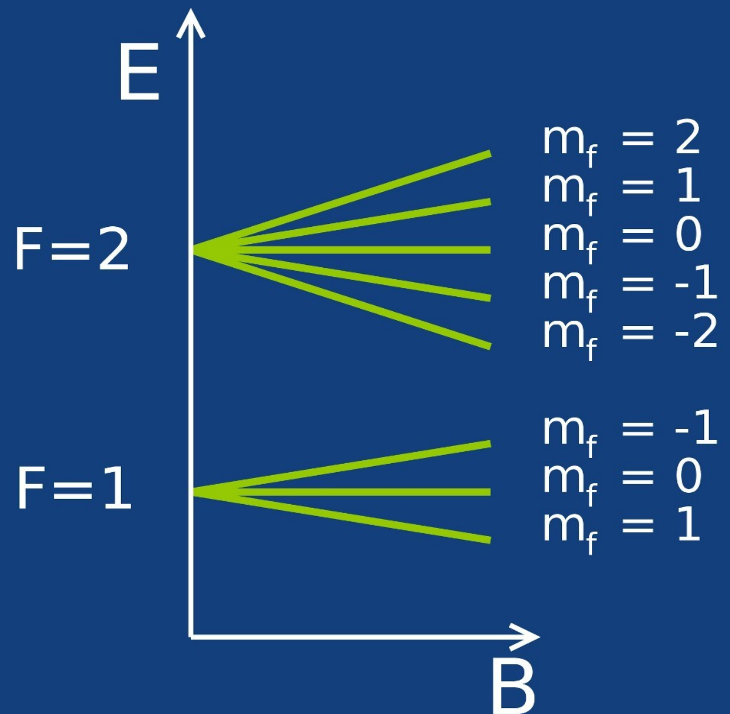


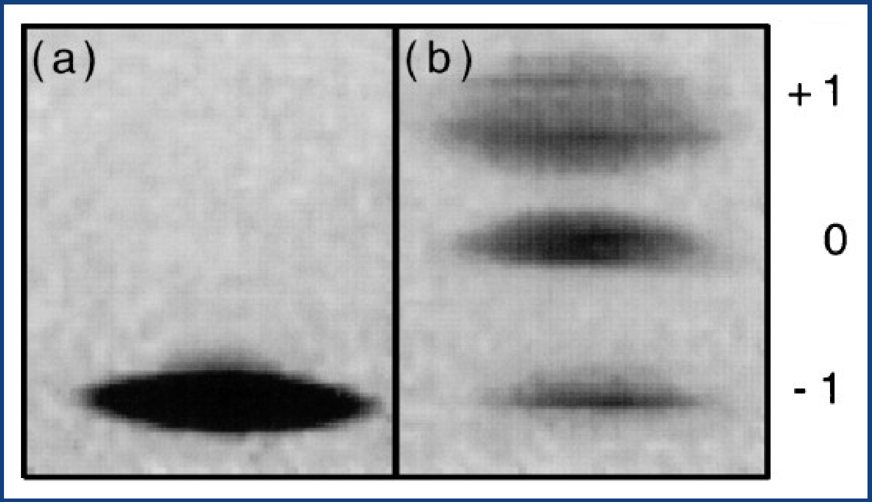
- Magnetic fields can be used for trapping of atoms in the lowest levels only



- Magnetic fields can be used for trapping of atoms in the lowest levels only
- In optical dipole traps, it is possible to trap atoms with arbitrary magnetic quantum number
- The condensate is described by a spinor wavefunction

$$\psi(\mathbf{r}) = \begin{pmatrix} \psi_+(\mathbf{r}) \\ \psi_0(\mathbf{r}) \\ \psi_-(\mathbf{r}) \end{pmatrix}$$



