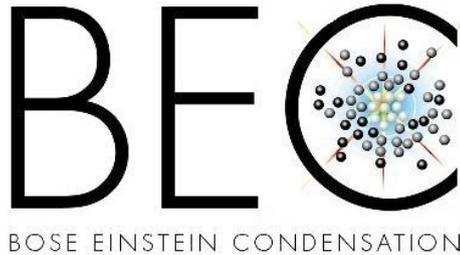


# Spin oscillations of normal Fermi gases at Unitarity

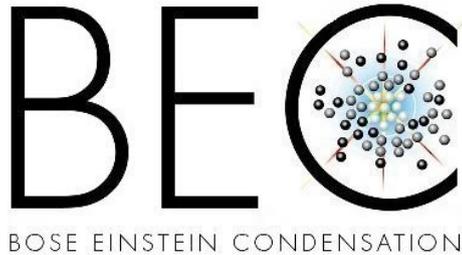


Alessio Recati & Sandro Stringari

CNR-INO BEC Center/  
Dip. Fisica, Univ. di Trento (I)



# Normal Fermi gases at Unitarity



Alessio Recati & Sandro Stringari

CNR-INO BEC Center/  
Dip. Fisica, Univ. di Trento (I)

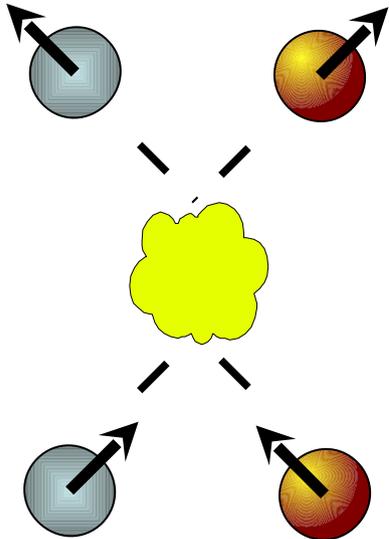


*In coll. with*  
*Stefano Giorgini (I), Carlos Lobo (UK)*  
*Roland Combescot, Fred Chevy (F), Chris Pethick (DK)*



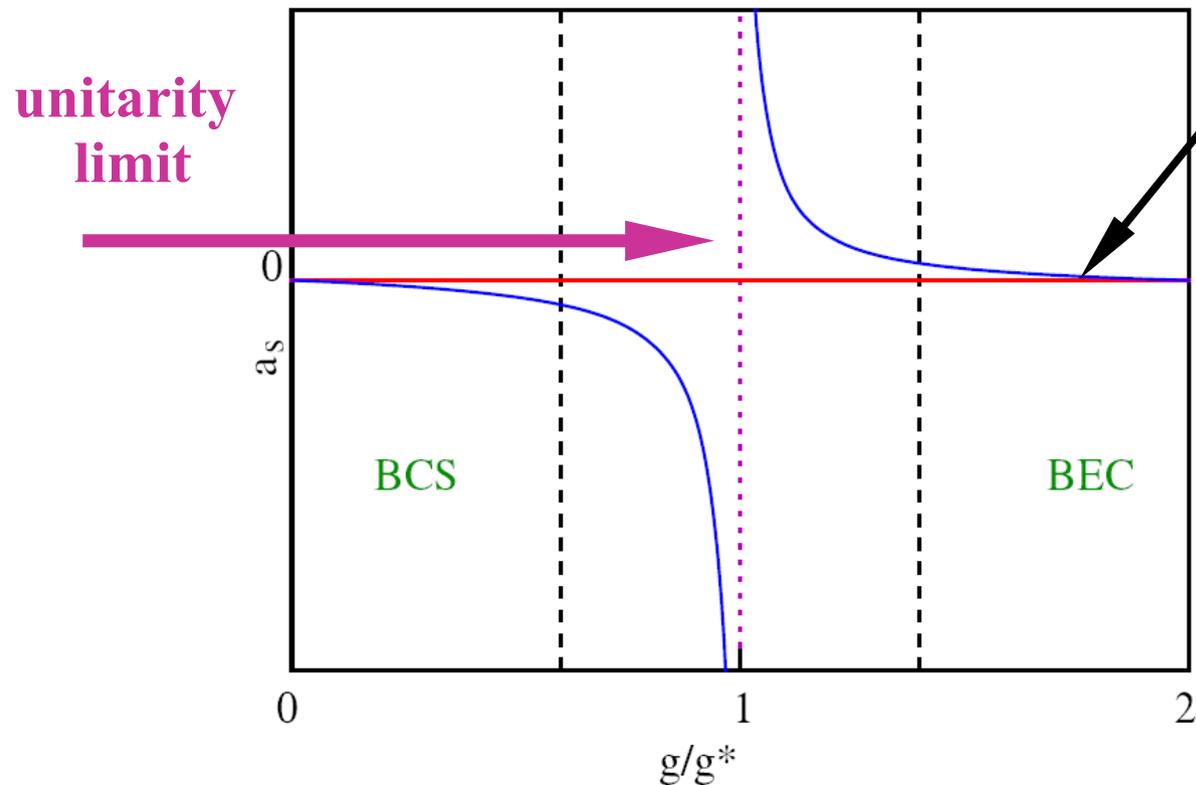
# BCS vs Bose-Einstein Condensation

The behaviour of the s-wave scattering length is *not continuous*



$$V(x - x') \rightarrow V_{eff}(x - x') \propto a\delta(x - x')(+reg.)$$

2-body bound state appears

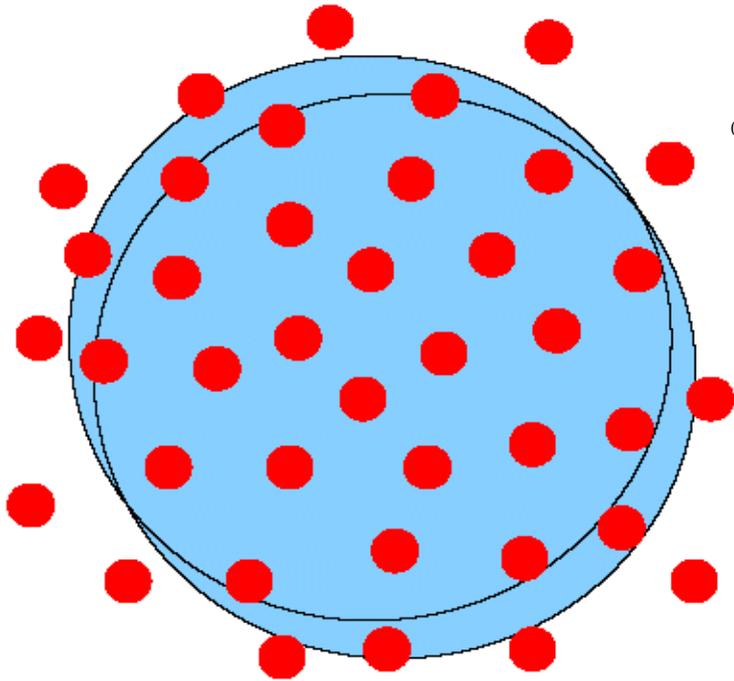


**Crossover postulate:** even though the scattering length changes abruptly in the many-body problem the *crossover is smooth*  
[Leggett; Nozieres/Schmitt-Rink]

