Dipolar resonances in oscillating magnetic fields

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Motivation

- condensation of ^{52}Cr atoms (magnetic moment $6\mu_B$)
  Phys. Rev. Lett. **94**, 160401 (2005)

  alkali atoms (magnetic moment $1/2\mu_B$)
- Einstein-de Haas (EdH) effect
  theory:
  Phys. Rev. Lett. **96**, 080495 (2006)
  Phys. Rev. Lett. **96**, 190404 (2006)
  Phys. Rev. Lett. **99**, 130401 (2007)
  experiment?

Eqation of motion

the eq. of motion for a spinor condensate in an $F = 1$ hyperfine state in the mean-field approximation

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix} = (\mathcal{H}_0 + \mathcal{H}_c + \mathcal{H}_d)\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix}$$

$$\mathcal{H}_0 = \begin{pmatrix} H_0 - \gamma\hbar B & 0 & 0 \\ 0 & H_0 & \\ 0 & 0 & H_0 + \gamma\hbar B \end{pmatrix}, \quad H_0 = E_{kin} + E_{trap}$$

$$\mathcal{H}_c = \begin{pmatrix} \mathcal{H}_{c11} & \mathcal{H}_{c10} & 0 \\ \mathcal{H}^{\star}_{c10} & \mathcal{H}_{c00} & \mathcal{H}_{c0-1} \\ 0 & \mathcal{H}^{\star}_{c0-1} & \mathcal{H}_{c-1-1} \end{pmatrix}$$

$$\mathcal{H}_d = \begin{pmatrix} \mathcal{H}_{d11} & \mathcal{H}_{d10} & 0 \\ \mathcal{H}^{\star}_{d10} & 0 & \mathcal{H}_{d10} \\ 0 & \mathcal{H}^{\star}_{d10} & -\mathcal{H}_{d11} \end{pmatrix}$$

$\psi = (\psi_1, \psi_0, \psi_{-1})^T$ is a condensate spinor wavefunction

$\mathcal{H}_c$ elements

$$\mathcal{H}_{c11} = (c_0 + c_2)|\psi_1|^2 + (c_0 + c_2)|\psi_0|^2 + (c_0 - c_2)|\psi_{-1}|^2$$

$$\mathcal{H}_{c00} = (c_0 + c_2)|\psi_1|^2 + c_0|\psi_0|^2 + (c_0 + c_2)|\psi_{-1}|^2$$

$$\mathcal{H}_{c-1-1} = (c_0 - c_2)|\psi_1|^2 + (c_0 + c_2)|\psi_0|^2 + (c_0 + c_2)|\psi_{-1}|^2$$

$$\mathcal{H}_{c10} = c_2\psi^\star_{-1}\psi_0$$

$$\mathcal{H}_{c0-1} = c_2\psi^\star_0\psi_1$$

$$\mathcal{H}_{c1-1} = 0$$

$$c_0 = 4\pi\hbar^2(a_0 + 2a_2)/3m \quad c_2 = 4\pi\hbar^2(a_2 - a_0)/3m$$

$a_0(a_2)$ is the scattering length of coliding atoms for the channel of total spin equal to $F = 0$ ($F = 2$)

$\mathcal{H}_d$ elements

$$\mathcal{H}_{d10} = -3\frac{\hbar^2\gamma^2}{\sqrt{2}}\int d^3r'\,\frac{[(x-x')-i(y-y')](z-z')}{|\mathbf{r}-\mathbf{r}'|^5}\times\left(|\psi_1|^2-|\psi_{-1}|^2\right)$$
$$-\frac{3}{2}\hbar^2\gamma^2\int d^3r'\,\frac{[(x-x')-i(y-y')]^2}{|\mathbf{r}-\mathbf{r}'|^5}\times\left(\psi_1^\star\psi_0+\psi_0^\star\psi_{-1}\right)$$
$$+\hbar^2\gamma^2\int d^3r'\left[\frac{1}{|\mathbf{r}-\mathbf{r}'|^3}-\frac{3}{2}\frac{(x-x')^2+(y-y')^2}{|\mathbf{r}-\mathbf{r}'|^5}\right]\times\left(\psi_0^\star\psi_1+\psi_{-1}^\star\psi_0\right)$$

