THEORY OF TWO-COMPONENT BEC INTERFEROMETRY

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OUTLINE

Introduction

- Why BEC interferometry is of interest.

Aim

- What the research focuses on.

Mean Field Theory for Two-Component BEC

- New self-consistent coupled GPE and matrix eqns.

Application to Ramsey Interferometry

- Specific mean field theory expressions.

Josephson Model

- Simplified theory of BEC interferometry.

♦ Numerical Study

- Find regimes where full mean field theory needed.

♦ Other Work

- Two other approaches to Ramsey interferometry.

♦ Summary and Outlook

♦ References

INTRODUCTION

♦ Bose-Einstein condensates in cold atomic gases

- All N bosons occupy *small* number of single particle states (or *modes*) – often only one mode $(T \ll T_c)$.
- Quantum system with long range *spatial coherence* on a *macroscopic* scale with *massive* particles $\lambda_{compton} \sim 10^{-30} m$.
- Controllable *experiments* - trap potentials, Feshbach resonances, one and two component BEC, 1D and 2D BEC,..
- Ideal for studying *quantum interferometry*, *decoherence*, *entanglement* and *non-classical states* in a *macroscopic* system of *massive* particles.
- Suitable system for *precision measurements*.

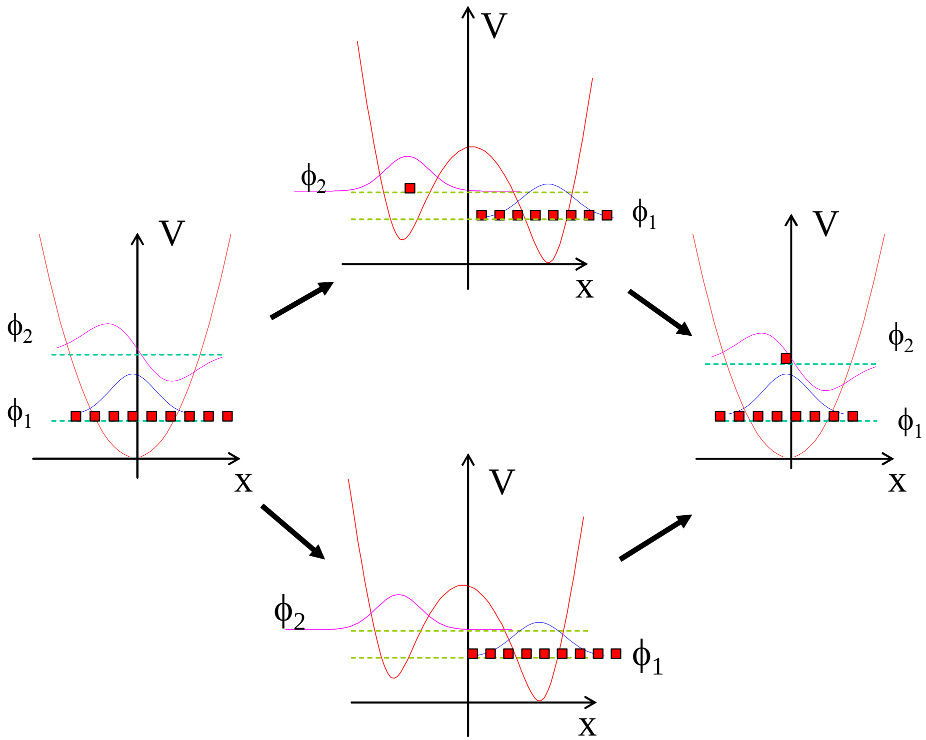
♦ BEC interferometry

- Based on *almost all* bosons in one (or two) modes.
- Many types - *Ramsey* interferometry, *Mach-Zender*, *Bragg*, ..
- Description - *quantum correlation functions* - expectation values of products of bosonic field operators - related to many-boson *position measurements*.
- All topics - *QInterf, Decoh, PrecM, Entang, Squeezing*.

♦ Quantum Interference

- Mach-Zender *double-well interferometry experiment* with *single-component* BEC shown.
- Starts with BEC in single well trap, changing trap to (possibly asymmetric) double-well trap and then back to single well.