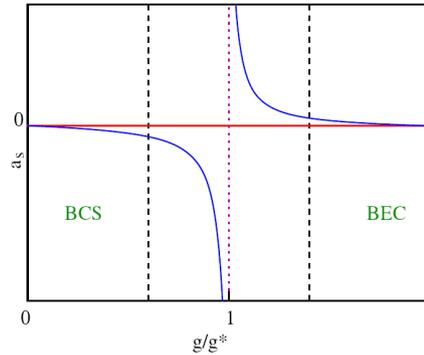
# BCS vs Bose-Einstein Condensation

Weak Coupling:  $k_F |a_s| \ll 1$   
Overlapping Cooper Pairs

$$\xi_b = \xi_0 = \frac{\hbar v_F}{\pi \Delta_0} \gg k_F^{-1}$$

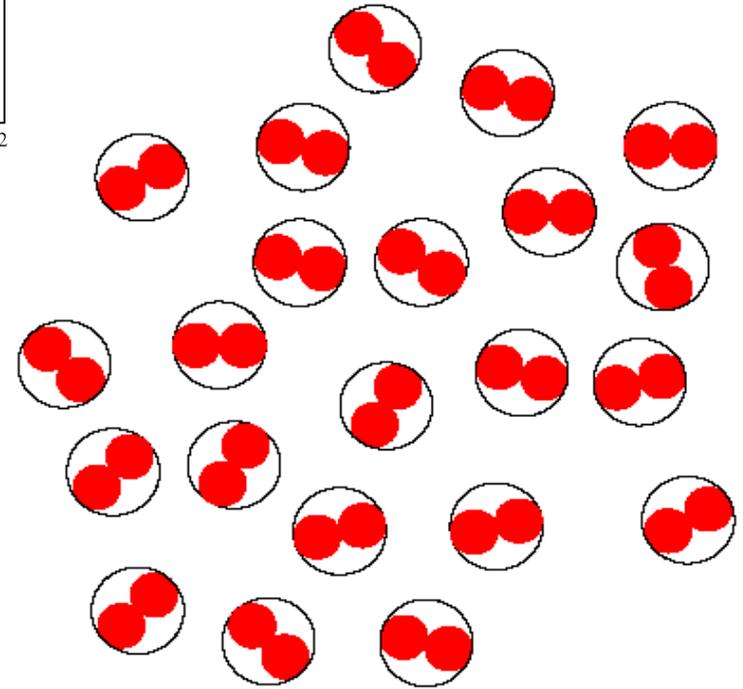


$$T^* = T_c^{(BCS)}$$



Strong Coupling:  $k_F a_s \ll 1$   
(Ideal) gas of molecules

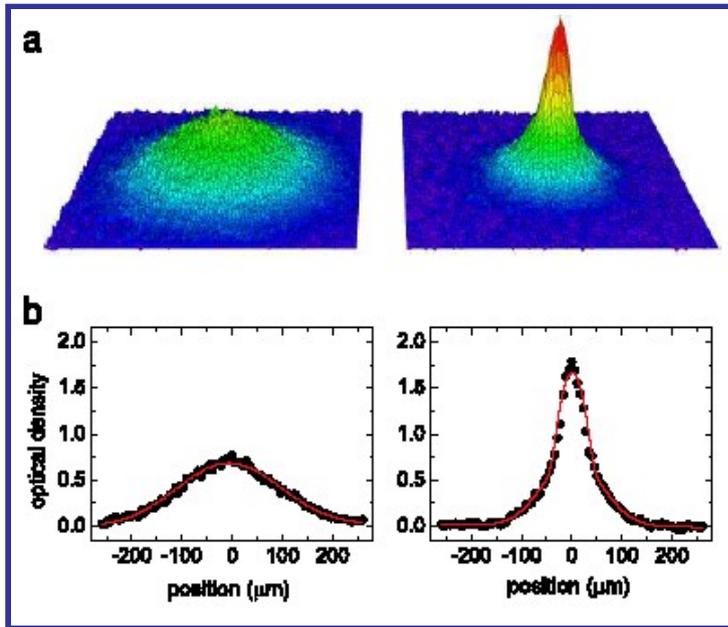
$$\xi_b \sim a_s \ll k_F^{-1} \quad E_b = \frac{\hbar^2}{m a_s^2}$$



$$T^* \gg T_c^{(BEC)}$$

**Note on finite  $T$ :** Except for very weak coupling (BCS) pairs form and condense at different temperature,  $T^*$  and  $T_c$

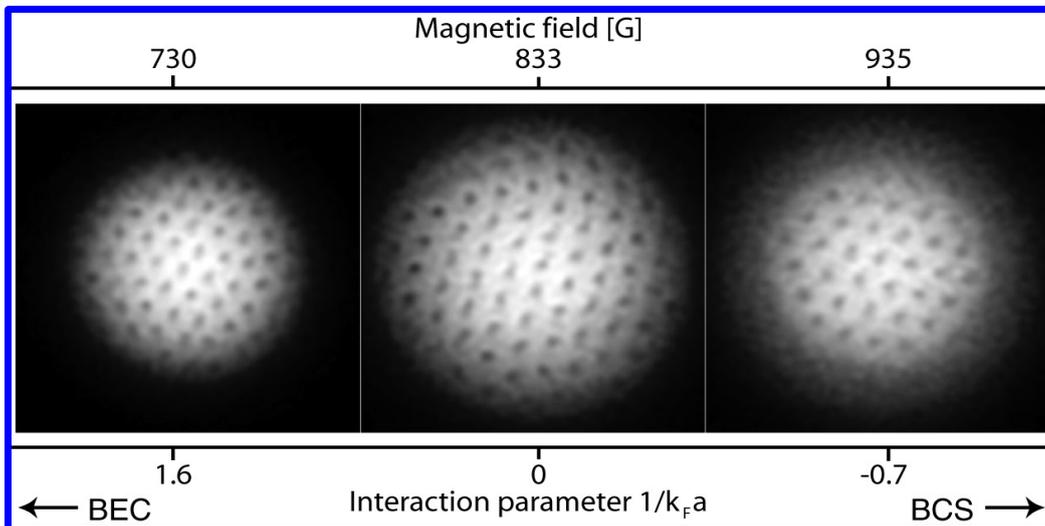
# Superfluid fermions



Molecular Bose-Einstein condensation  
from a fermionic gas

[JILA, Innsbruck, MIT, ENS, RICE,  
2003]

## Vortex lattice on the BCS-BEC crossover [MIT, 2005]



Observation of High- $T_c$  superfluidity  
Indeed  $T_c/T_F$

- 1)  $10^{-5}$ - $10^{-4}$  conventional SC and  $^3\text{He}$
- 2)  $10^{-2}$  high- $T_c$  SC
- 3) 0.15 unitary superfluid Fermi gas

# *Superfluid fermions at unitarity*

- ◆ The only scales at unitarity are the Fermi energy and the temperature.
- ◆ The thermodynamic properties have an “universal” form.

