Superluminal pulse propagation in multi-level optically dressed atomic system

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Table of contents

Propagation velocities

Phase velocity

$$v_f = \frac{\omega}{k}$$

Group velocity

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$$

Energy transfer velocity

Information transfer velocity

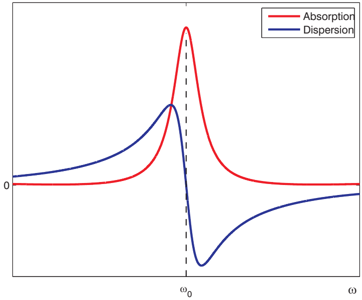
It can be proved that

$$v_g = \frac{c}{1 + \frac{\omega}{2}\frac{\mathrm{d}\chi'(\omega)}{\mathrm{d}\omega}\big|_{\omega=\omega_0}} = \frac{c}{n_g}$$

Analysis of group velocity index

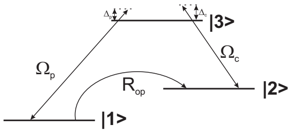
The most interesting case is for $n_g \in (-\infty, 1)$.
In this case we obtain a negative group velocity or group velocity greater than c.
To have $n_g \in (-\infty, 1)$ we should find such an atomic system in which the dispersion is normal while absorption (or gain) being not too strong.

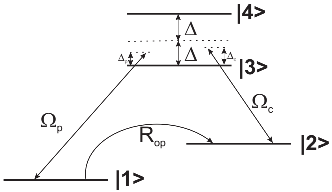
The Electric susceptibility



Rysunek: A typical plot of $\chi(\omega)$ (real part - dispersion and imaginary part - absorption)

Atomic systems





Hamiltonian

Hamiltonian of the system

$$\mathcal{H} = \begin{bmatrix} \Delta_p & 0 & \Omega_{p3} & \Omega_{p4} \\ 0 & \Delta_c & \Omega_{c3} & \Omega_{c4} \\ \Omega_{p3}^* & \Omega_{c3}^* & \Delta & 0 \\ \Omega_{p4}^* & \Omega_{c4}^* & 0 & -\Delta \end{bmatrix}$$

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Von Neumann's equation:

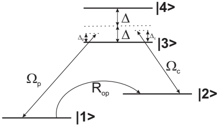
$$i\hbar \frac{\mathrm{d}\rho}{\mathrm{d}t} = [\mathcal{H}, \rho] + \Lambda$$

The electric susceptibility in atomic systems

The electric susceptibility in atomic systems can be written as:

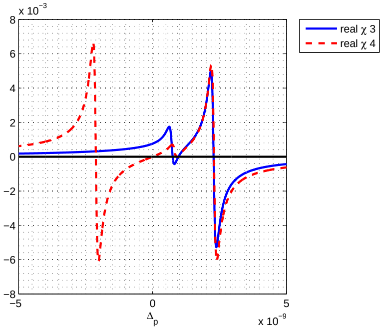
$$\chi = \frac{N\left(|d_{13}|^2\rho_{13} + |d_{14}|^2\rho_{14}\right)}{\varepsilon_0 \hbar \Omega_p}$$

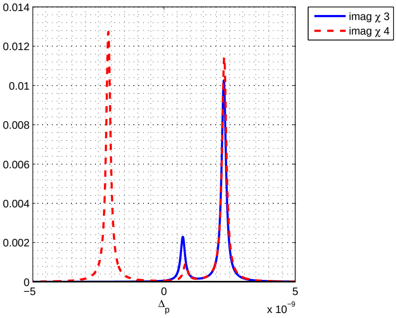
By solving the system of equations of motion for density matrix and putting the solutions to the expression above we obtain graphs for dispersion and absorption depending on following parameters: $\Delta, \Delta_c, R_{op}, \Omega_c, \Omega_p$ as a function of Δ_p.