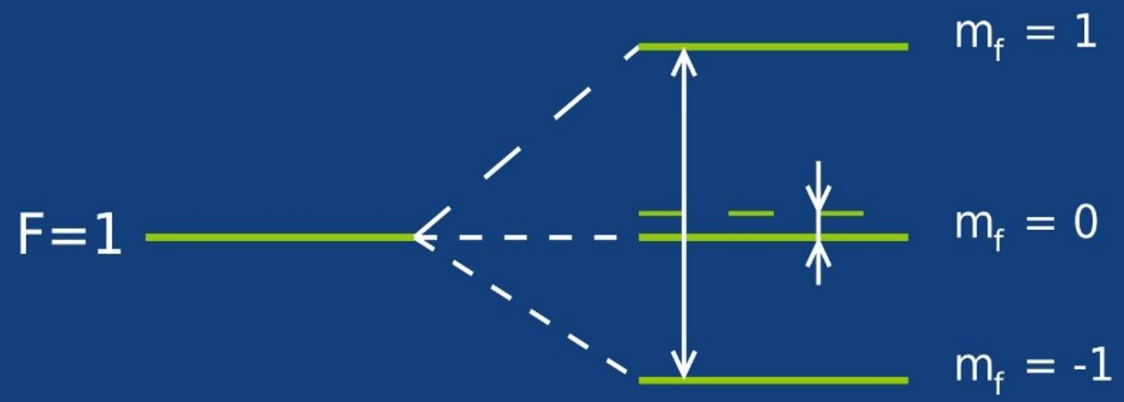


Stern-Gerlach experiment
D. Stamper-Kurn et al., PRL **80**, 2027 (1998)

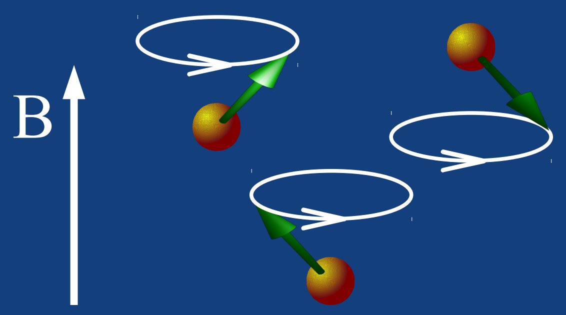
Motivation

- Spin squeezing and entanglement – possible applications in quantum information, quantum measurements, quantum simulators
- Magnetic phenomena
- Spin domains and vortices, skyrmions, monopoles
- Quantum phase transitions

Magnetic field – the Zeeman effect



$$E_{Zeeman} \approx E_0 + m_f \Delta E + m_f^2 \delta E + \ldots$$



Larmor precession

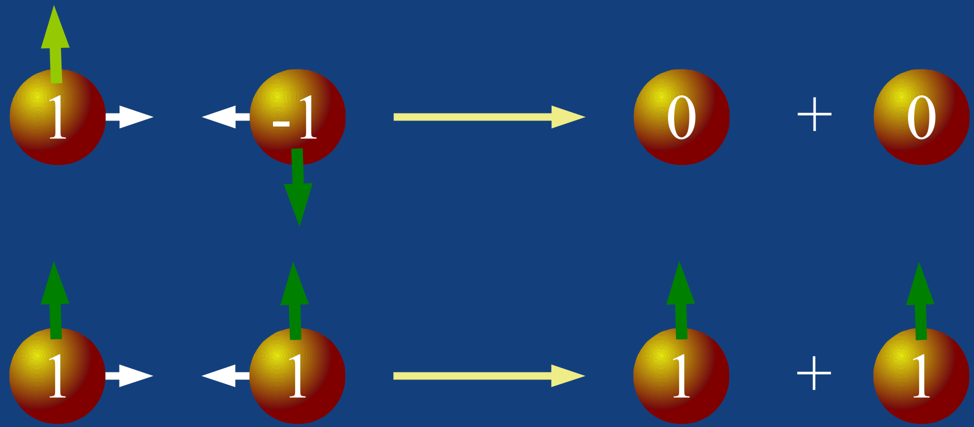
- Linear Zeeman effect: ΔE
  The precession frequency is constant, rotation of the reference frame

- Quadratic Zeeman effect: δE
  The precession frequency depends on the atomic state, nontrivial effect

Atomic interactions

- Contact interactions

F = 1



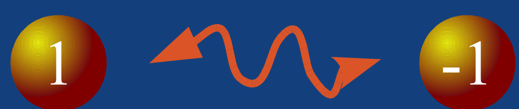
Scattering lengths a_0, a_2

Scattering length a_2

$$E_{\text{interaction}} = \frac{1}{2}\int d\mathbf{r}\, c_0 n + c_2 \mathbf{F}^2 \qquad c_2 \sim a_2 - a_0$$