In particular at  $T=0$

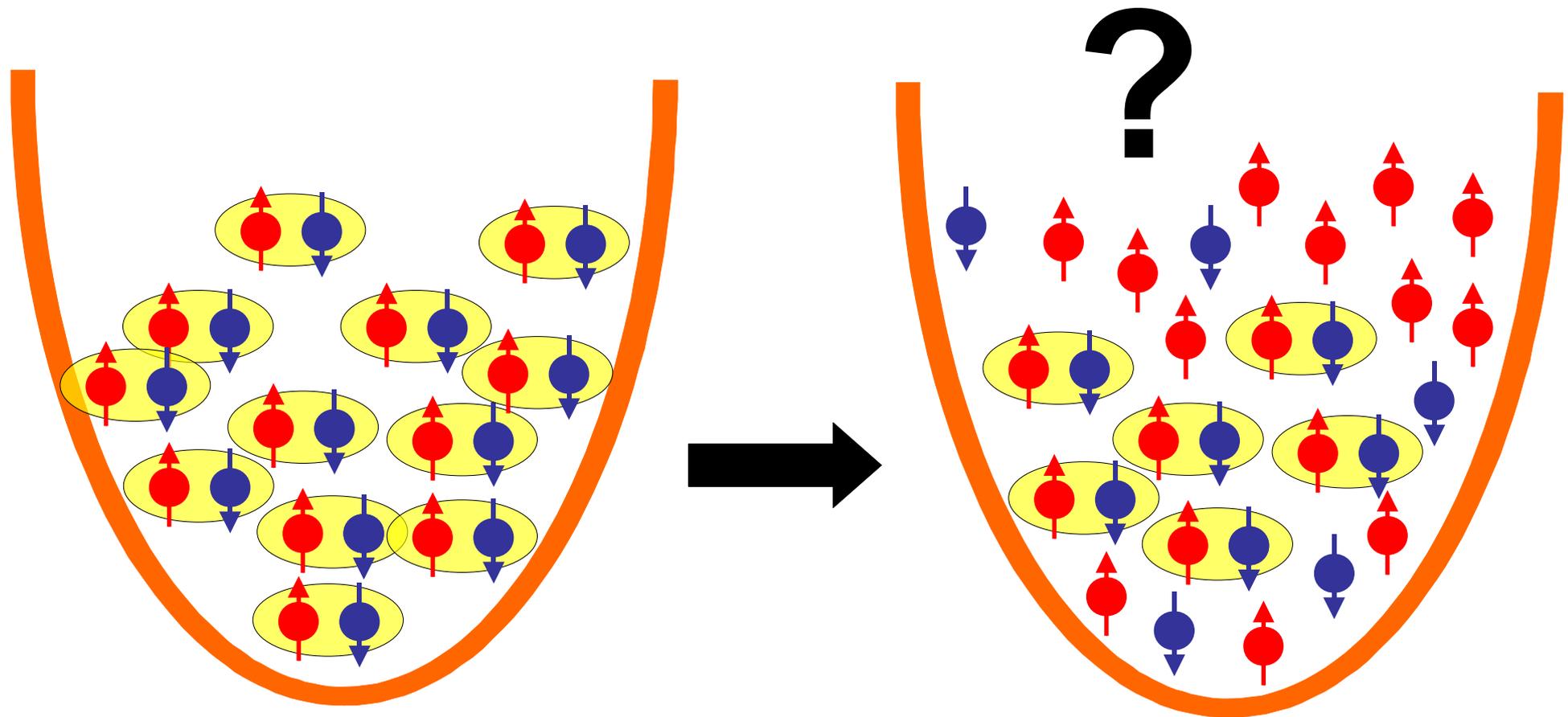
energy density, pressure, chemical potential are *proportional* to the ones of an ideal Fermi gas with a density equal to the superfluid one.

The universal parameter (via Montecarlo & Experiments)

$$\xi_S \simeq 0.42$$

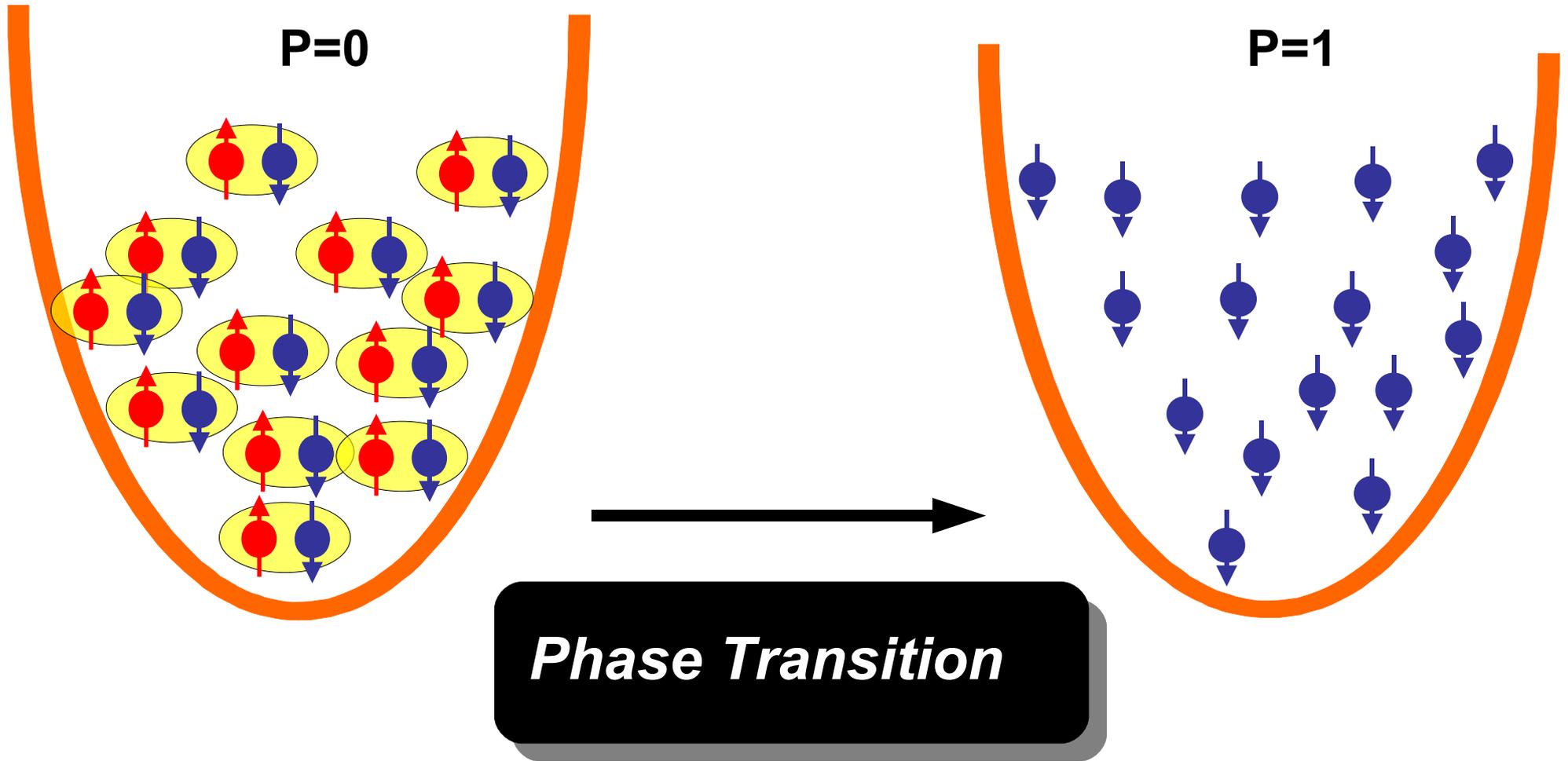
$$\frac{E_S}{N_S} = 2\xi_S \frac{3}{5} \frac{\hbar^2}{2m} (6\pi^2 n_S)^{2/3} \equiv 2\epsilon_S(n_S)$$

# Imbalanced Fermi gases at unitarity



Polarization: 
$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

# Balanced Fermi gases at unitarity



[Phase Transition to a normal phase for large magnetic field  
B. S. Chandrasekhar (1962), A. M. Clogston (1962)]