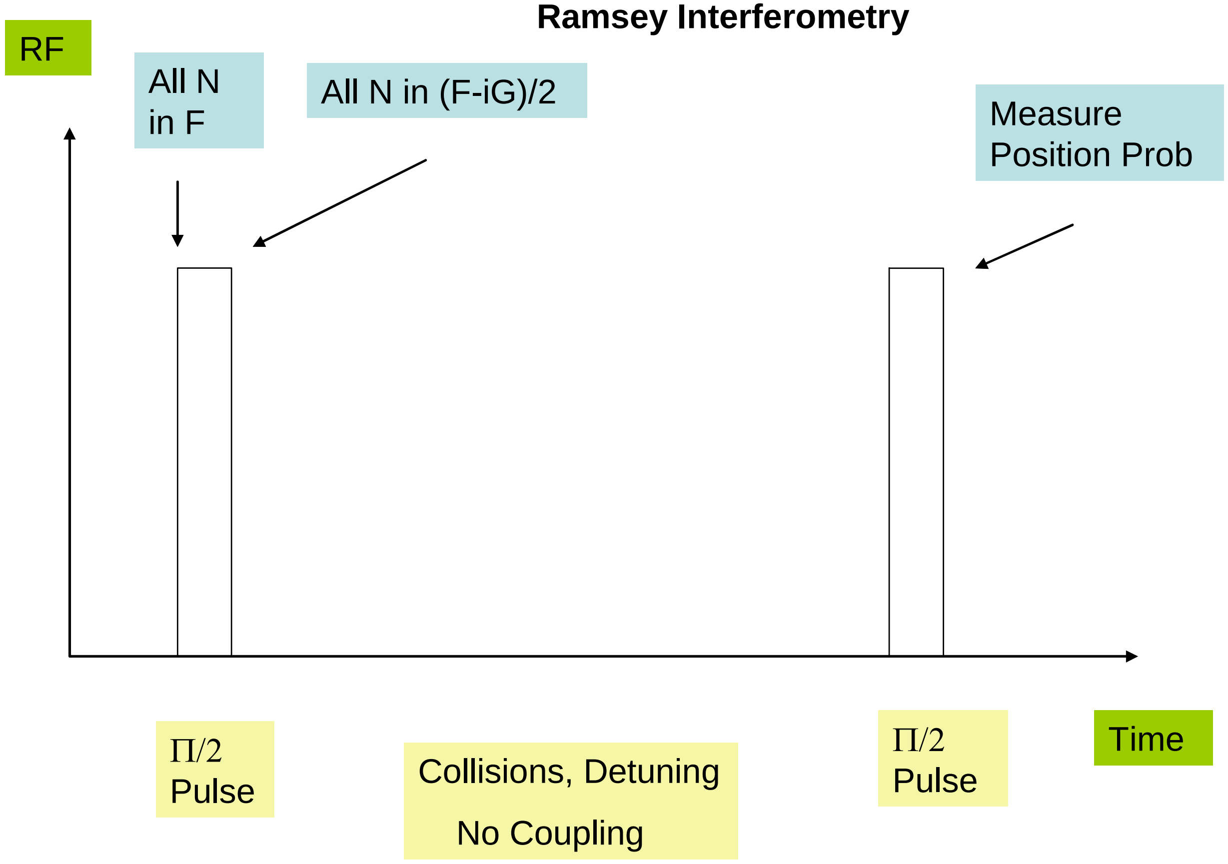
- *Process* of one boson excitation shown with two *quantum pathways*, both involving intermediate double well trap.



- Asymmetry could lead to *excitation* of some bosons to higher energy states of final trap.

- *Near degeneracy* of energy levels for asymmetric double well facilitates boson transfer to excited state.
- Two *non-observed* quantum pathways with boson transfer in *different* halves.
- *Superposition* of quantum *transition amplitudes* gives *interference* effects.
- *Ramsey interferometry* with *two-component* BEC shown.
- BEC in single well trap with all bosons *initially* in internal state F, applying $\Pi/2$ *pulse* (which changes internal states $F \rightarrow F - iG$, $G \rightarrow -iF + G$), then *free evolution* followed by another $\Pi/2$ *pulse* and final *detection* in state F.
- Two quantum pathways $F \rightarrow F \Rightarrow F \rightarrow F$ and $F \rightarrow G \Rightarrow G \rightarrow F$ with *different* phase factors in free evolution stage $\Rightarrow$.
- *Superposition* of *transition amplitudes* gives *interference*.

Ramsey Interferometry



♦ Decohence and Dephasing

- Even where *external* environmental effects are absent, *internal* boson-boson interactions can still result in *dephasing* (due to interactions within condensate modes) and *decoherence* effects (due to interactions causing transitions from condensate modes) that *degrade* interference pattern.

♦ Precision Measurement

- *BEC interferometry* (such as by splitting trapped BEC into two traps and then allowing BECs to recombine) offers possible *precision improvements* over *standard quantum limit* by a factor given by $\sqrt{N}$ (Kasevich (2002); Dunningham, Barnett, Burnett (2002)) - *Heisenberg limit*.
- Dunningham et al (2004) based on *collapse, revival* effects.

♦ Entanglement and Non-Classical States

- Two mode $(\hat{a}, \hat{b})$ *entangled*, *non-entangled* states

$$|\Phi\rangle_E = (1/\sqrt{2})^N \sum_{n=0}^{N} \sqrt{C_n^N}\,|n\rangle_a |N-n\rangle_b \qquad |\Phi\rangle_{NE} = |N\rangle_a |0\rangle_b$$

- Apply 50:50 *beam splitter* process - *interferometry*.
- Find for *entangled* state $\langle \hat{a}^\dagger \hat{a} \rangle = N, \langle \hat{b}^\dagger \hat{b} \rangle = 0$ and for *non-entangled* state $\langle \hat{a}^\dagger \hat{a} \rangle = N/2, \langle \hat{b}^\dagger \hat{b} \rangle = N/2$.
- Two mode *spin squeezing* occurs due to collisions between bosons - *non-classical* states.
- Interferometry involves *entanglement* and *spin squeezing*.

AIM

♦ Develop a general theory of BEC interferometry

- Allow for *dephasing* and *decoherence* effects.
- Treat *two mode* cases including *one-component* BEC in double wells and *two-component* BEC in single wells.
- Base theory on *mean field* and *phase space* methods.
- Single component BEC in *double wells* - hybrid *Wigner*, *P+* phase space *distribution functional*.

B J Dalton; Cond-mat.quant-gas 1007.0100, 2010.

♦ Present work

- *Mean field* theory of *dephasing* for *two-component* BECs.

MEAN FIELD THEORY

♦ Features

- Theory of *dephasing* effects on *quantum correlation functions* describing *two-component* BEC interferometry.
- Generalised mean field theory for *two* modes with *macroscopic* occupancy.
- *Dephasing* transitions *within* condensate modes - *decoherence* processes *not* included.
- Wide range of states allowed including *single mode* BEC and BEC *fragmented* into *two modes*.
- Elementary *collective excitations* (Bogoliubov) and *single boson excitations* (thermal modes) *not* included.