F - spin density

- Dipole interactions – long range



Much weaker than contact interactions, but detectable

Ferromagnetic and antiferromagnetic condensates (F=1)

$$E_{interaction} = \frac{1}{2}\int d\vec{r}\, c_0 n + c_2 |\vec{F}|^2$$

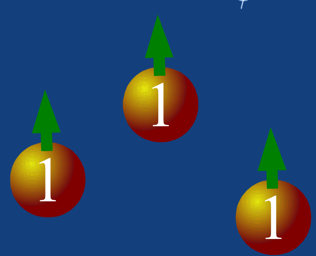
Spin-independent interaction

Small correction

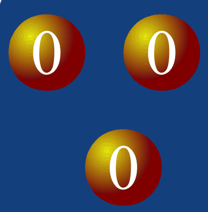
- In the ground state, the density profile n(r) is similar as in the scalar condensate
- Internal spin state depends on the coefficient c_2

B = 0

$$\psi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

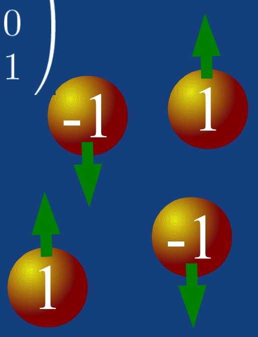


$$\psi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



or

$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$



Ferromagnetic condensate $c_2 < 0$

Antiferromagnetic condensate $c_2 > 0$

Theoretical model (F = 1)

- Spinor order parameter

$$\psi(\mathbf{r}) = \begin{pmatrix} \psi_+(\mathbf{r}) \\ \psi_0(\mathbf{r}) \\ \psi_-(\mathbf{r}) \end{pmatrix}$$

- The Hamiltonian can be split into symmetric (spin-independent) and asymmetric part

$$\hat{H}_0 = \sum_{j=-,0,+} \int d\mathbf{r}\, \hat{\psi}_j^\dagger \Big(-\frac{\hbar^2}{2m}\nabla^2 + \frac{c_0}{2}\hat{n} + V(\mathbf{r}) \Big) \hat{\psi}_j,$$

$$\hat{H}_A = \int d\mathbf{r} \Big(\sum_j E_j \hat{n}_j + \frac{c_2}{2} : \hat{\mathbf{F}}^2 : \Big),$$

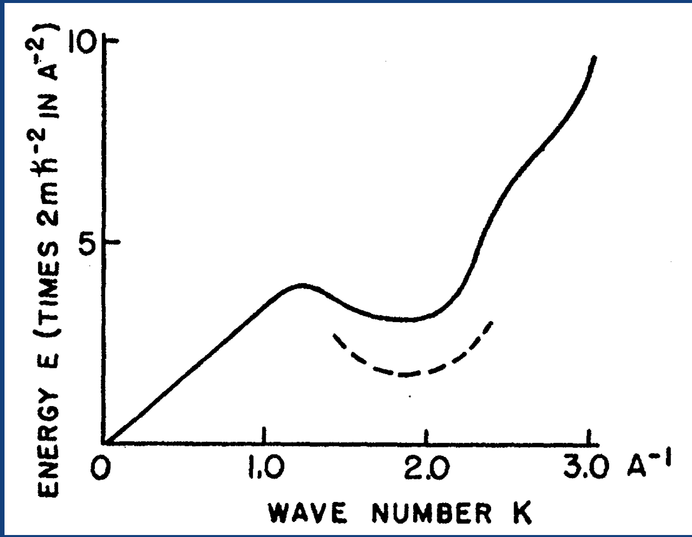
$$n_j = |\psi_j|^2$$

$F_{x,y,z} = \psi^\dagger \hat{F}_{x,y,z} \psi$ Spin-1 matrices

E_j Zeeman energy levels

Roton excitations

- Characterized by a minimum of the excitation energy at a finite wavelength
- Introduced by Landau in the context of superfluid helium, crucial for the understanding of its physical properties
- Present also in quantum Hall systems, strongly correlated fermions
- Predicted for condensates with long-range interactions (dipolar, Rydberg BECs)



R.P. Feynman Phys. Rev. **94**, 262 (1954)