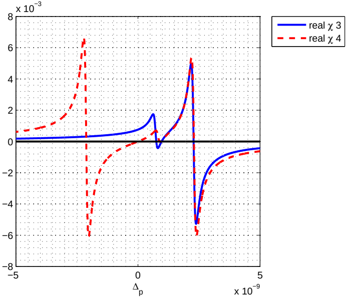
Results - without optical pump

$$\Delta = 2 \cdot 10^{-9}, \Delta_c = 1 \cdot 10^{-9}, E_c = 6 \cdot 10^{-10}, E_p = 3 \cdot 10^{-12}$$

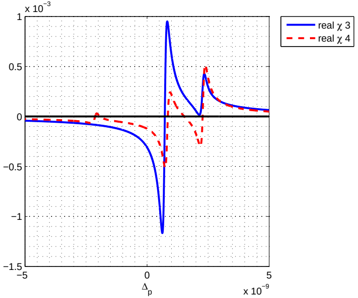




Optical pump influence on dispersion

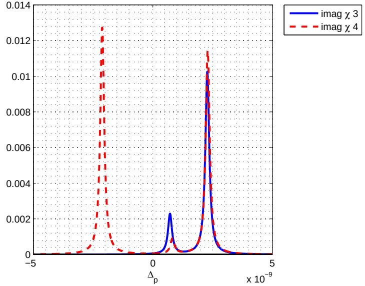


Rysunek: Without optical pump



Rysunek: With optical pump $R_{op} = 2 \cdot 10^{-9}$

Optical pump influence on absorption



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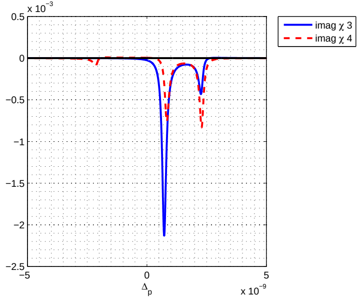


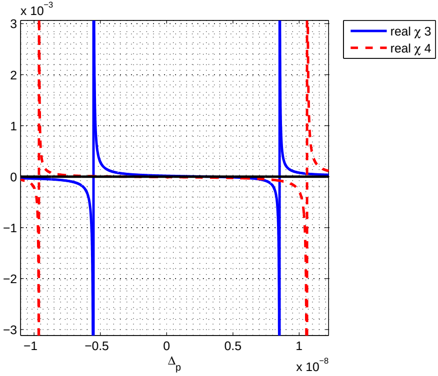
Rysunek: With optical pump $R_{op} = 2 \cdot 10^{-9}$

$$v_g \in (-\infty, 0)$$

Results - $v_g \in (-\infty, 0)$

Analysis of dispersion

$\Delta = 2 \cdot 10^{-9}$, $\Delta_c = 1 \cdot 10^{-9}$, $E_c = 7 \cdot 10^{-9}$, $E_p = 3 \cdot 10^{-12}$, $Rop = 2 \cdot 10^{-11}$



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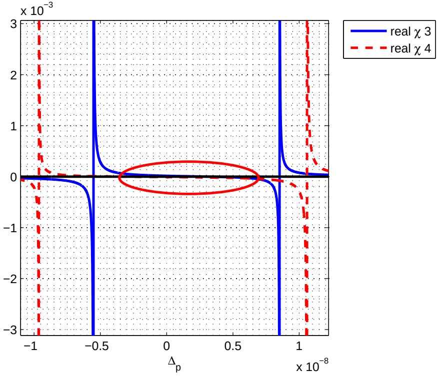

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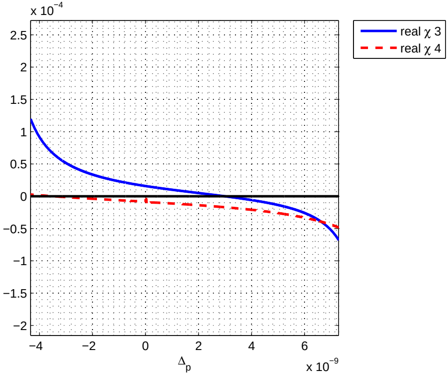

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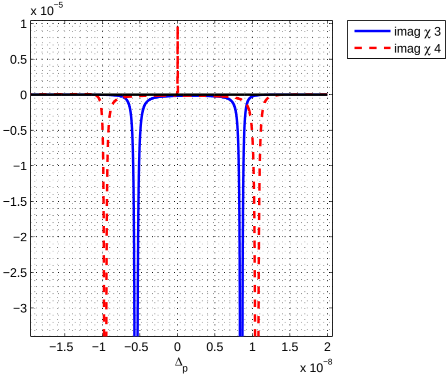

