The superfliud fountain effect in a Bose-Einstein condensate

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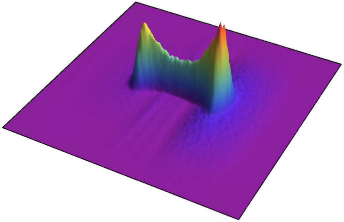
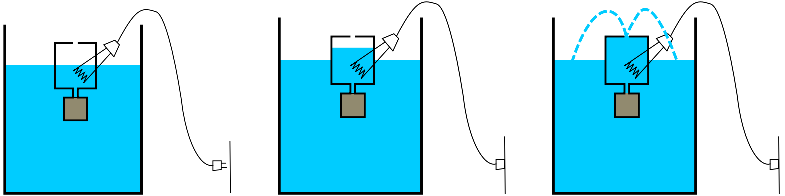
Outline

1. Superfluid fountain effect
2. Classical Field Approximation
3. Fountain in BEC



Helium fountain

J.F. Allen and H. Jones, Nature (London) **141**, 243 (1938)



Normal component:

- $S \neq 0$ (heat transport)
- viscous

Superfluid component:

- $S = 0$ (no heat transport)
- no viscosity

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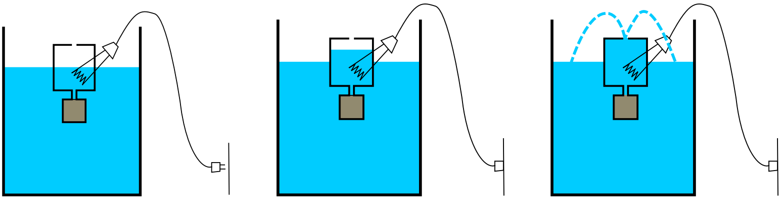
Helium fountain



- The system cannot achieve thermal equilibrium

$$T_1 \neq T_2$$

- The system can achieve mechanical equilibrium

$$\mu_1 = \mu_2$$

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System

- We theoretically investigate a possibility of an experimental implementation of the helium fountain effect in a Bose-Einstein condensate of alkali atoms
- We use the classical field approximation of the version described in J. Phys. B **40**, R1 (2007) and optimized for an arbitrary trapping potential in Phys. Rev. A **81**, 013629 (2010)
- We work with a cloud of Na atoms in the $|3S_{1/2}, F = 1, m_F = -1\rangle$ state confined in an harmonic trap with the frequencies $\omega_x = \omega_y = 2\pi \times 51$Hz and $\omega_z = 2\pi \times 25$Hz similarly like in Phys. Rev. Lett. **99**, 260401 (2007)

Classical Field Approximation

Hamiltonian in the second quantization framework:

$$\mathcal{H} = \int d^3r\, \hat{\Psi}^+(\vec{r},t)\mathcal{H}_0(\vec{r})\hat{\Psi}(\vec{r},t) + \frac{1}{2}\iint d^3r\, d^3r'\, \hat{\Psi}^+(\vec{r},t)\hat{\Psi}^+(\vec{r}',t)U(\vec{r}-\vec{r}')\,\hat{\Psi}(\vec{r}',t)\hat{\Psi}(\vec{r},t)$$

where

$$\mathcal{H}_0(\vec{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) \quad ; \quad U(\vec{r}-\vec{r}') = g\delta^3(\vec{r}-\vec{r}')$$

Commutation relations for bosons:

$$[\hat{\Psi}(\vec{r},t), \hat{\Psi}^+(\vec{r}',t)] = \delta^3(\vec{r}-\vec{r}')$$

$$[\hat{\Psi}(\vec{r},t), \hat{\Psi}(\vec{r}',t)] = [\hat{\Psi}^+(\vec{r},t), \hat{\Psi}^+(\vec{r}',t)] = 0$$

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Classical Field Approximation

The Heisenberg equation of motion for the bosonic field operator reads:

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi}(\vec{r},t) = \mathcal{H}_0(\vec{r})\hat{\Psi}(\vec{r},t) + g\hat{\Psi}^{+}(\vec{r},t)\hat{\Psi}(\vec{r},t)\hat{\Psi}(\vec{r},t)$$

We expand the field operator in the basis of one-particle wave functions:

$$\hat{\Psi}(\vec{r},t) = \sum_k \psi_k(\vec{r})\hat{a}_k(t)$$

We assume that some modes are macroscopically occupied and replace operators by c-numbers:

$$\hat{a}_k(t) \longrightarrow a_k(t)$$

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Classical Field Approximation