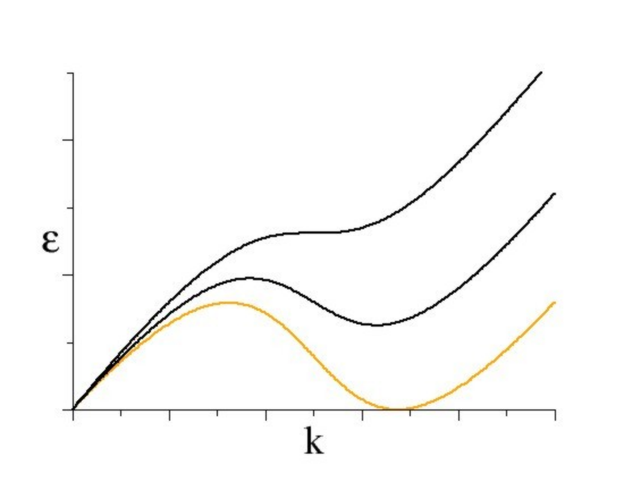
L. Santos et. al., PRL 2003
D.H.J. Dell et al., PRL 2003

Roton instability

- If the roton gap can be decreased by changing the system parameters, the softening of the roton mode can eventually lead to an instability

- It is characterized by unstable modes with wave vector lengths close to the roton minimum

- It was suggested that it can lead to the transition to the supersolid state, in which the superfluid and crystalline orders coexist



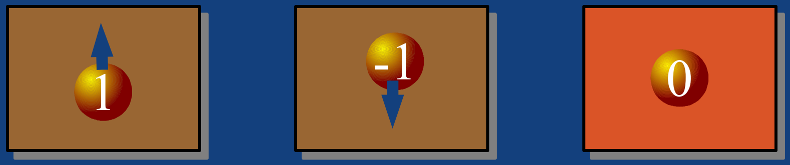
What are the possible phases of a spin-1 condensate?

States that can exist independently – homogeneous stationary states

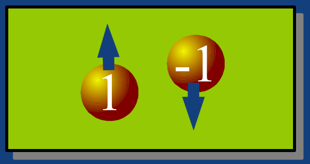
$V(\mathbf{r}) = 0 \qquad \psi_j(\vec{r}, t) = \sqrt{n_j}\, e^{-i\mu_j t + i\theta_j} \qquad j = m_f \qquad \theta = \theta_{+1} + \theta_{-1} - 2\theta_0$

1) No magnetic field – ferromagnetic ($\vec{F} = 1$) and polar ($\vec{F} = 0$) phases (Ho, 1998)
2) In the presence of a magnetic field, the classification is different

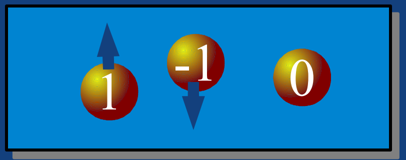
- One component, magnetized or nematic



- Two component state (2C)

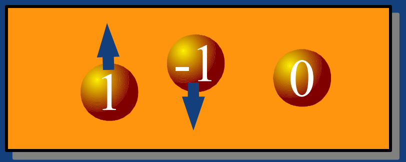


- Phase-matched state (PM)
  broken transverse symmetry



$\theta = 0$

- Anti-phase-matched state (APM)
  broken transverse symmetry



$\theta = \pi$

M. Matuszewski et al., Phys. Rev. A (2008)

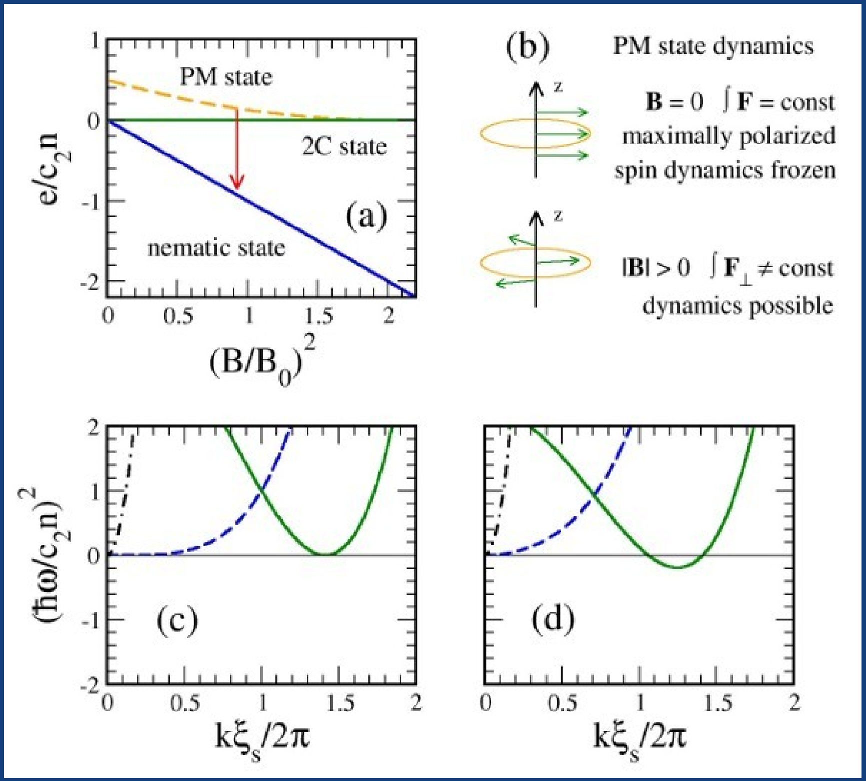
$\mathbf{F}_\perp^2 = 2\left|\psi_+\psi_0^* + \psi_0\psi_-^*\right|^2$