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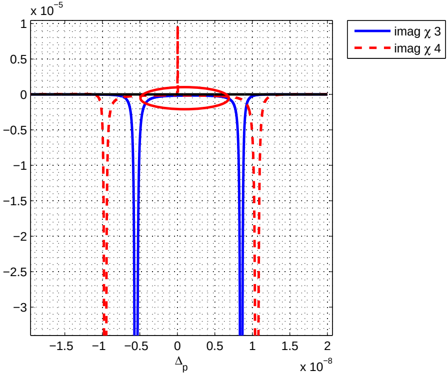

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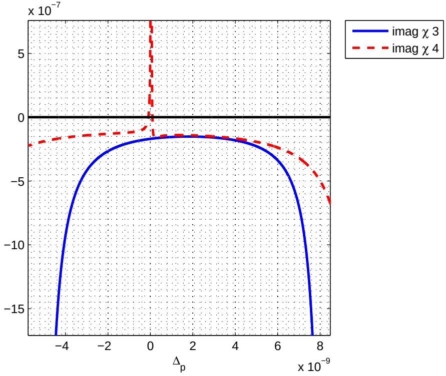

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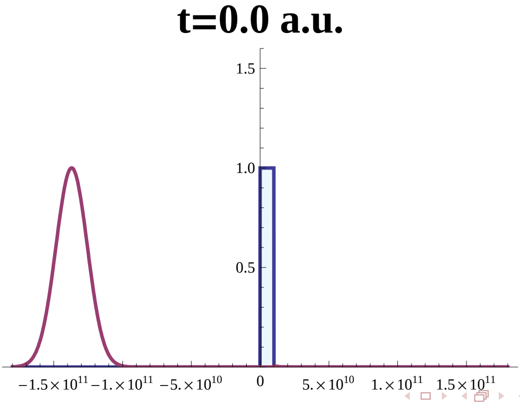

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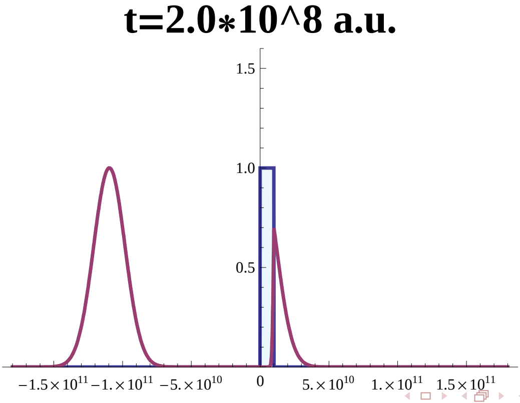

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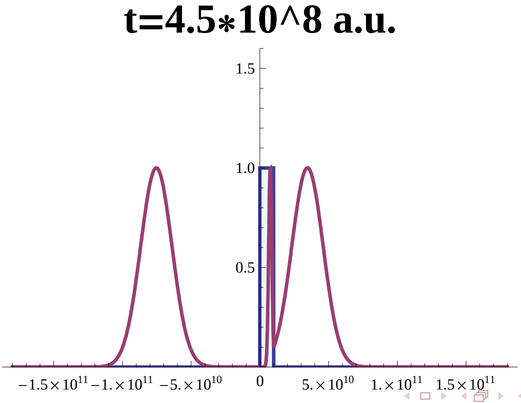

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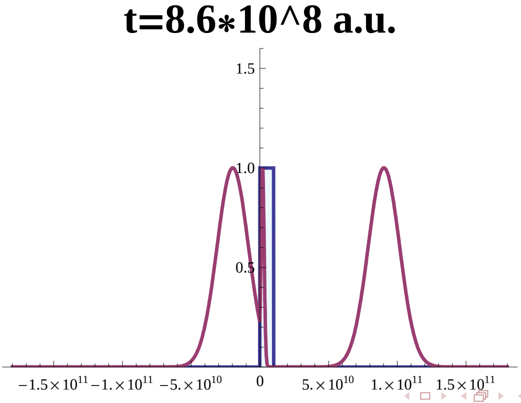