$$\mathcal{H}_{d11} = \hbar^2\gamma^2\int d^3r'\left[\frac{1}{|\mathbf{r}-\mathbf{r}'|^3}-3\frac{(z-z')^2}{|\mathbf{r}-\mathbf{r}'|^5}\right]\times\left(|\psi_1|^2-|\psi_{-1}|^2\right)$$
$$-3\frac{\hbar^2\gamma^2}{\sqrt{2}}\int d^3r'\,\frac{z-z'}{|\mathbf{r}-\mathbf{r}'|^5}[(x-x')-(y-y')]\times\left(\psi_1^\star\psi_0+\psi_0^\star\psi_{-1}\right)$$
$$-3\frac{\hbar^2\gamma^2}{\sqrt{2}}\int d^3r'\,\frac{z-z'}{|\mathbf{r}-\mathbf{r}'|^5}[(x-x')+(y-y')]\times\left(\psi_0^\star\psi_1+\psi_{-1}^\star\psi_0\right)$$

note

$\mathcal{H}_{d10}$ and $\mathcal{H}_{d11}$ initially dependend on $m_F = 1$ state density

Properties of interactions

contact V_c

$$V_c = \frac{1}{3}(V_0 + 2V_2) + \frac{1}{3}(V_2 - V_0)\mathbf{F}_1\mathbf{F}_2$$

$$V_0 = \frac{4\pi\hbar^2 a_0}{m}\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$V_2 = \frac{4\pi\hbar^2 a_2}{m}\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$[V_c, F_{1z} + F_{2z}] = 0$$

$$[V_c, L_{1z} + L_{2z}] = 0$$

dipolar V_d

$$V_d = \frac{\boldsymbol{\mu}_1\boldsymbol{\mu}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

$$-3\frac{[\boldsymbol{\mu}_1(\mathbf{r}_1 - \mathbf{r}_2)][\boldsymbol{\mu}_2(\mathbf{r}_1 - \mathbf{r}_2)]}{|\mathbf{r}_1 - \mathbf{r}_2|^5}$$

$\boldsymbol{\mu} = \gamma\mathbf{F}$, γ gyromagnetic coef.

$$[V_d, F_{1z} + F_{2z}] \neq 0$$

$$[V_d, L_{1z} + L_{2z}] \neq 0$$

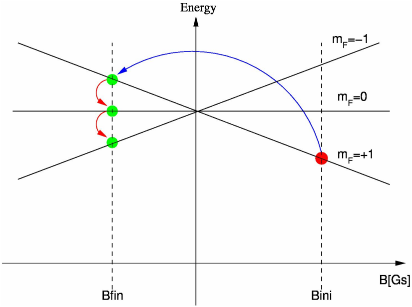
$$[V_d, F_{1z} + F_{2z} + L_{1z} + L_{2z}] = 0$$

Einstein-de Haas (EdH) effect

$$[V_d, F_{1z} + F_{2z} + L_{1z} + L_{2z}] = 0$$

$$\mathbf{B} = (0, 0, B_z), \quad B_z = const$$

scheme of the experiment



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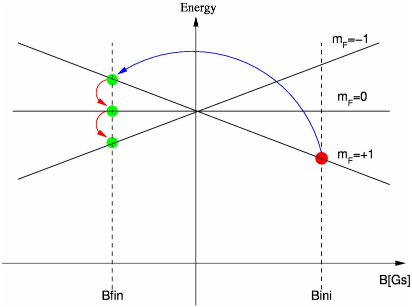
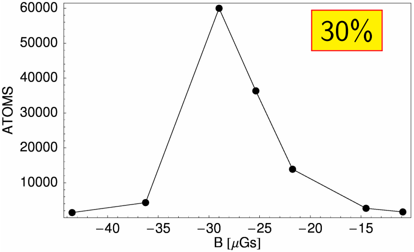