The instability of antiferromagnetic condensates

- The PM phase is an excited state at low magnetic fields. Despite this, it is stable at B = 0 due to spin conservation, which ensures that the condensate remains fully polarized

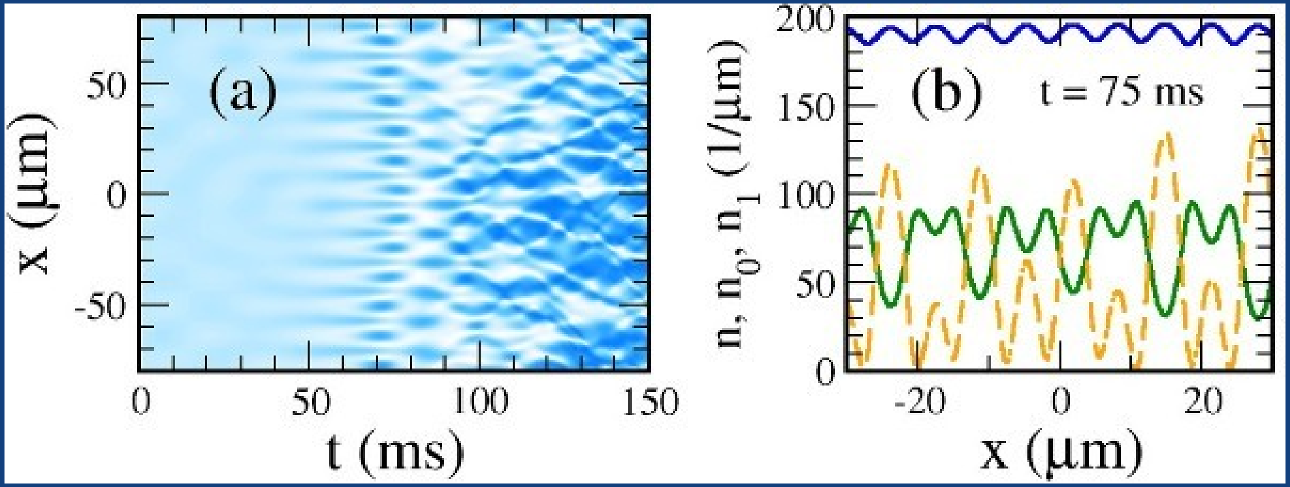
- After introducing the magnetic field, the perpendicular part of the spin is no longer conserved and the spin quadrupole Bogolubov modes become unstable

- The minimum corresponds to rotons at k > 0.