Taking into account only macroscopically occupied modes we approximate the field operator by the complex wave function

$$\hat{\Psi}(\vec{r},t) = \sum_k \psi_k(\vec{r})\hat{a}_k(t) \rightarrow \Psi(\vec{r},t) = \sum_k^{k_{max}} \psi_k(\vec{r})a_k(t)$$

The classical field obeys the following equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \mathcal{H}_0(\vec{r})\Psi(\vec{r},t) + g|\Psi(\vec{r},t)|^2\Psi(\vec{r},t)$$

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Classical Field Approximation

To split the classical field into the condensed and non-condensed fractions we use Penrose and Onsager idea. The condensate corresponds to the eigenvector of the one-particle density matrix with the dominant eigenvalue.

$$\rho^{(1)}(\vec{r},\vec{r}\,';t) = \frac{1}{N}\Psi^*(\vec{r},t)\Psi(\vec{r}\,',t)$$

The mixed state appears after the averaging procedure:

$$\bar{\rho} = \langle \rho^{(1)}(\vec{r},\vec{r}\,';t)\rangle_{T,R}$$

To obtain the averaged density matrix we integrate along some direction:

$$\bar{\rho}(x,y,x',y';t) = \int dz\, \Psi^*(x,y,z;t)\Psi(x',y',z;t)$$

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Classical Field Approximation

We solve the eigenvalue problem:

$$\bar{\rho}(x, y, x', y'; t) = \sum_k \frac{N_k}{N} \varphi_k^*(x, y, t) \varphi_k(x', y', t)$$

The functions corresponding to macroscopically occupied modes:

$$\psi_k(x, y, t) = \sqrt{\frac{N_k}{N}} \varphi_k(x, y, t)$$

The condensate wave function:

$$\Psi_0(x, y, t) = \sqrt{\frac{N_0}{N}} \varphi_0(x, y, t)$$

The non-condensate density:

$$\rho_T(x, y, t) = \bar{\rho}(x, y, t) - |\Psi_0(x, y, t)|^2$$

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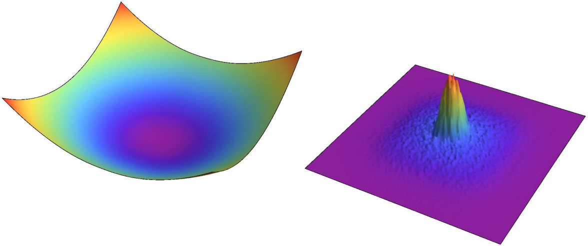
Our procedure

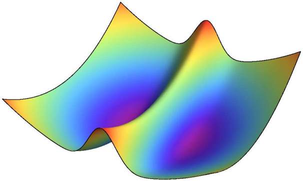
$$V(\vec{r}, t) = V_0(\vec{r}) = \frac{1}{2} m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

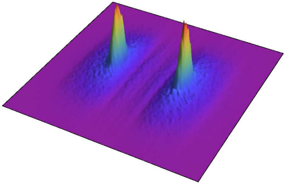


Our procedure

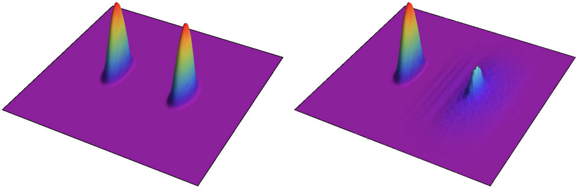
$$V(\vec{r},t) = V_0(\vec{r}) + V_1(\vec{r},t) = V_0(\vec{r}) + f_1(t)\; v_1 e^{-2x^2/w_1^2}$$