♦ Hamiltonian

$$\widehat{H} = \sum_i \int d\mathbf{r}\left(\frac{\hbar^2}{2m}\nabla\widehat{\Psi}_i(\mathbf{r})^\dagger \bullet \nabla\widehat{\Psi}_i(\mathbf{r}) + \widehat{\Psi}_i(\mathbf{r})^\dagger V_i \widehat{\Psi}_i(\mathbf{r})\right)$$

$$+ \sum_{ij} \int d\mathbf{r}\,\frac{g_{ij}}{2}\widehat{\Psi}_i(\mathbf{r})^\dagger \widehat{\Psi}_j(\mathbf{r})^\dagger \widehat{\Psi}_j(\mathbf{r})\widehat{\Psi}_i(\mathbf{r})$$

$$+ \sum_{i\neq j} \int d\mathbf{r}\,\widehat{\Psi}_i(\mathbf{r})^\dagger \Lambda_{ij}(\mathbf{r},t)\widehat{\Psi}_j(\mathbf{r})$$

- *Components* F, G, *zero range* approximation, *trap* potentials $V_i(\mathbf{r},t)$, *inter-component* coupling $\Lambda_{ij}(\mathbf{r},t)$.

♦ Field operators

- Approximation - *one spatial mode* per component.
- Single boson states (*modes*) $\left|\tilde{\phi}_F\right\rangle, \left|\tilde{\phi}_G\right\rangle$.

- Mode *annihilation* operators $\widehat{c}_F(t), \widehat{c}_G(t)$.

$$\widehat{\Psi}_i(\mathbf{r}) = \widehat{c}_i(t)\phi_i(\mathbf{r},t) \qquad \left\langle \mathbf{r} | \widetilde{\phi}_i \right\rangle = \phi_i(\mathbf{r},t)|i\rangle \qquad (i = F, G)$$

$$[\widehat{\Psi}_i(\mathbf{r}), \widehat{\Psi}_j^{\dagger}(\mathbf{r}')] = \delta_{ij}\delta(r - r') \qquad \int d\mathbf{r}\, \phi_i^*(\mathbf{r},t)\phi_i(\mathbf{r},t) = 1$$

♦ General quantum state

- Superposition of *fragmented* states $|k\rangle$, *amplitudes* b_k.

$$|\Phi(t)\rangle = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}} b_k(t) \frac{(\widehat{c_F}^{\dagger})^{(\frac{N}{2}-k)}}{[(\frac{N}{2}-k)!]^{\frac{1}{2}}} \frac{(\widehat{c_G}^{\dagger})^{(\frac{N}{2}+k)}}{[(\frac{N}{2}+k)!]^{\frac{1}{2}}} |0\rangle$$

- Quantum *Dirac-Frenkel principle of least action* gives *self consistent* coupled eqns for mode functions (*generalised Gross-Pitaevskii eqns*) and for amplitudes (*matrix eqns*).

♦ Amplitude eqns

$$i\hbar\frac{\partial b_k}{\partial t} = \sum_l (H_{kl} - \hbar U_{kl}) b_l$$

- Describes BEC state *dynamics*.
- *Hamiltonian* and *rotation* matrices

$$H_{kl} = \langle k|\widehat{H}|l\rangle \qquad U_{kl} = ((\partial_t\langle k|)|l\rangle - \langle k|(\partial_t|l\rangle))/2i$$

- *Modes* ϕ_i affect amplitudes via H_{kl}, U_{kl}.

♦ Generalised GPE for two condensate modes

$$i\hbar X_{ii}\frac{\partial}{\partial t}\phi_i = \left\{X_{ii}(-\frac{\hbar^2}{2m}\nabla^2 + V_i) + \sum_j g_{ij} Y_{ijji}\,\phi_j^*\,\phi_j\right\}\phi_i + \sum_{j\neq i} X_{ij}\Lambda_{ij}\phi_j$$

- Describes *non-adiabatic behavior* of modes.

- Generalised *mean field* terms $Z_{ii} = \sum_j g_{ij} Y_{ijji} \phi_j^* \phi_j$.
- Inter-component *coupling* terms $\sum_{j \neq i} X_{ij} \Lambda_{ij} \phi_j$.
- One and two-body *correlation functions*

$$X_{ij} = \langle \Phi | \hat{c}_i^\dagger \hat{c}_j | \Phi \rangle \sim N \qquad Y_{ijji} = \langle \Phi | \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_j \hat{c}_i | \Phi \rangle \sim N^2$$

- Coefficients X_{ij}, Y_{ijji} depend quadratically on *amplitudes* $b_k(t)$.
- *Relative amplitudes* of fragmented states $|k\rangle$ affect modes.

♦ Bloch vector

- *Spin* operators

$$\hat{S}_x = (\hat{a}_G^\dagger \hat{a}_F + \hat{a}_F^\dagger \hat{a}_G)/2 \qquad \hat{S}_y = (\hat{a}_G^\dagger \hat{a}_F - \hat{a}_F^\dagger \hat{a}_G)/2i$$

$$\hat{S}_z = (\hat{a}_G^\dagger \hat{a}_G - \hat{a}_F^\dagger \hat{a}_F)/2 \qquad (\underrightarrow{\hat{S}})^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$$

- *Spin* states

$$(\hat{\vec{S}})^2 \left| \frac{N}{2}, k \right\rangle = \frac{N}{2}\left(\frac{N}{2}+1\right) \left| \frac{N}{2}, k \right\rangle \qquad \hat{S}_z \left| \frac{N}{2}, k \right\rangle = k \left| \frac{N}{2}, k \right\rangle$$