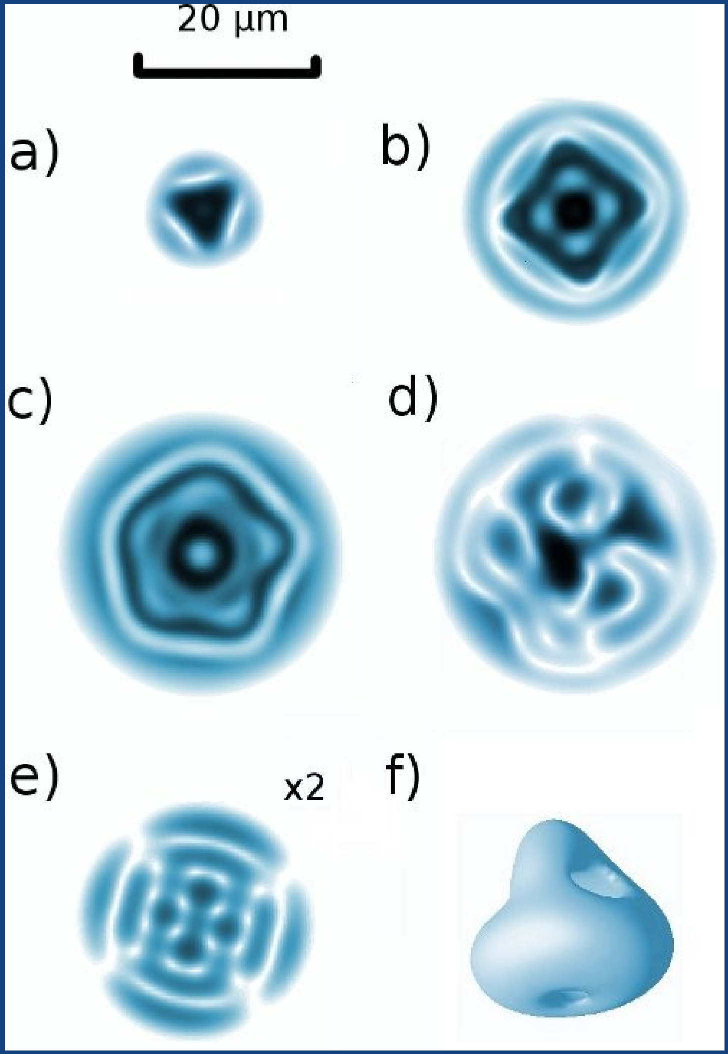
Pattern formation

- Dynamics within the truncated Wigner approximation
- The instability leads to spontaneous appearance of transient periodic patterns
- The spatial period and the structure of patterns correspond to the unstable quadrupole modes



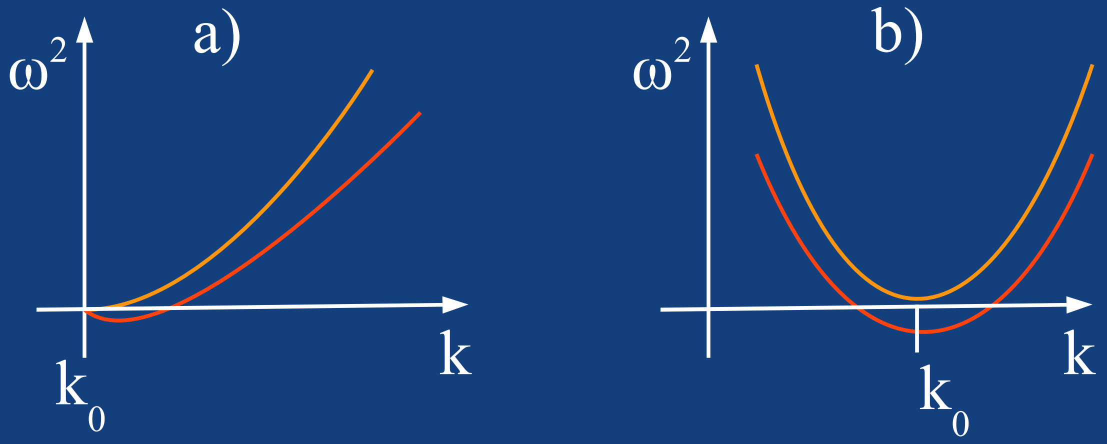
^{23}Na, no trapping potential, periodic (ring) boundary conditions

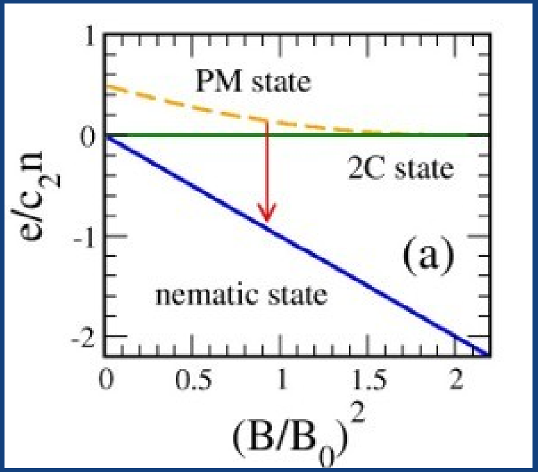
- Patterns that can be observed in trapped systems in two and three dimensions include polygonal, polyhedral and crystal-like structures



- Spontaneous symmetry breaking
- The patterns emerge from the roton-like instability only
- The characteristic length scale is the spin healing length, which is related to the wavelength of roton modes

Why does the rotonlike instability appear?