Experimental evidence of a critical value of  $P$

# Normal phase of polarized Fermi gas at unitarity

## **Assumption:**

at high polarization homogeneous phase,

NORMAL FERMI LIQUID: consider a very dilute mixture of spin- $\downarrow$  atoms immersed in non-interacting gas of spin- $\uparrow$  atoms

Energy expansion for small concentration  $x = \frac{n_{\downarrow}}{n_{\uparrow}}$

$$\frac{E(x)}{N_{\uparrow}} = \frac{3}{5} \epsilon_{F\uparrow} \left( 1 - Ax + \frac{m}{m^*} x^{5/3} + \dots \right) = \frac{3}{5} \epsilon_{F\uparrow} \epsilon(x)$$

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Non interacting gas

single-particle energy

quantum pressure  
of a Fermi gas of quasi-particles  
with an effective mass

>  $A$  and  $m^*$  determined by solving the  $N+1$  problem:  
good approximation is obtained  
by single particle-hole excitations

# Superfluid-Normal phase coexistence at unitarity

$$\frac{E(x)}{N_{\uparrow}} = \frac{3}{5} \epsilon_{F\uparrow} \left( 1 - Ax + \frac{m}{m^*} x^{5/3} + \underbrace{Bx^2}_{\text{interaction between quasi-particles}} \right) = \frac{3}{5} \epsilon_{F\uparrow} \epsilon(x)$$

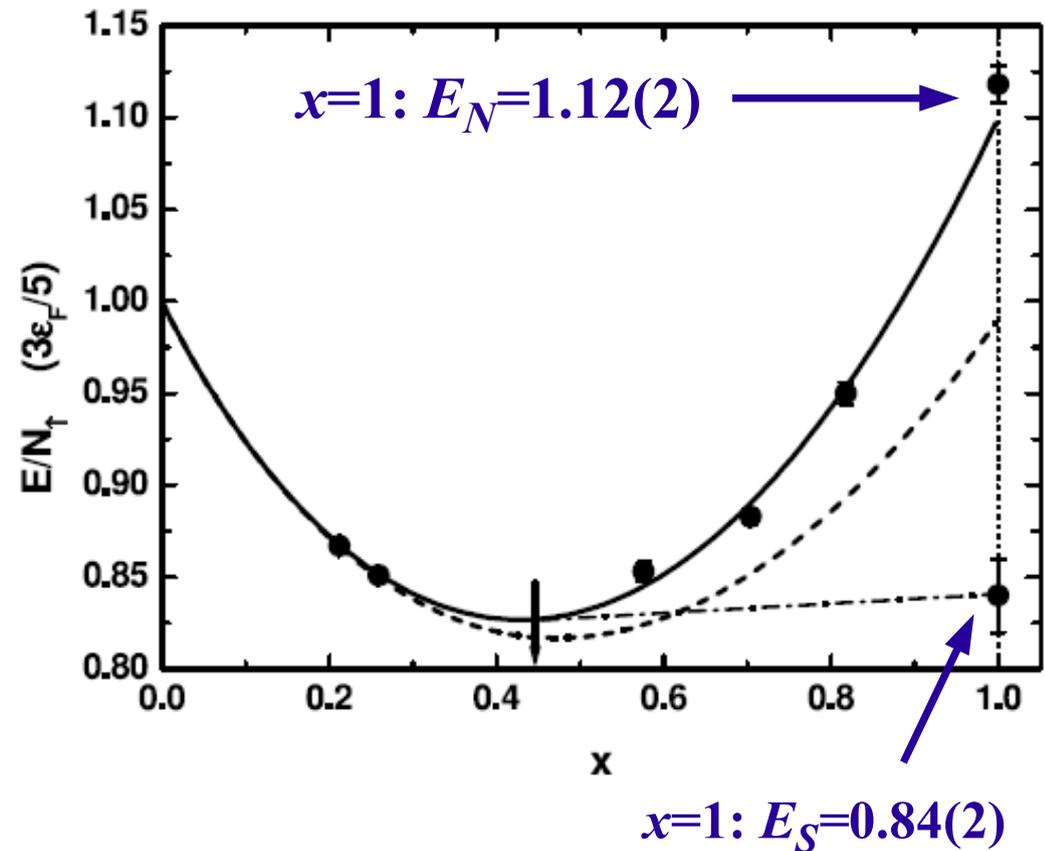
Most recent values using FN-QMC

$$A = 0.99(2)$$

$$m^*/m = 1.09(3)$$

$$B = 0.14$$

[S. Pilati and S. Giorgini,  
*Phys. Rev. Lett.* **100**, 030401 (2008)]



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$$\frac{E(x)}{N_{\uparrow}} = \frac{3}{5} \epsilon_{F\uparrow} \left( 1 - Ax + \frac{m}{m^*} x^{5/3} + \underbrace{Bx^2}_{\text{interaction between quasi-particles}} \right) = \frac{3}{5} \epsilon_{F\uparrow} \epsilon(x)$$

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Critical concentration  $x_c$ :

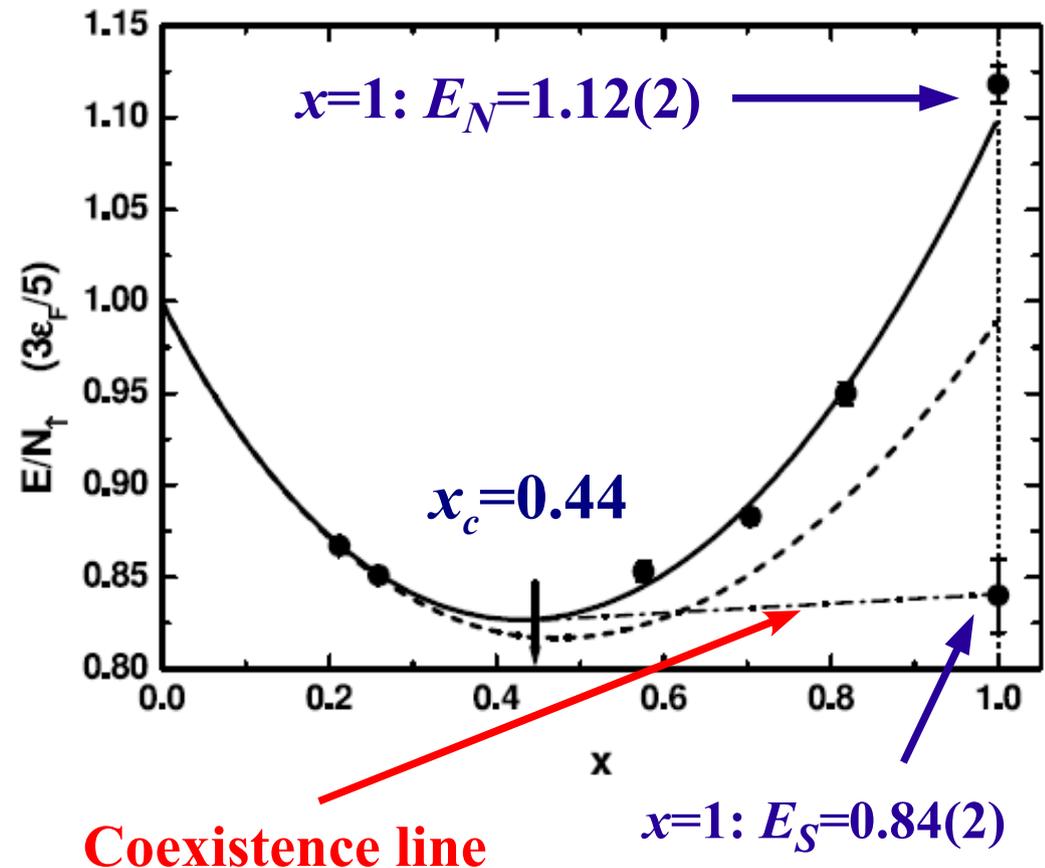
$$P_{\text{SF}} = P_{\text{N}}$$

$$\frac{\epsilon'(x_c)}{\epsilon(x_c)} = \frac{5}{3} \frac{\epsilon(x_c)^{3/5} - (2\xi_S)^{3/5}}{x_c - 1}$$