- *Bloch vector*

$$\sigma_a = \frac{1}{N} Tr(\hat{\rho}\hat{S}_a) = \langle \hat{\sigma}_a \rangle \qquad (a = x, y, z)$$

- Inside or on the *Bloch sphere* of radius $\frac{1}{2}$.
- Bloch vector *components*

$$\sigma_\pm = \frac{1}{N} \sum_k b_k^* b_{k\mp 1} \sqrt{\left(\frac{N}{2}\left(\frac{N}{2}+1\right) - k(k \mp 1)\right)}$$

$$\sigma_x = (\sigma_+ + \sigma_-)/2 \qquad \sigma_y = (\sigma_+ - \sigma_-)/2i$$

$$\sigma_z = \frac{1}{N} \sum_k b_k^* b_k k$$

♦ Application to interferometry

- *Spatial interferometry* - First order correlation functions

$$G_{ij}^{(1)}(\mathbf{r},\mathbf{r}',t) = Tr(\widehat{\rho}(t)\widehat{\Psi}_i(\mathbf{r})^{\dagger}\widehat{\Psi}_j(\mathbf{r}'))$$

$$G_{FF}^{(1)}(\mathbf{r},\mathbf{r}',t) = N\phi_F(\mathbf{r})^*\phi_F(\mathbf{r}')(\frac{1}{2} - \sigma_z)$$

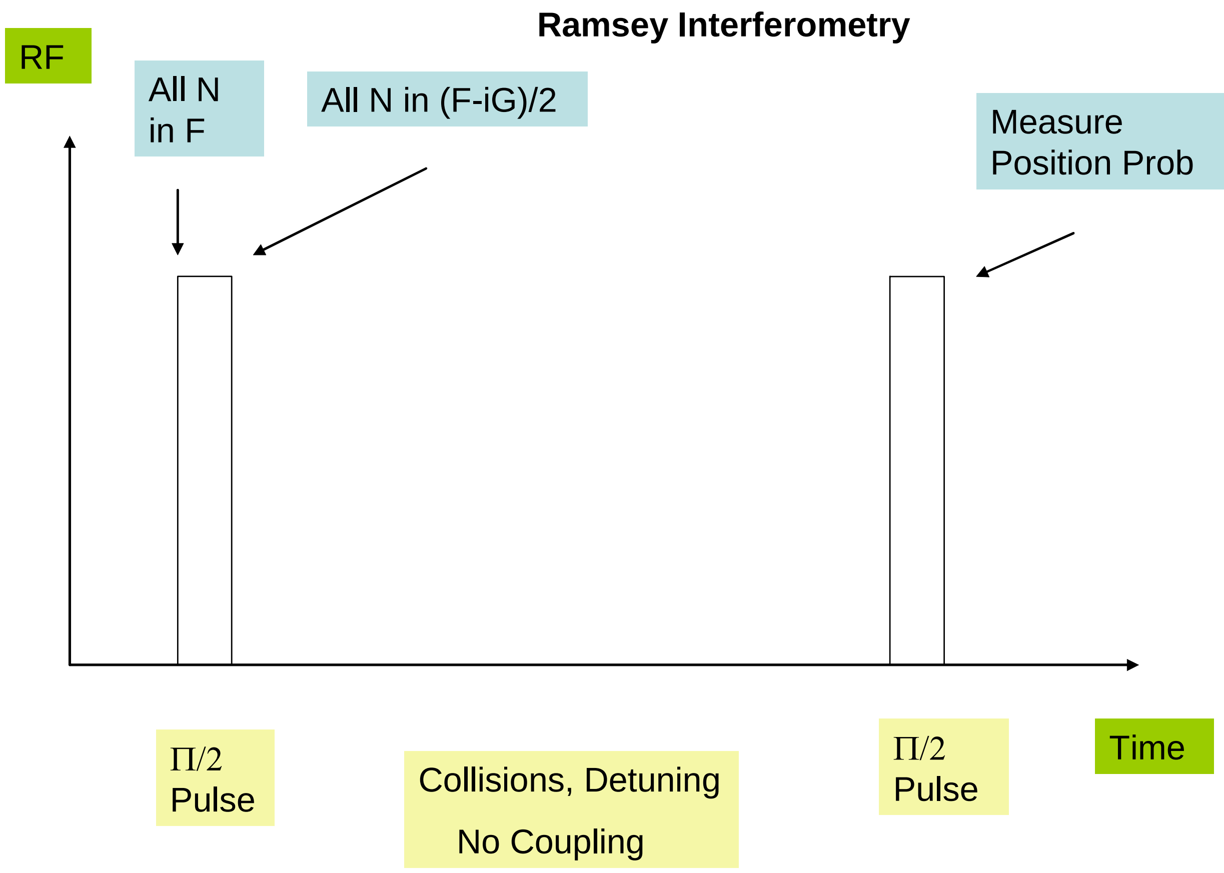
$$G_{FG}^{(1)}(\mathbf{r},\mathbf{r}',t) = N\phi_F(\mathbf{r})^*\phi_G(\mathbf{r}')(\sigma_x - i\sigma_y)$$

$$G_{GF}^{(1)}(\mathbf{r},\mathbf{r}',t) = N\phi_G(\mathbf{r})^*\phi_F(\mathbf{r}')(\sigma_x + i\sigma_y)$$

$$G_{GG}^{(1)}(\mathbf{r},\mathbf{r}',t) = N\phi_G(\mathbf{r})^*\phi_G(\mathbf{r}')(\frac{1}{2} + \sigma_z)$$

- Relation between *QCF* and *Bloch vector*.
- *Spatial interferometric patterns* and existence of *long range order* in BECs determined from correlation functions.

RAMSEY INTERFEROMETRY



- *Ramsey interferometry* with *two-component* BEC shown.