$$f_1(t) = \frac{t - t_1}{t_2 - t_1}.$$





Our procedure

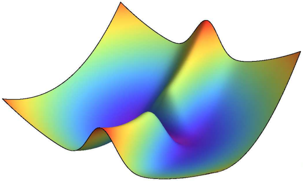


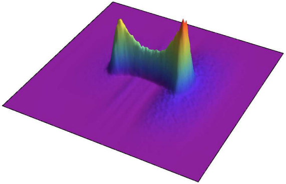
Our procedure

$$V(\vec{r},t) = V_0(\vec{r}) + V_1(\vec{r},t)\; V_2(\vec{r},t)$$

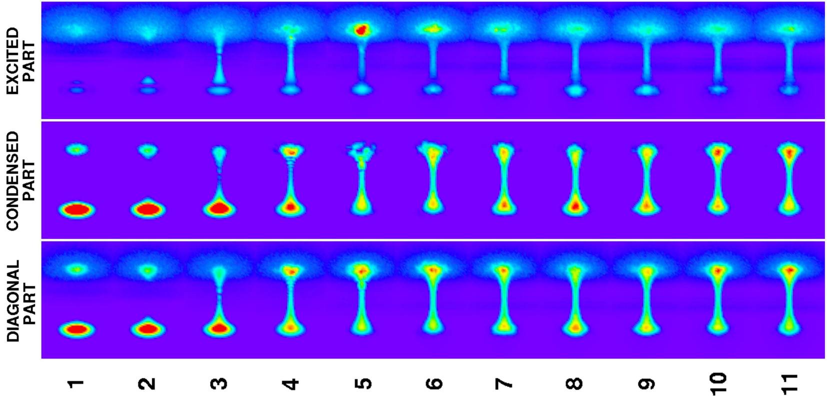
$$V_2(\vec{r},t) = 1 - f_2(t)\; e^{-(y^2+z^2)/w_2^2}$$

$$f_2(t) = \frac{t - t_3}{t_4 - t_3}.$$





Column density — condensate and non-condensate



Relative occupations — condensate and non-condensate

Relative numbers of atoms:

$$n^L(t) = \int_{-\infty}^{0} dx \int_{-\infty}^{\infty} dy \; \Psi^*(x,y,t)\Psi(x,y,t)$$
$$n^R(t) = \int_{0}^{\infty} dx \int_{-\infty}^{\infty} dy \; \Psi^*(x,y,t)\Psi(x,y,t)$$

Relative occupations of modes:

$$n_k(t) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \; \psi_k^*(x,y,t)\psi_k(x,y,t) = \frac{N_k}{N}$$
$$n_k^L(t) = \int_{-\infty}^{0} dx \int_{-\infty}^{\infty} dy \; \psi_k^*(x,y,t)\psi_k(x,y,t)$$
$$n_k^R(t) = \int_{0}^{\infty} dx \int_{-\infty}^{\infty} dy \; \psi_k^*(x,y,t)\psi_k(x,y,t)$$

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Relative occupations — condensate and non-condensate

Relative occupations of condensate:

$$n_0(t)$$
$$n_0^L(t)$$
$$n_0^R(t)$$

Relative occupations of non-condensate:

$$1 - n_0(t)$$
$$n_L(t) - n_0^L(t)$$
$$n_R(t) - n_0^R(t)$$

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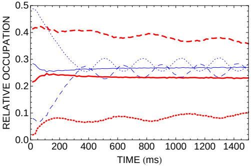
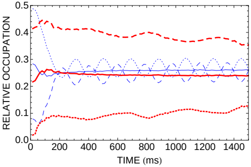
Relative occupations — condensate and non-condensate

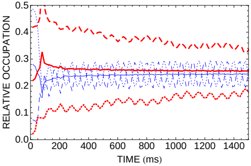
$W = FWHM$ of pipe,
$W = 4\mu m$



$W = 5\mu m$

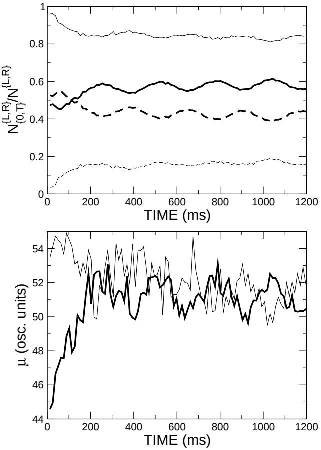


$W = 10\mu m$



- thin blue — condensate
- **thick red** — thermal
- solid — total/2
- d.o.t.t.e.d — left trap
- d_a_s_h_e_d — right trap

Chemical potential and condensate fraction



- Thin lines — left trap
- Thick lines — right trap
- Solid lines — condensate
- Dashed lines — thermal component

$$N_0^L/N^L \neq N_0^R/N^R$$

$$T_L \neq T_R$$

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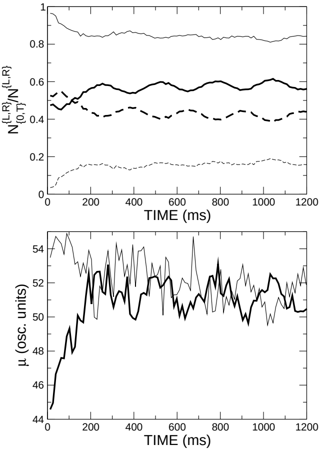
Chemical potential and condensate fraction