SF

N with  
 $x_c = 0.44$

Phase Separation



# Superfluid-Normal phase coexistence at unitarity

$$\frac{E(x)}{N_{\uparrow}} = \frac{3}{5} \epsilon_{F\uparrow} \left( 1 - Ax + \frac{m}{m^*} x^{5/3} + Bx^2 \right) = \frac{3}{5} \epsilon_{F\uparrow} \epsilon(x)$$

interaction between quasi-particles

Most recent values using FN-QMC

$$A = 0.99(2)$$

$$m^*/m = 1.09(3)$$

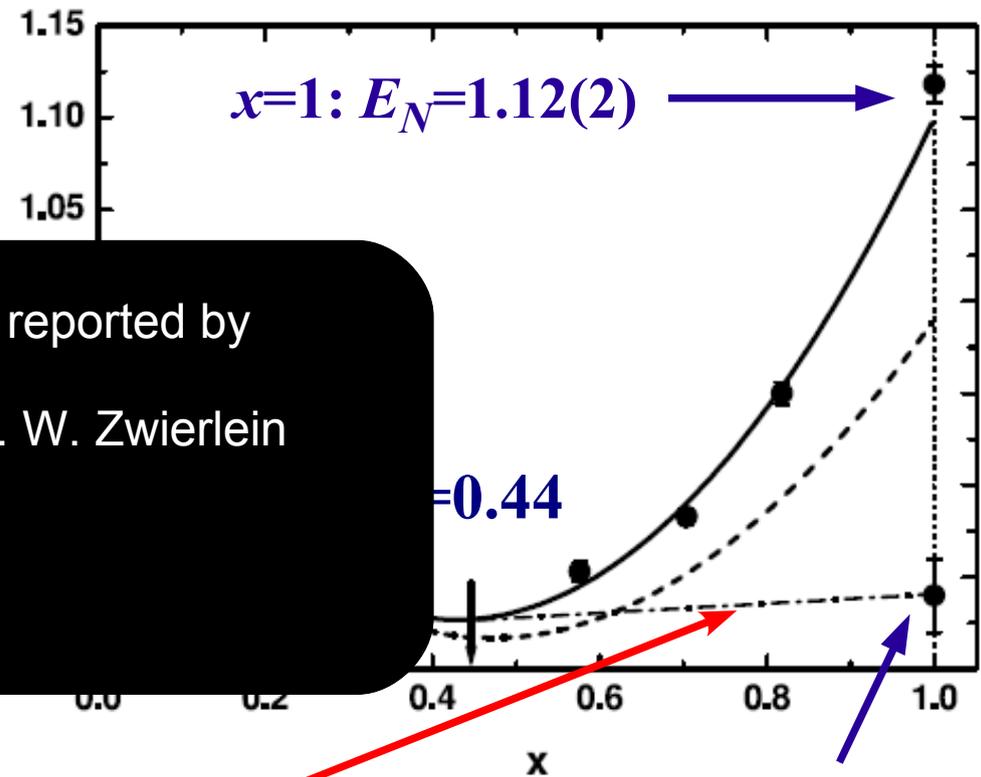
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[S. Pilati and S. Giorgini,  
*Phys. Rev. Lett.* **100**, 030401 (2008)]

First measurements of the coefficient A reported by

A. Schirotzek, C. Wu, A. Sommer, and M. W. Zwierlein  
*PRL* **102**, 230402 (2009)

$$A = 1.06(7)$$



Coexistence line

$$x=1: E_S = 0.84(2)$$

SF

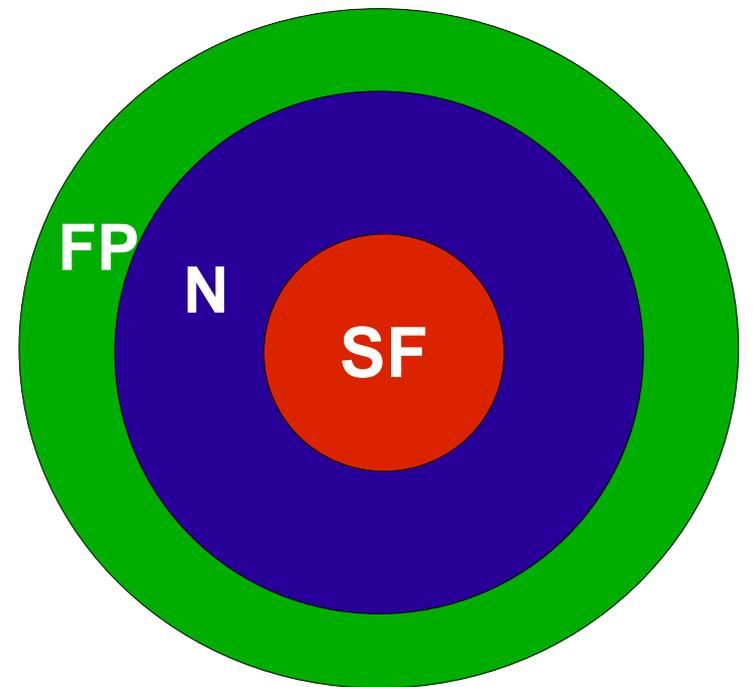
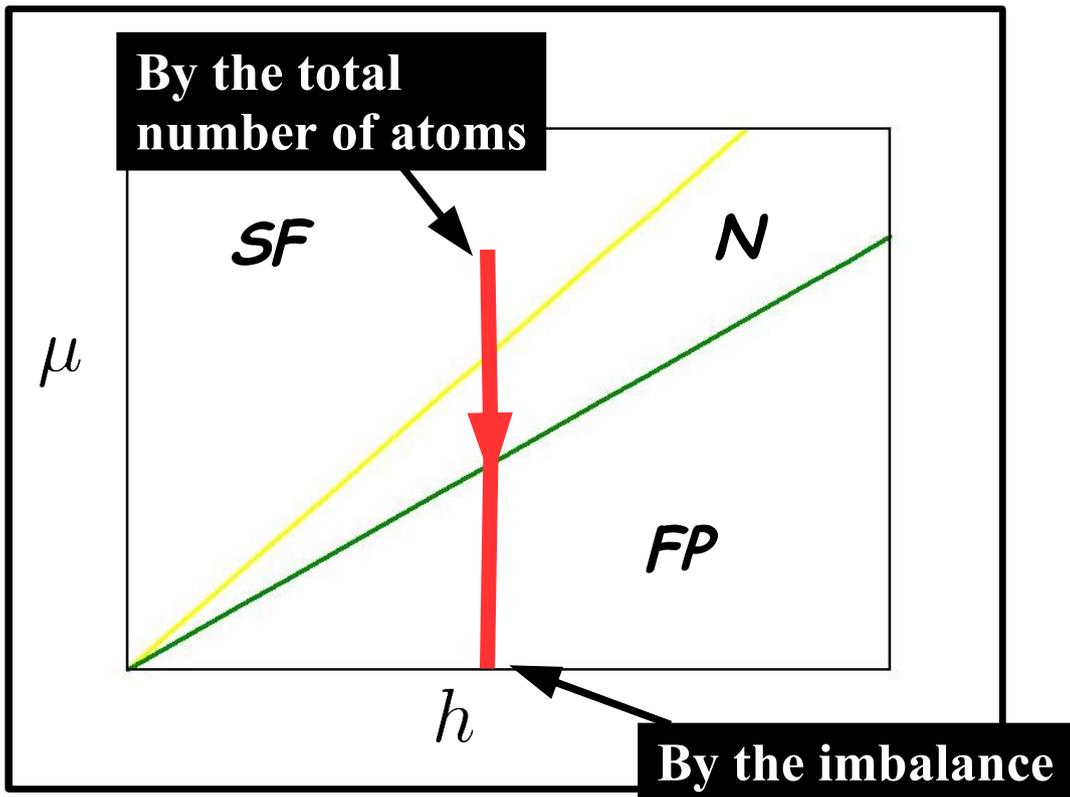
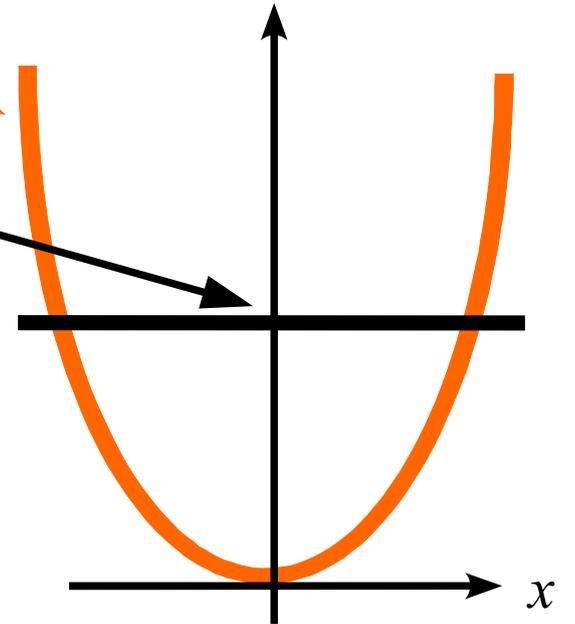
N with  
 $x_c = 0.44$