♦ Coupling period

- Collisions *ignored*.
- *Coupling* term $W_{GF} = \int dr\, \phi_G^* (\Lambda_{GF}) \phi_F = \hbar A(t) \exp(-i(\Omega t + \phi)) = W_{FG}^*$
- Amplitude $A(t)$, resonant frequency Ω, phase ϕ.
- Equivalent to *rotation* in *spin space*

$$b_k(t(s)) = \exp(-i(\alpha_k t(s) + k\phi)) \exp(ik\frac{\pi}{2}) \sum_l \left\langle \frac{N}{2}, k \right| \exp(-i2\hat{S}_y s) \left| \frac{N}{2}, l \right\rangle$$

$$\times \exp(-il\frac{\pi}{2})\, b_l(t(0)) \exp(+i(\alpha_l t(0) + l\phi))$$

- Pulse *area* $s(t) = \int_{-\infty}^{t} A(\tau) d\tau$, *energy* $W_{ii} = \int dr\, \phi_i^* \left(-\frac{\hbar^2}{2m} \nabla^2 + V_i\right) \phi_i$,

oscillation *frequency* $\alpha_k = (\frac{N}{2} - k) W_{FF}/\hbar + (\frac{N}{2} + k) W_{GG}/\hbar$.

- *All* bosons in F - $|N\rangle_F |0\rangle_G$ - find $|b_k(t(s))|^2 = |d^{\frac{N}{2}}_{k,-\frac{N}{2}}(2s)|^2$.
- Bosons in *fragmented* state $|N/2\rangle_F |N/2\rangle_G$ - find $|b_k(t(s))|^2 = |d^{\frac{N}{2}}_{k,0}(2s)|^2$

♦ No coupling period

- *Amplitudes* given by $b_k(t) = b_k(0)\exp(-iA_k(t))$
- *Single pair* of coupled GPE for *modes*

$$i\hbar\frac{\partial}{\partial t}\phi_i = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_i\right)\phi_i + \left(\frac{\langle N_i(N_i-1)\rangle}{\langle N_i\rangle} g_{ii}\phi_i^*\phi_i + \sum_{j\neq i}\frac{\langle N_i N_j\rangle}{\langle N_i\rangle} g_{ij}\phi_j^*\phi_j\right)\phi_i$$

- $\langle F(k)\rangle \equiv \sum_k |b_k(0)|^2 F(k)$ involves *initial* $b_k(0)$, $N_{F,G}(k) = \frac{N}{2} \mp k$.

- *Phase* equation

$$\frac{\partial}{\partial t} A_k(t) = \sum_i N_i \left(W_{ii} - \hbar T_{ii} \right) + \sum_i N_i \left(N_i - 1 \right) \frac{g_{ii}}{2} \int dr \left| \phi_i(r,t) \right|^4$$

$$+ N_F N_G \, g_{FG} \int dr \left| \phi_F(r,t) \right|^2 \left| \phi_G(r,t) \right|^2$$

- *Rotation* matrix $\hbar T_{ii} = \int dr \frac{\hbar}{2i} \left((\partial_t \phi_i^*(r,t)) \phi_i(r,t) - \phi_i^*(r,t) (\partial_t \phi_i(r,t)) \right)$

JOSEPHSON MODEL

- *Josephson Hamiltonian*

$$\hat{H}_{Jos} = -J_x \hat{S}_x - J_y \hat{S}_y + \delta \hat{S}_z + U \hat{S}_z^2$$

- *Tunneling* h_x, $G - F$ *transition* energy δ, *collision* U

$$-J_x = \{W_{GF} + W_{FG}\} \qquad -J_y = i\{W_{GF} - W_{FG}\}$$

$$\delta = \left\{W_{GG} + (N-1)\frac{g_{GG}}{2}\int dr\,|\phi_G(r,t)|^4\right\} - \left\{W_{FF} + (N-1)\frac{g_{FF}}{2}\int dr\,|\phi_F(r,t)|^4\right\}$$

$$U = \left\{\sum_i \frac{g_{ii}}{2}\int dr\,|\phi_i(r,t)|^4 - g_{FG}\int dr\,|\phi_F(r,t)|^2|\phi_G(r,t)|^2\right\}$$

- Assume mode functions *constant*.

♦ No coupling period

- *Amplitudes* for $J_x = J_y = 0$

$$b_k(t) = \exp\left\{-\frac{i}{\hbar}(\delta k + U k^2)t\right\} b_k(0)$$

- *Rabi oscillations* of Bloch vector at frequency $\delta/\hbar$.
- *Dephasing* for x, y components due *collision* term $U/\hbar$.
- *Collapse* and *revival effects* on x, y compts of Bloch vector.
- At collapse stage Bloch vector *not* on Bloch sphere - *fragmentation* occurs.
- For mode fns *same* $U \propto (g_{FF} + g_{GG} - 2g_{FG})$ - small except near *Feshbach* resonance.
- For mode fns *same* but *displaced* $U \propto (g_{FF} + g_{GG})$ - large.