transfer to the $m_F = 0$ state



$N_+ = 200\ 000 \quad {}^{87}\mathrm{Rb}$

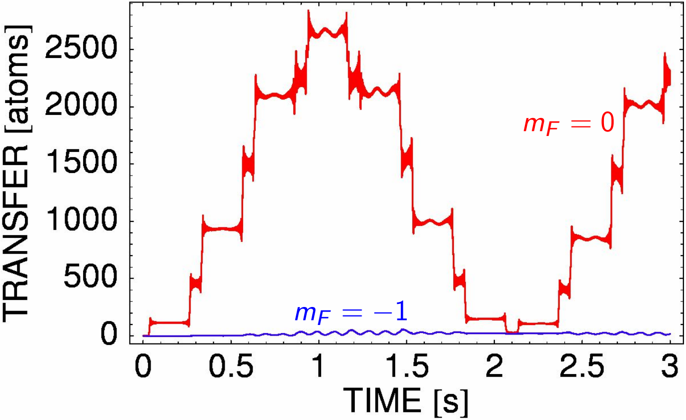
an optical trap $\omega = 2\pi \times 100$ Hz

resonanse condition: $\mu B_z = E_{kin}/N_0$

K. Gawryluk, M. Brewczyk, K. Bongs, and M. Gajda Phys. Rev. Lett. **99**, 130401 (2007)

EdH in oscillating magnetic fields

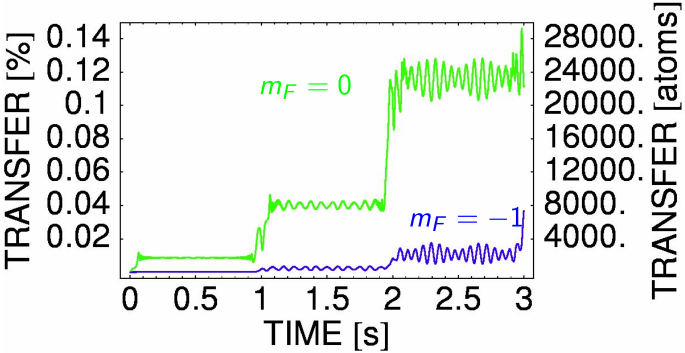
$$\mathbf{B} = (0, 0, B_z), \quad B_z = B_0 + A\cos(\omega_B t)$$



initially $N = 200\,000$ ^{87}Rb atoms in $m_F = +1$ state
in an optical trap $\omega_{trap} = 2\pi \times 100$ Hz
$B_0 = -1.42$ mG, $A = 1.85$ mG, $\omega_B = 1/30\ \omega_{trap}$

EdH in oscillating magnetic fields

$$\mathbf{B} = (0, 0, B_z), \quad B_z = B_0 + A\cos(\omega_B t)$$



initially $N = 200\,000$ ^{87}Rb atoms in $m_F = +1$ state
in an optical trap $\omega_{trap} = 2\pi \times 100$ Hz
$B_0 = -0.92$ mG, $A = 0.93$ mG, $\omega_B = 1/100\ \omega_{trap}$

Two level model

$$i\frac{\partial}{\partial t}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} \epsilon(t) & \Delta \\ \Delta & -\epsilon(t) \end{pmatrix}\begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

$$\epsilon(t) = \epsilon_0 + A\cos(\omega t)$$

S. Ashhab, J. R. Johansson, A. M. Zagorskin, and F. Nori, Phys. Rev. A **75**, 063414 (2007)

Two level model

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(in the interaction picture)

$$H'(t) = \frac{\Delta}{2}\begin{pmatrix} 0 & H_{10} \\ H_{10}^{\star} & 0 \end{pmatrix}, \quad H_{10} = \sum_{n=-\infty}^{\infty} J_n\left(\frac{A}{\omega}\right) e^{-i(n\omega+\epsilon_0)t}$$