Phase Separation

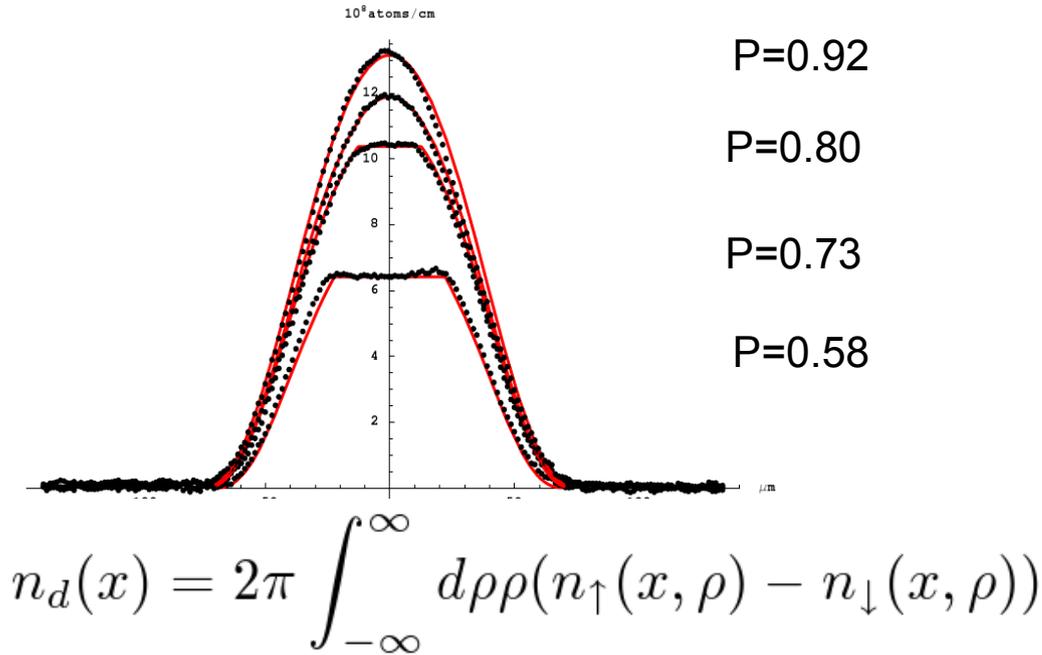
# Exploring Phase diagram in the Trap: LDA

LDA:  $\mu_\sigma(\mathbf{x}) = \mu_\sigma^0 - V(\mathbf{x}) = \mu_\sigma^0 - \frac{1}{2}m\omega x^2$

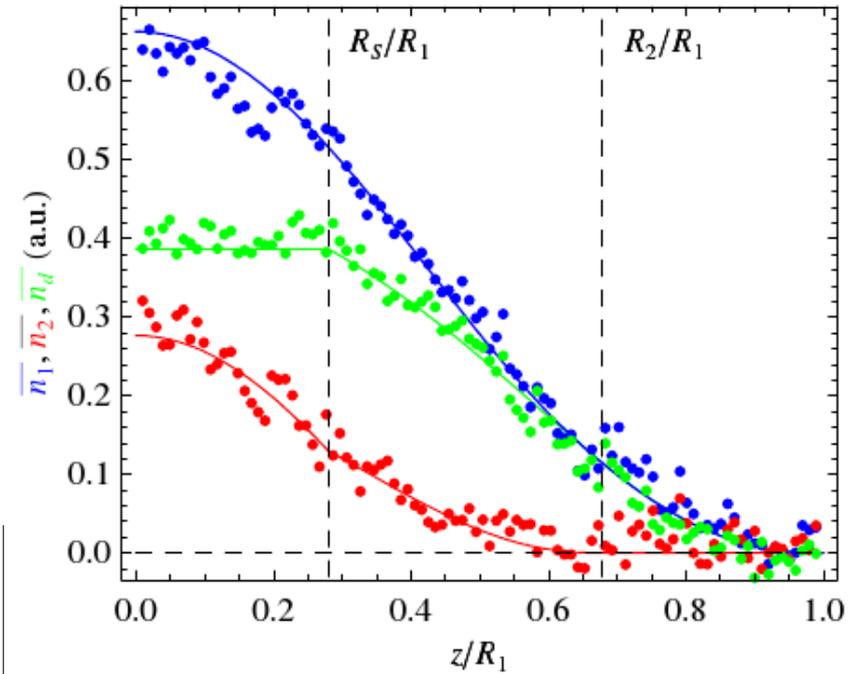
$\mu(\mathbf{x}) = \mu^0 - \frac{1}{2}m\omega x^2$  Decreasing outward  
 $h(\mathbf{x}) = h^0$  Constant also inside the trap