NUMERICAL STUDY

- Graphs show Bloch vector *components* and Bloch vector *magnitude* as function of *time* during *no-coupling* period.
- Ramsey interferometry for *all* bosons initially in F - $|N\rangle_F |0\rangle_G$ and $\Pi/2$ pulse.
- For a *spherical* trap of frequency ω and QHO *ground state* mode functions with dimension $A = \sqrt{\hbar/m\omega}$

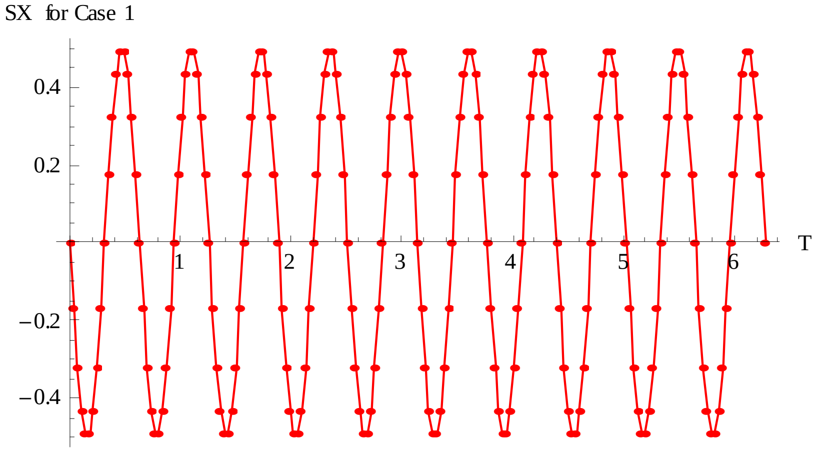
$$\frac{U}{\delta} = \sqrt{\frac{2}{\pi}} \frac{(a_{FF} + a_{GG} - 2a_{FG})/2}{A} \frac{\hbar\omega}{\delta}$$

♦ *Case 1* - No collisions - $\delta = 10.0, U = 0.0, N = 40$.

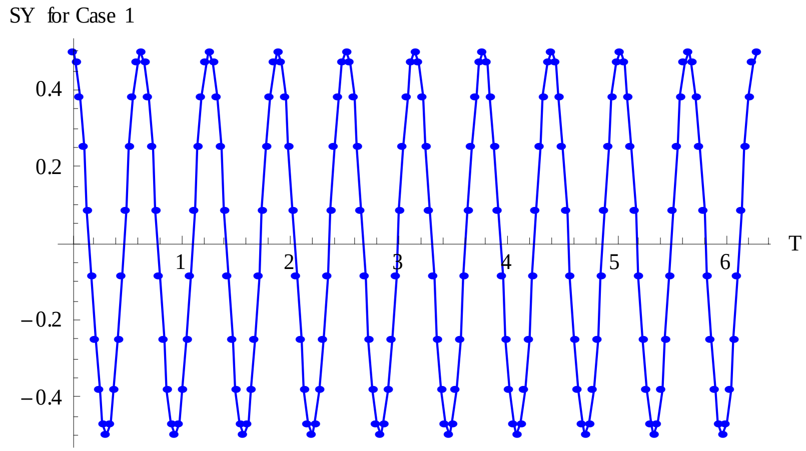
- Undamped Rabi oscillations, Bloch vector on Bloch sphere.