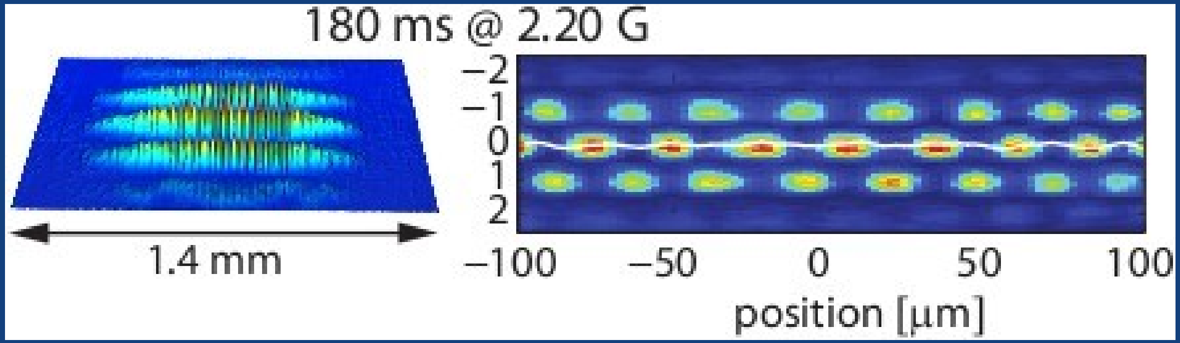




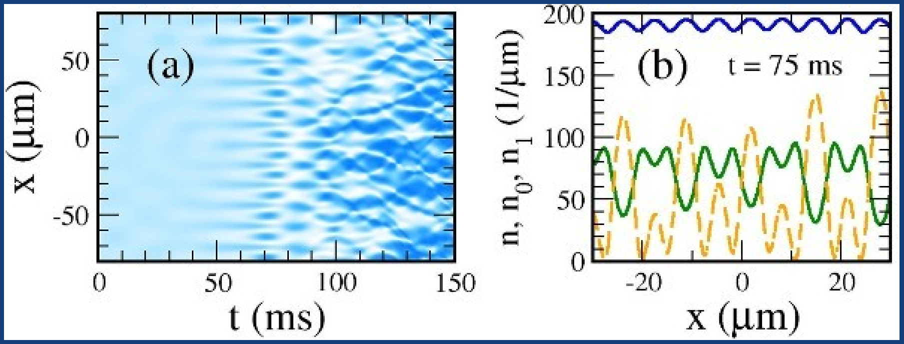
- Regular patterns emerge when the spectrum of unstable modes appears in a narrow range of wavevectors close to a finite value k_0, as in b)
- This kind of spectrum is associated with the crossing of a roton minimum into the unstable domain
- For finite k_0, energy conservation requires an energy gap even at the critical point (B = 0)
- Despite the gap, the relaxation of the excited PM state at B = 0 is prevented by the spin conservation

Experimental results

- Recent experiments in the antiferromagnetic F=2 ^{87}Rb condensate demonstrated the emergence of similar periodic patterns in the 1D case
- The structure of the patterns corresponds to quadrupole modes
- At relatively strong magnetic fields, the F=2 condensate may behave like an F=1 BEC due to the quadratic Zeeman effect



J. Kronjäger et al., arxiv:0904.2339 to appear in PRL



Conclusions

- The excitation spectrum of an antiferromagnetic condensate in a simple model with pure contact interactions can exhibit a rotonlike minimum
- Under the influence of a magnetic field, this minimum gives rise to an instability and can lead to spontaneous emergence of regular patterns
- Theoretical considerations suggest that the appearance of rotonlike instability is related to spin conservation, which inhibits the relaxation of excess energy at low magnetic fields

Phys. Rev. Lett. **105**, 020405 (2010)

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