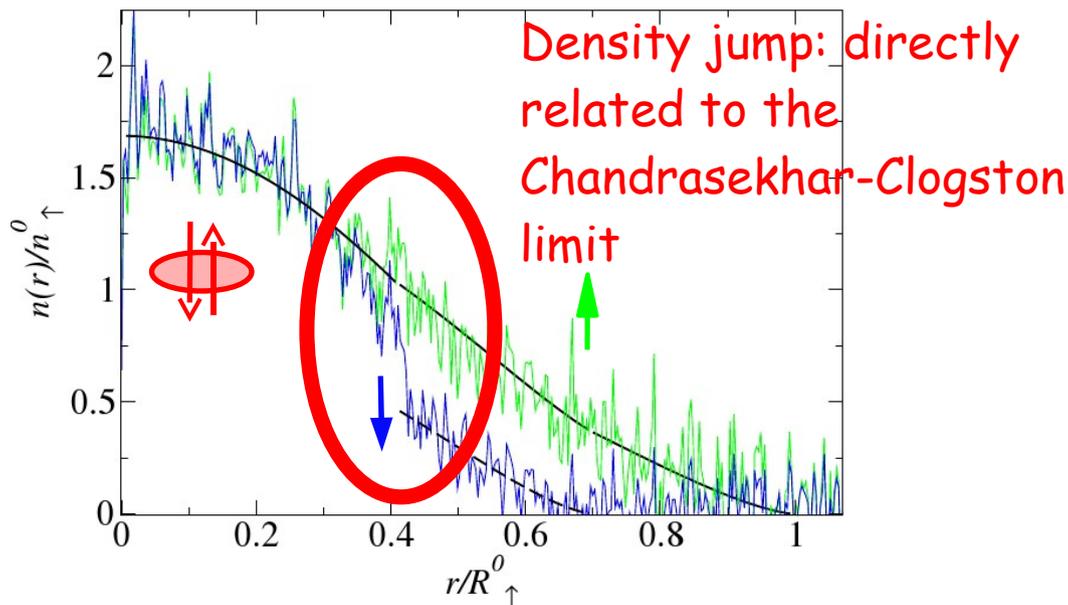
# Some Experimental results for trapped gases



[Exp. data from MIT]



[N. Navon et al. PRL 103, 18 (2009)]



Experiments agree (very) well with the description (smooth lines in the figures) of the polarized normalized phase in terms of polarons

[A. Recati, C. Lobo, S. Stringari, PRA 78, 023633 (2008)]

# *Polaron modes frequencies*

**Polaron effective Hamiltonian:**

*renormalized mass and trapping potential*

$$H_{sp} = \frac{p^2}{2m^*} + V(\mathbf{r}) \left( 1 + \frac{3}{5}A \right)$$

# *Polaron modes frequencies*

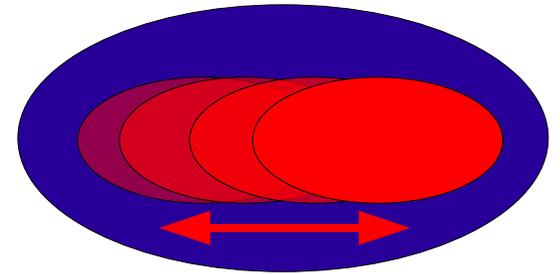
**Polaron effective Hamiltonian:**  
*renormalized mass and trapping potential*

$$H_{sp} = \frac{p^2}{2m^*} + V(\mathbf{r}) \left( 1 + \frac{3}{5}A \right)$$

Spin-dipole  
mode

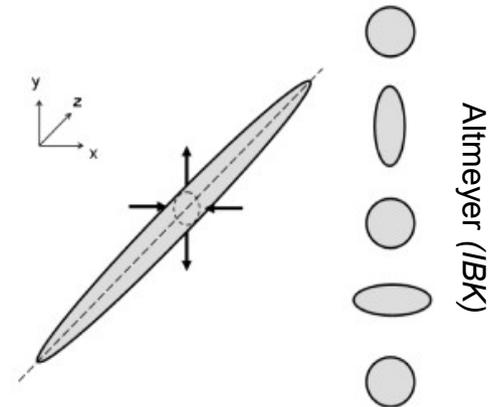
$$\omega_D^{(s)} = \omega_i \sqrt{\frac{m}{m^*} (1 + (3/5)A)}$$

$$\simeq 1.26\omega_i$$



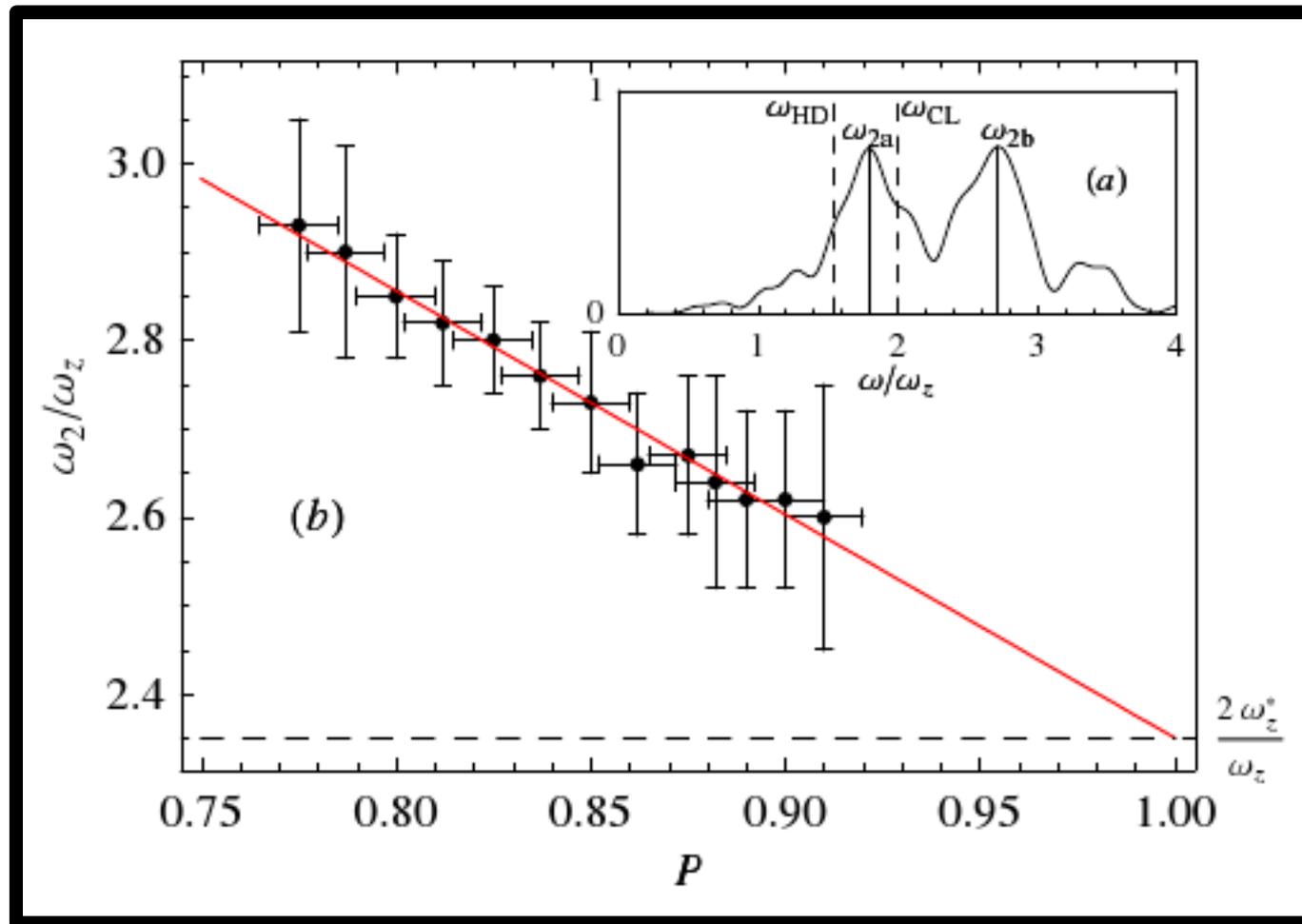
Spin-transverse-  
Quadrupole mode

$$\omega_Q^{(s)} = 2\omega_{\perp} \sqrt{\frac{m}{m^*} (1 + (3/5)A)}$$



# Finite $P$ (many-Polaron) modes frequencies

## Quadrupole compressional mode



[S. Nascimbene, et al. PRL **103**, 107402 (2009)]