CASE 1 – No Collisions

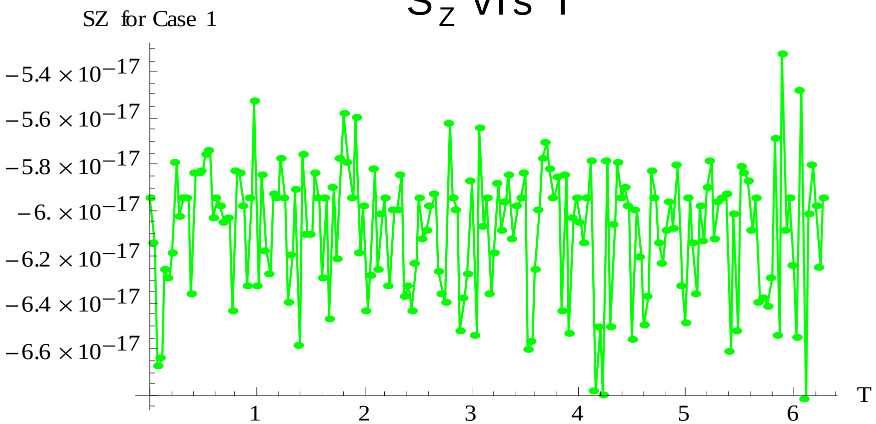
S_X vrs T



S_Y vrs T

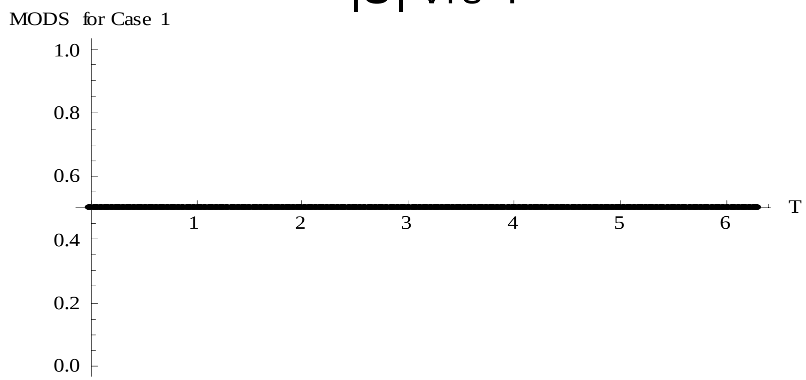


S_Z vrs T



Calcn noise

|**S**| vrs T



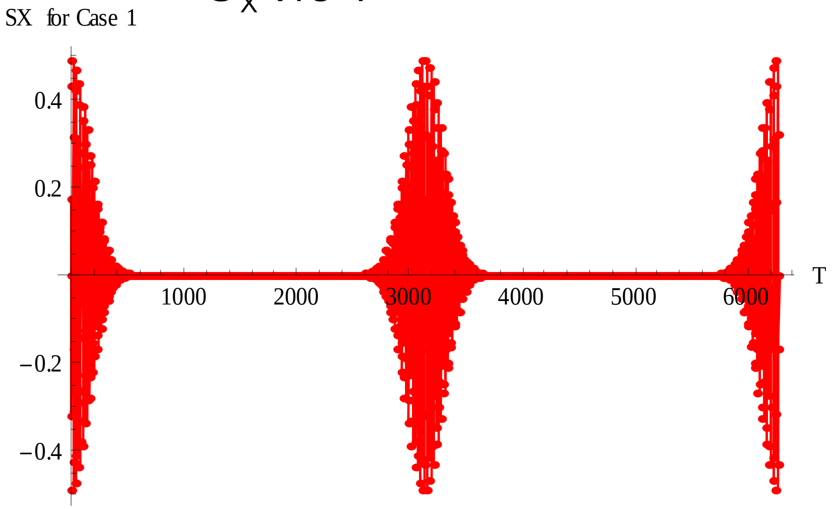
|**S**| =0.50

♦ *Case 2* - Weak collisions - $\delta = 10.0, U = 1.0 \times 10^{-4}, N = 40$.

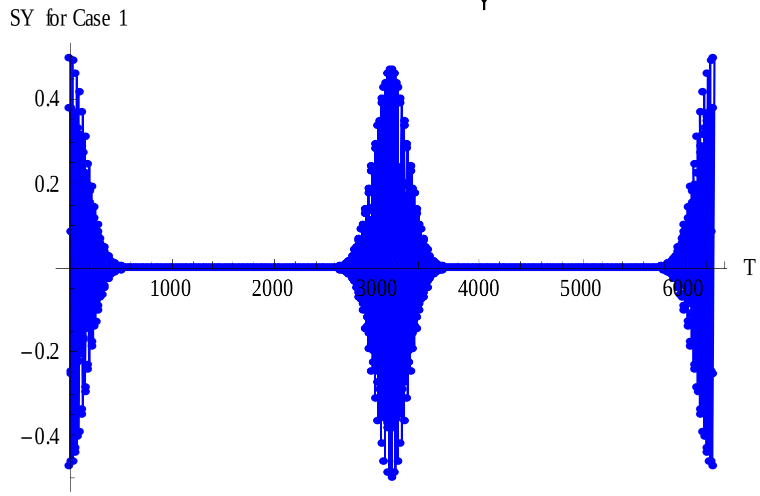
- Treat case of Na^{23} in isotropic *optical* trap, where δ is *Zeeman* splitting ~Mhz.
- Similar to Treutlein expt for Rb^{87} in *magnetic* trap, where δ is *hyperfine* splitting ~Ghz, and $a_{FF} + a_{GG} - 2a_{FG} \rightarrow a_{FF} + a_{GG}$.
- Parameters used are: $a_{FF} = a_{GG} = 51.9 r_B; \omega = 2\pi.8000; \delta = 2\pi\hbar.0.7.10^6 J$ at $B = 1G$ with $g_F = -1/2$.
- Collapses and revivals of Rabi oscillations after ~10^2 and ~10^3 oscillations - *observable ?*

CASE 2 – Weak Collisions

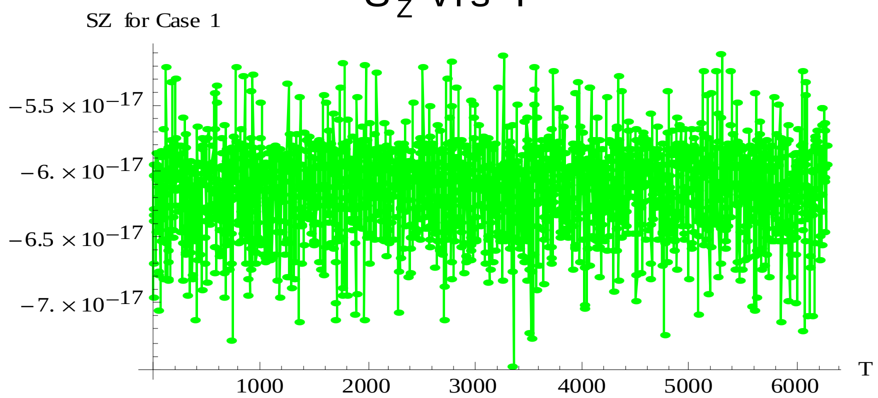
S_X vrs T



S_Y vrs T

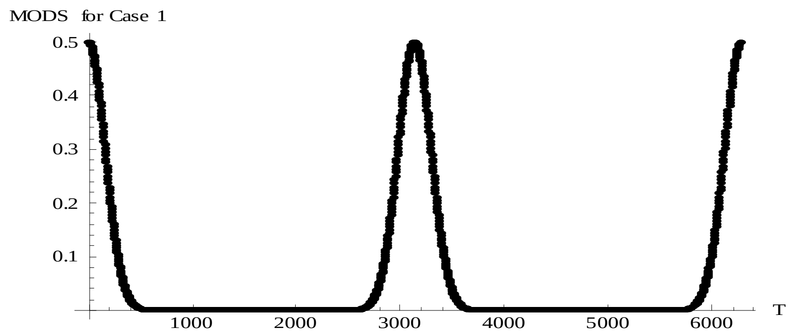


S_Z vrs T



Calcn noise

|**S**| vrs T



|**S**| varies from 0.50 to 0.0

OTHER WORK

♦ Features

- Simplest mean field theory treatment of two component BEC interferometry based on an *unfragmented* state [1,2] where *all* bosons occupy *same* single particle state, leads to coupled GPEs for spatial modes.
- Alternative mean field theory [3,4] allowing for fragmentation based on applying *variational* principle to *separate fragmented states* with occupancies $N_F = (N/2 - k), N_G = (N/2 + k)$, leads to coupled GPEs *specific* to these occupancies.
- Coupled GPEs for *no-coupling* period *differ* from present work.