$J_n(x)$ Bessel function of the first kind

Two level model

$$i\frac{\partial}{\partial t}\left(\begin{array}{c} a_+ \\ a_- \end{array}\right) = -\frac{1}{2}\left(\begin{array}{cc} \epsilon(t) & \Delta \\ \Delta & -\epsilon(t) \end{array}\right)\left(\begin{array}{c} a_+ \\ a_- \end{array}\right)$$

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$J_n(x)$ Bessel function of the first kind

$\boxed{\text{RWA: } n\omega = \epsilon_0}$

$$|a_+|^2 \propto \sin^2(\Omega t), \quad \Omega = \Delta |J_n(\tfrac{A}{\omega})|$$

Two level model

$$i\frac{\partial}{\partial t}\left(\begin{array}{c} a_+ \\ a_- \end{array}\right) = -\frac{1}{2}\left(\begin{array}{cc} \epsilon(t) & \Delta \\ \Delta & -\epsilon(t) \end{array}\right)\left(\begin{array}{c} a_+ \\ a_- \end{array}\right)$$

$$\epsilon(t) = \epsilon_0 + A\cos(\omega t)$$

$$RWA: \quad n\omega = \epsilon_0$$

assumption: ψ_{-1} is ruled out, then

$$i\hbar\frac{\partial}{\partial t}\left(\begin{array}{c} \psi_1 \\ \psi_0 \end{array}\right) = -\frac{1}{2}\left(\begin{array}{cc} \mu\tilde{B}(\mathbf{r},t) & 2H_{10} \\ 2H_{10}^{\star} & \mu\tilde{B}(\mathbf{r},t) \end{array}\right)\left(\begin{array}{c} \psi_1 \\ \psi_0 \end{array}\right)$$

$$\tilde{B}(\mathbf{r},t) = B_0 - \delta B(\mathbf{r},t) + A\cos(\omega_B t)$$

$$\delta B(\mathbf{r},t) = (c_2|\psi_0|^2 + \mathcal{H}_{d11})/\mu$$

$$H_{10} = \mathcal{H}_{d10}$$

Two level model - reduction

dynamics of a system

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix} = (\mathcal{H}_0 + \mathcal{H}_c + \mathcal{H}_d)\begin{pmatrix}\psi_1\\ \psi_0\\ \psi_{-1}\end{pmatrix}$$

is reduced to a set of coupled two-level systems

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}\mu\tilde{B}(\mathbf{r},t) & 2H_{10}\\ 2H_{10}^{\star} & \mu\tilde{B}(\mathbf{r},t)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}$$

where H_{10} and $\tilde{B}$ are time and space dependent

Two-level system

RWA

assumptions (not realistic):

$$H_{10} = const$$

$$\delta B(\mathbf{r}, t) = 0$$

$\boxed{\text{RWA: } n\hbar\omega_B = \mu B_0}$

$$|\psi_0|^2 \propto \sin^2(\Omega t)$$

$$\hbar\Omega = |H_{10}||J_n\left(\frac{\mu A}{\hbar\omega}\right)|$$

Two-level system

RWA

assumptions (not realistic):

$$H_{10} = const$$

$$\delta B(\mathbf{r}, t) = 0$$

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$$|\psi_0|^2 \propto \sin^2(\Omega t)$$

$$\hbar\Omega = |H_{10}||J_n\left(\frac{\mu A}{\hbar\omega}\right)|$$

Off-resonance behavior

$$B_0 \rightarrow B_0 + \delta B_0$$

then the least oscillating term in the off-diagonal element is given by

$$H_{10} J_n\left(\frac{\mu A}{\hbar\omega}\right) \exp\left(-i\mu\delta B_0 t/\hbar\right)$$

analytical solution:

$$\psi_0(t) = -i\frac{C}{\Omega} e^{i\tilde{\omega}t/2} \sin\Omega t$$

$$C = H_{10} J_n\left(\frac{\mu A}{\hbar\omega}\right)/\hbar, \quad \tilde{\omega} = \mu\delta B_0/\hbar$$

$$\Omega = \sqrt{|C|^2 + \left(\frac{\tilde{\omega}}{2}\right)^2}$$

Off-resonance behavior

maximal transfer under off-resonance condition

$$|\psi_0(t)|^2_{max} = \frac{1}{1 + \left(\frac{\mu \delta B_0}{2|H_{10}|J_n(\mu A/\hbar\omega_B)}\right)^2}$$

maximal transfer increases when:

- H_{10} increases
- ω_B decreases and $B_0 = const$ ($nJ_n(n) \to \infty$ when $n \to \infty$)

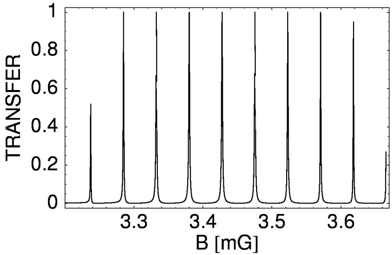
Strong-driving limit: $A \approx B_0$

$$J_n(\mu A/\hbar\omega_B) \approx J_n(\mu B_0/\hbar\omega_B) = J_n(n\hbar\omega_B/\hbar\omega_B) = J_n(n)$$

Two-level system

$$B_z = B_0 + A\cos(\omega_B t)$$

$\omega_B = 1/3\ \omega_{trap}$



Two-level system

$$B_z = B_0 + A\cos(\omega_B t)$$

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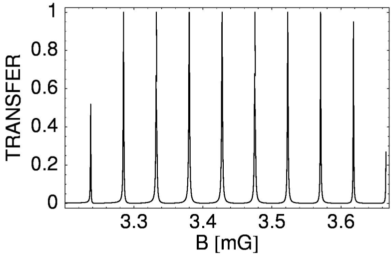
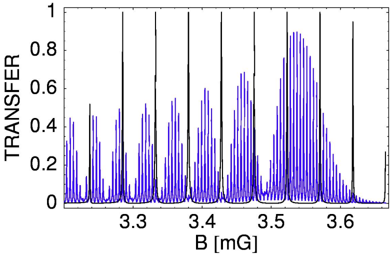