- Thin lines — left trap
- Thick lines — right trap

$$\mu(\mathbf{r}) = g\,\rho_0(\mathbf{r}) + 2\,g\,\rho_{th}(\mathbf{r}) + V_{tr}(\mathbf{r})$$

$$\mu_L \sim \mu_R$$

Summary

- We have shown that the thermomechanical effect may be also observed in present-day experiments with alkali atoms
- We have proposed a possible setup based on the harmonic trap widely available in many labs
- The examined system exhibit two main features present in the superfluid fountain i.e. the lack of thermal equilibrium and the presence of the mechanical equilibrium at once
- We have shown that the superfluid component in this system contains the major condensate part and a few minor excited modes
- The border line between the superfluid modes and the normal modes is drawn by the competition among the healing length of the particular mode and the pipe width
- The superfluid flow is at least one order of magnitude faster than the flow of the normal component.
- The slow flow of the normal component is the phase space effect

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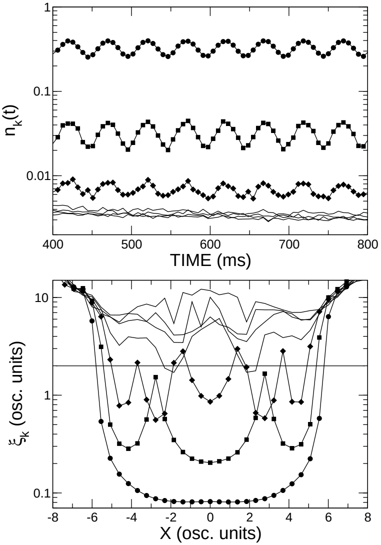
Relative occupation and healing length



- Circles, squares, diamonds — three highest occupied modes
- Lines without symbols — lower lying modes
- The healing length along the pipe direction

$$\xi_k(x,0,0) = 1/\sqrt{8\pi a \rho_k(x,0,0)}$$

- Superfluid condition

$$\xi_k(0,0,0) < \frac{W}{2}$$

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Relative occupations — superfluid and non-superfluid

Functions corresponding to macroscopically occupied modes:

$$\psi_k(x,y,t) = \sqrt{\frac{N_k}{N}}\,\varphi_k(x,y,t)$$

Relative occupation of superfluid:

$$n_S(t) = \sum_{k=0}^{k_S} n_k(t)$$

$$n_S^L(t) = \sum_{k=0}^{k_S} n_k^L(t); \quad n_S^R(t) = \sum_{k=0}^{k_S} n_k^R(t)$$

Relative occupation of non-superfluid:

$$n_N(t) = 1 - n_S(t)$$

$$n_N^L(t) = n^L(t) - n_S^L(t); \quad n_N^R(t) = n^R(t) - n_S^R(t)$$

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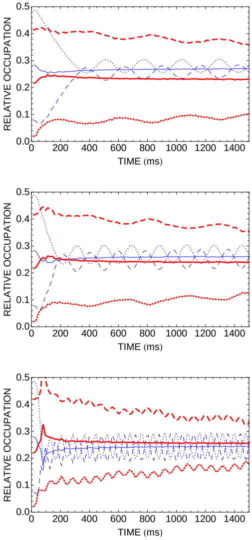
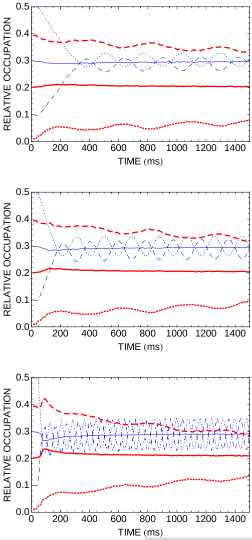
Relative occupations — superfluid and non-superfluid

BEC — thermal



superfluid — normal



- thin blue — condensate, superfluid
- **thick red** — thermal, normal
- solid — total
- d.o.t.t.e.d — left trap
- d_a_s_h_e_d — right trap

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Column density — superfluid and non-superfluid

Superfluid density:

$$\rho_S(x,y,t) = \sum_{k=0}^{k_S} |\psi_k(x,y,t)|^2$$

Non-superfluid density:

$$\rho_N(x,y,t) = \bar{\rho}(x,y,x,y;t) - \rho_S(x,y,t)$$

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Column density — superfluid and non-superfluid