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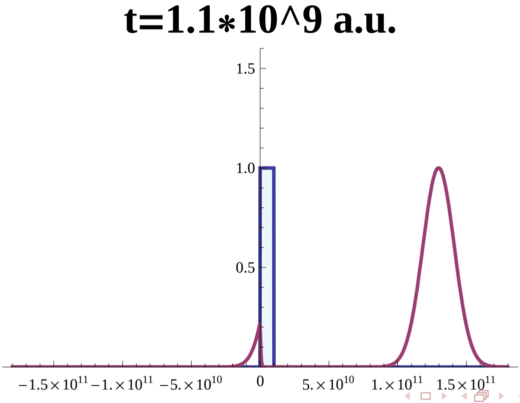

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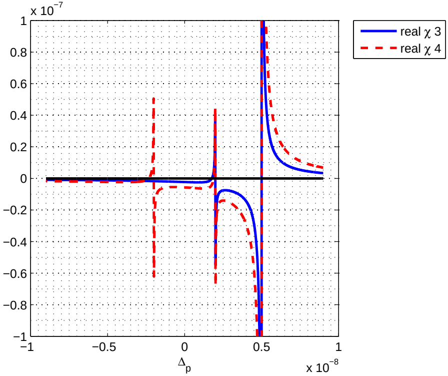

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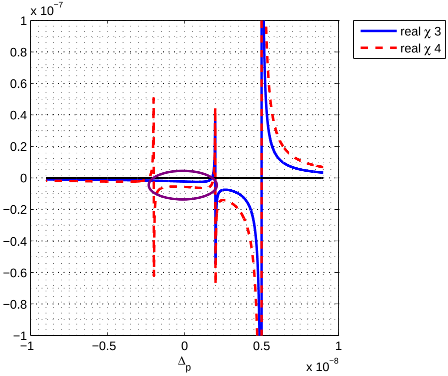

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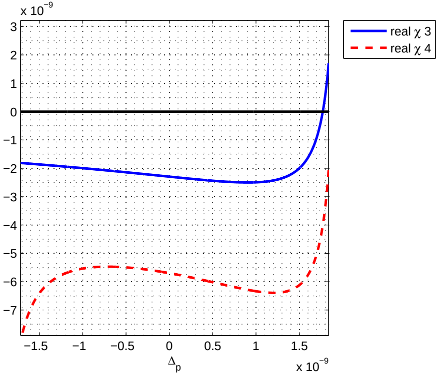

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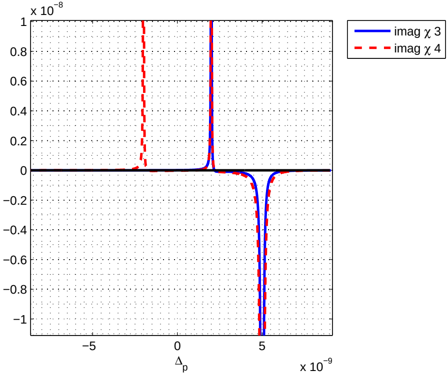

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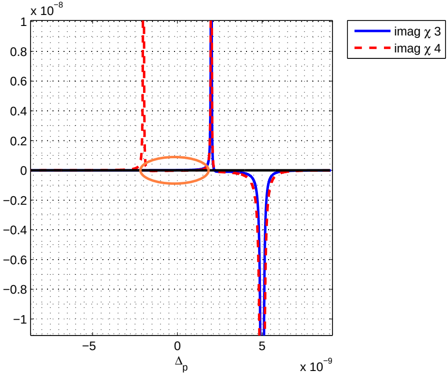

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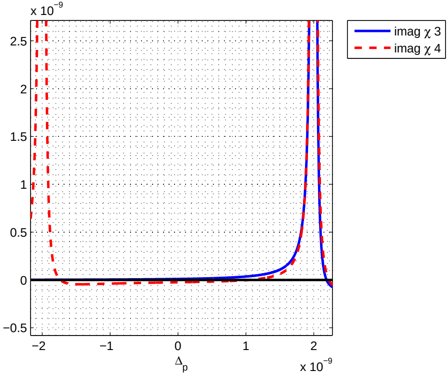

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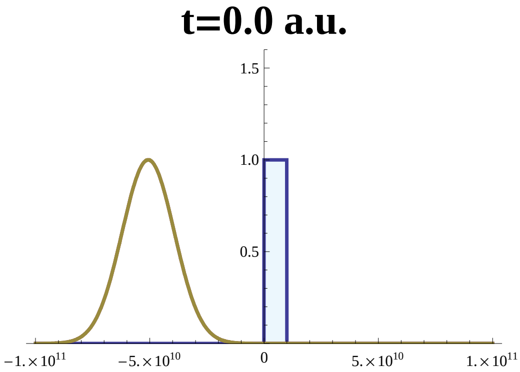