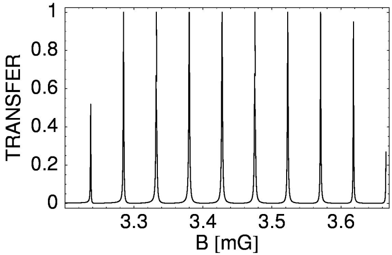
$\omega_B = 1/30\ \omega_{trap}$



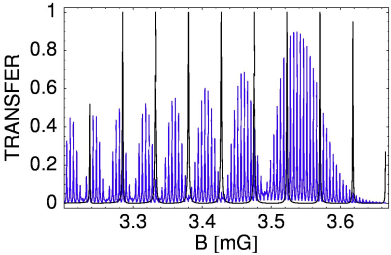
Two-level system

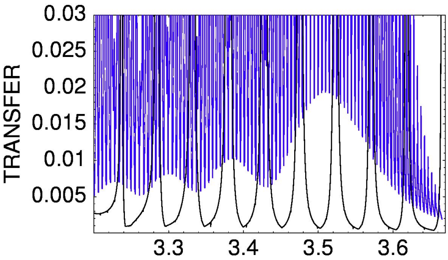
$$B_z = B_0 + A\cos(\omega_B t)$$

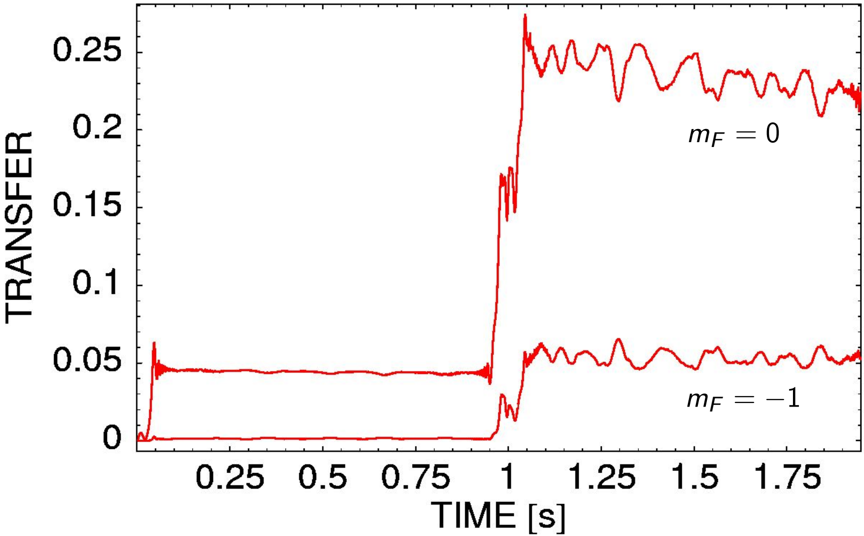
$\omega_B = 1/3\ \omega_{trap}$



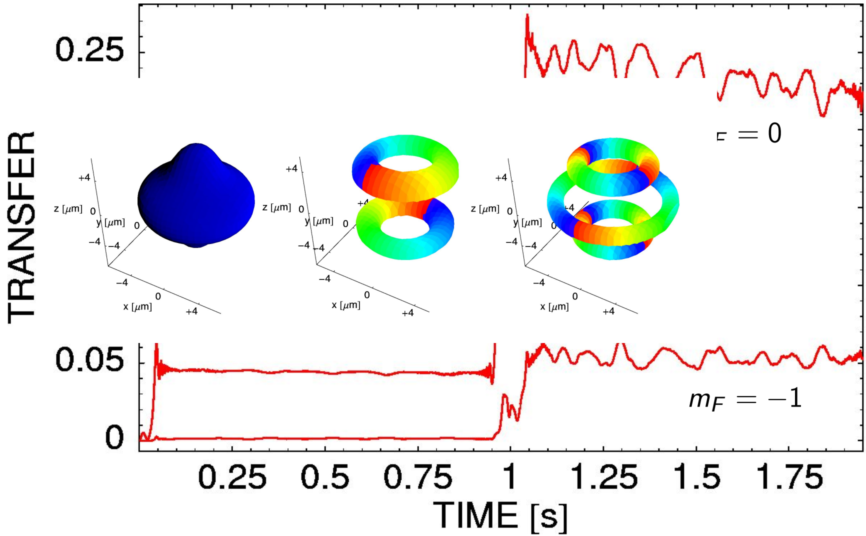
$\omega_B = 1/30\ \omega_{trap}$







100×10^6 ^{23}Na atoms, $B_0 \approx A = 3.5$ mG, $\omega_B = 1/100\omega_{trap}$

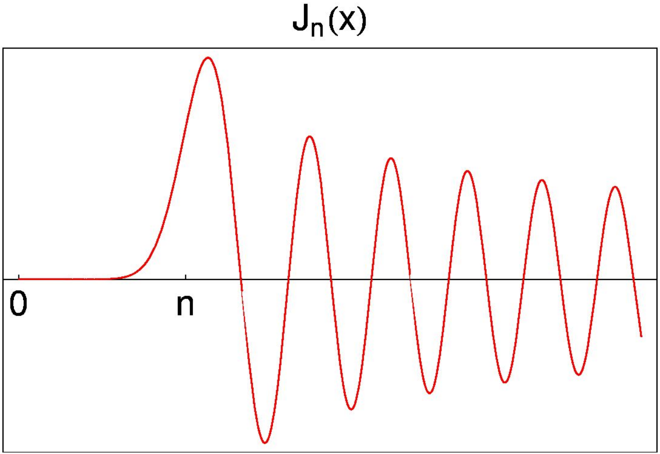


100×10^6 ^{23}Na atoms, $B_0 \approx A = 3.5$ mG, $\omega_B = 1/100\omega_{trap}$

Summary

- dynamics of an $F = 1$ spinor condensate can be reduced to a set of coupled two-level systems
- two-level system is indeed helpful while looking for dipolar resonances
- the resonances induced by the oscillating magnetic fields occur at larger magnetic fields (mG regime) as compared to resonances at constant magnetic fields
- the resonances are much wider (up to $0.5mG$) than at constant fields