♦ Unfragmented state

- Quantum superposition of *hyperfine states*

$$\langle r|\tilde{\phi}_{\theta,\chi}\rangle = \cos\theta\ \exp(-i\chi/2)\phi_F(r,t)|F\rangle + \sin\theta\ \exp(i\chi/2)\phi_G(r,t)|G\rangle$$

$$= \chi_F(r,t)|F\rangle + \chi_G(r,t)|G\rangle$$

- Mode *annihilation* operator

$$\hat{c}_{\theta,\chi}(t) = \cos\theta\ \exp(i\chi/2)\hat{c}_F(t) + \sin\theta\ \exp(-i\chi/2)\hat{c}_G(t)$$

- Unfragmented state with *all* bosons in $|\tilde{\phi}_{\theta,\chi}\rangle$

$$|\Phi_{\theta,\chi}(t)\rangle = \frac{(\hat{c}^{\dagger}_{\theta,\chi})^{(N)}}{[(N)!]^{\frac{1}{2}}}|0\rangle$$

- Special case of $|\Phi(t)\rangle$ with *binomial coefficient* amplitudes.
- *On* Bloch sphere, spherical polar angles $\Theta = \pi - 2\theta, \Phi = 2\pi - \chi$.

- Equal *fluctuations* in new perpendicular Bloch vector components $\tilde{\sigma}_x$, $\tilde{\sigma}_y$ - *no squeezing*.
- Quantum *Dirac-Frenkel principle of least action* gives coupled GPEs for mode functions

$$i\hbar\frac{\partial}{\partial t}\chi_i = \left\{\left(-\frac{\hbar^2}{2m}\nabla^2 + V_i\right) + (N-1)\sum_j g_{ij}\chi_j^*\chi_j\right\}\chi_i + \sum_{j\neq i}\Lambda_{ij}\chi_j$$

- *Normalisation*

$$\sum_i \int dr\, \chi_i^*(r,t)\chi_i(r,t) = 1$$

- Coupled GPE *differ* from those for general quantum state where *fragmentation* included.

♦ Separate fragmented states

- State *after* coupling pulse is *unfragmented*

$$|\Phi_{\theta,\chi}(0)\rangle = \frac{(\widehat{c}^{\dagger}_{\theta,\chi}(0))^{(N)}}{[(N)!]^{\frac{1}{2}}}|0\rangle$$

- State during *no-coupling* period.

$$|\Phi(t)\rangle = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}} b_k(0)\exp(-iA_k(t))\frac{(\widehat{c_F}^{\dagger})^{(\frac{N}{2}-k)}}{[(\frac{N}{2}-k)!]^{\frac{1}{2}}}\frac{(\widehat{c_G}^{\dagger})^{(\frac{N}{2}+k)}}{[(\frac{N}{2}+k)!]^{\frac{1}{2}}}|0\rangle$$

- *Initial state* after pulse determines *fixed* $b_k(0)$.
- *Mode functions* $\phi_{F,G}(r,t)$ and *phase factor* $A_k(t)$ obtained by applying variational formulation of time-dependent *Schrodinger* equation to *separate* state vector

$$|\Phi_k(t)\rangle = \exp(-iA_k(t)) \frac{(\widehat{c_F}^{\dagger})^{(\frac{N}{2}-k)}}{[(\frac{N}{2}-k)!]^{\frac{1}{2}}} \frac{(\widehat{c_G}^{\dagger})^{(\frac{N}{2}+k)}}{[(\frac{N}{2}+k)!]^{\frac{1}{2}}} |0\rangle$$

- Coupled GPEs for *mode* functions $\phi_i^k(r,t)$

$$i\hbar\frac{\partial}{\partial t}\phi_i^k = \left\{\left(-\frac{\hbar^2}{2m}\nabla^2 + V_i\right) + g_{ii}(N_i(k)-1)\phi_i^{k*}\phi_i^k\right\}\phi_i^k + \left\{\sum_{j\neq i} g_{ij}N_j(k)\phi_j^{k*}\phi_j^k\right\}\phi_i^k$$

depend on *occupancies* $N_F(k) = \frac{N}{2} - k, N_G(k) = \frac{N}{2} + k$.

- Equation for *phase*

$$\frac{d}{dt}A_k = -\sum_i N_i(k)(N_i(k)-1)\frac{g_{ii}}{2}\int dr\,|\phi_i^k(r,t)|^4 - N_F N_G\, g_{FG}\int dr\,|\phi_F^k(r,t)|^2|\phi_G^k(r,t)|^2$$

- Coupled GPEs, phase eqns and correlation fns *differ* from when variational principle is applied to *complete* state vector.

SUMMARY

- Self-consistent matrix eqns for *amplitudes* and generalised GPE for *modes* obtained for *two-component* BEC based on two-mode *mean field* theory allowing for *fragmentation*.
- Application made to *Ramsey interferometry*.
- Equations differ from *other work*.
- *Josephson* model shows *collapse, revival* effects on *Rabi oscillations* of Bloch vector.
- Bloch vector leaves *Bloch sphere* - fragmentation *occurs*.
- Effects *observable* in dipole traps ?
- *Spin squeezing* effects also expected.

♦ Further work

- *Spin squeezing* numerics.
- Numerical solution of *mean field theory* self-consistent GPE and matrix eqns for *Ramsey* and other *Heisenberg limited* interferometry.
- Development of *full phase space theory* based on mean fields for *two-component* BEC.

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