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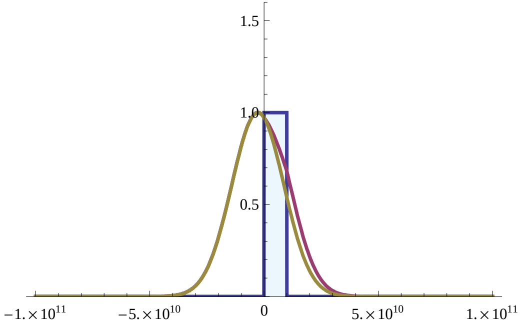

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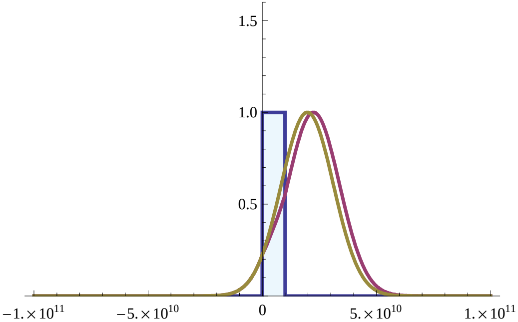

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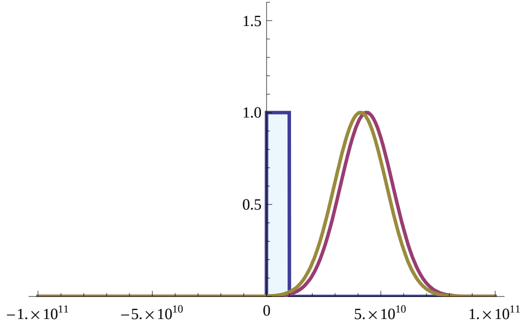
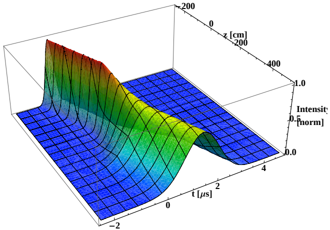


Second derivative approximation

An example of an impulse propagating through a sample with a negative group velocity for electric susceptibility approximated by:

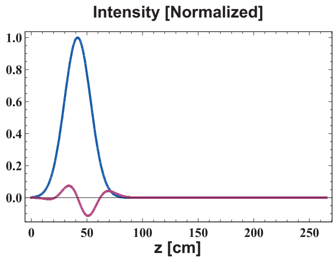
$$\tilde{\chi}(\Delta_p)=\chi(0)+\frac{d\chi(0)}{d\omega}(\Delta_p)+\frac{1}{2}\frac{d^2\chi(0)}{d\Delta_p^2}(\Delta_p)^2$$



Third derivative approximation

A pulse propagating in a subluminal regime. The Brillouin precursor magnified by the factor of 5000 is shown. The precursor was obtained for electric susceptibility approximated by:

$$\tilde{\chi}(\Delta_p)=\chi(0)+\frac{d\chi(0)}{d\omega}(\Delta_p)+\frac{1}{2}\frac{d^2\chi(0)}{d\Delta_p^2}(\Delta_p)^2+\frac{1}{6}\frac{d^3\chi'(0)}{d\Delta_p^3}(\Delta_p)^3$$



Interpretation on superluminal propagation

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- Effects are observed only for analytical pulses.

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    - In both high and weakly detuning regimes
    - For relatively strong coupling field $E_c = 3 \cdot 10^{-9}$ for $E_p = 3 \cdot 10^{-10}$
    - In a wide range of frequency

In both cases we can control the value of the group velocity index by changing the amplitude of coupling field or incoherent optical pump.

All the results have been obtained in cooperation with:

- prof. A. Raczyński[1]
- prof. J. Zaremba[1]
- dr S. Zielińska-Kaniasty[2]

[1]Institute of Physics, Nicolaus Copernicus University, Toruń
[2]Institute of Mathematics and Physics, University of Technologies and Life Sciences, Bydgoszcz

Thank you for your attention