Bessel function of the first kind



Reduction to a set of two-level systems

$$\begin{pmatrix} H_{11} & H_{10} \\ H_{10}^{*} & H_{00} \end{pmatrix} = \begin{pmatrix} H_0^{'} & 0 \\ 0 & H_0^{'} \end{pmatrix} + \begin{pmatrix} -\tilde{H}_{11} & H_{10} \\ H_{10}^{*} & \tilde{H}_{11} \end{pmatrix}$$

$$H_0^{'} = H_{00} - \frac{1}{2}\mu\tilde{B}(\vec{r},t)$$

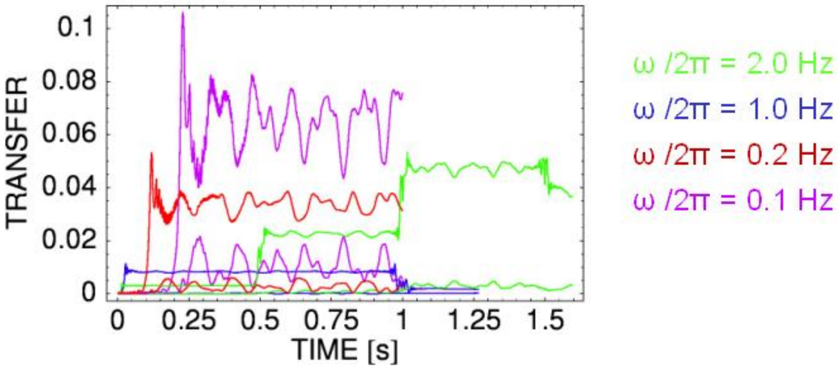
$$\tilde{H}_{11} = \frac{1}{2}\mu\tilde{B}(\vec{r},t)$$

$$\begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} = e^{-i/\hbar\int_0^t H_0^{'}(t)\,dt\,I} \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_0 \end{pmatrix}$$

Transfer of ^{87}Rb atoms to m_F=0,-1 states

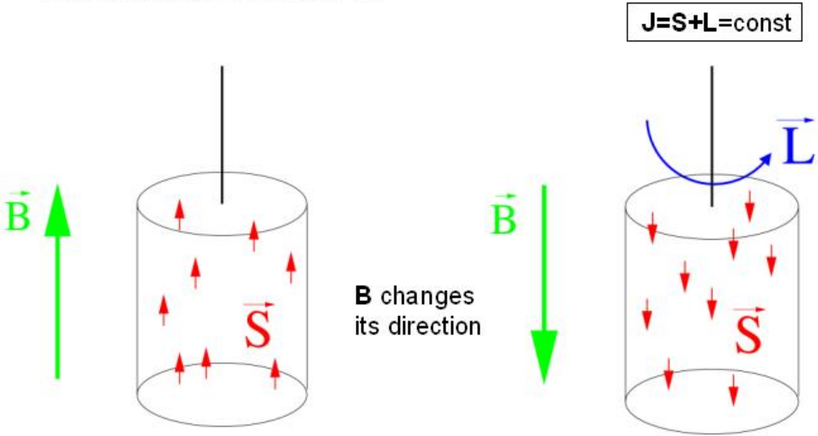
$N_{+1}(t=0)=2\times10^6$, $\rho_{max}= 7.5\times10^{14}\ cm^{-3}$

B_0=-18.4 mG , A=18.5 mG



Einstein-de Haas effect

Einstein A. und de Haas W. J., *„Experimenteller Nachweis der Ampereschen Molekularstrome"* Deutsche Physikalische Gesellschaft Verhandlungen, **17** (1915) 152 - 170



Transfer of ^{23}Na atoms to m_F=0,-1 states

B_0=-14 mG , A=14.3 mG , $\omega/2\pi$=3.3 Hz

B_0=+14 mG , A=14.3 mG , $\omega/2\pi$=3.3 Hz