# Finite P (many-Polaron) modes frequencies

Method:

Collective oscillation via variational principle  $\delta S = 0$  applied to the action

$$S = \int dt \langle \Psi | H - i\hbar \partial_t | \Psi \rangle = \int dt (E - \langle \Psi | i\hbar \partial_t | \Psi \rangle)$$

**We take the scaling ansatz (with 4+4 time dependent parameters):**

$$\psi_\sigma(r, z, t) = e^{-1/2(2\alpha_\sigma + \beta_\sigma)} \psi_\sigma^0(e^{-\alpha_\sigma} r, e^{-\beta_\sigma} z) e^{i(\chi_\sigma r^2 + \xi_\sigma z^2)}$$

- 1) Axially symmetric
- 2) Compressional modes of axial/radial nature

**Collective modes: equation of motion given by the second order expansion of S w.r.t. the scaling parameters (4-by-4 linear system).**

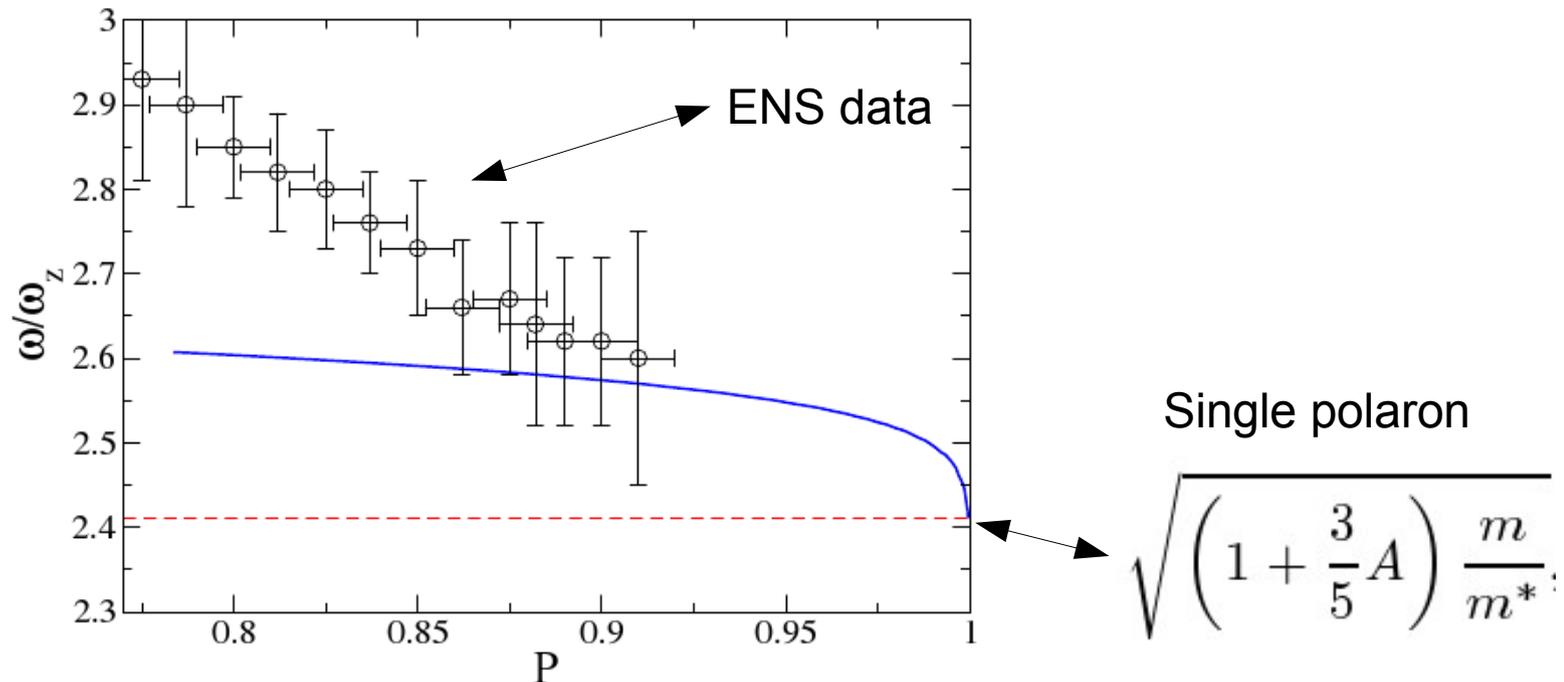
# Finite $P$ (many-Polaron) modes frequencies

First order expansion (virial- like expressions)

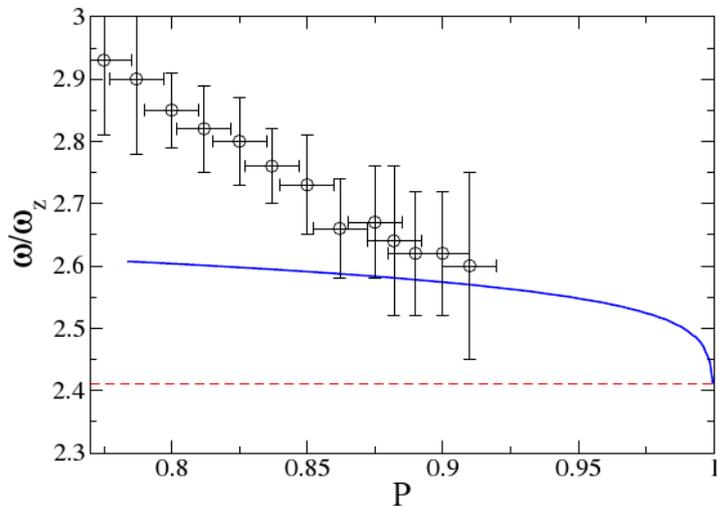
$$-\frac{4}{3} \int \frac{\tau_{\uparrow}}{2m} + N_{\uparrow} m \omega_{\perp}^2 \langle r^2 \rangle_{\uparrow} - N_{\downarrow} \frac{\hbar^2 (6\pi^2)^{2/3}}{2m} A \left( \langle r \partial_r n_{\uparrow}^{2/3} \rangle_{\downarrow} + \frac{4}{3} \langle n_{\uparrow}^{2/3} \rangle_{\downarrow} \right) = 0$$

$$-\frac{4}{3} \int \frac{\tau_{\downarrow}}{2m^*} + N_{\downarrow} m \omega_{\perp} \langle r^2 \rangle_{\downarrow} + N_{\downarrow} \frac{\hbar^2 (6\pi^2)^{2/3}}{2m} A \langle r \partial_r n_{\uparrow}^{2/3} \rangle_{\downarrow} = 0$$

Results for the axial compressional mode



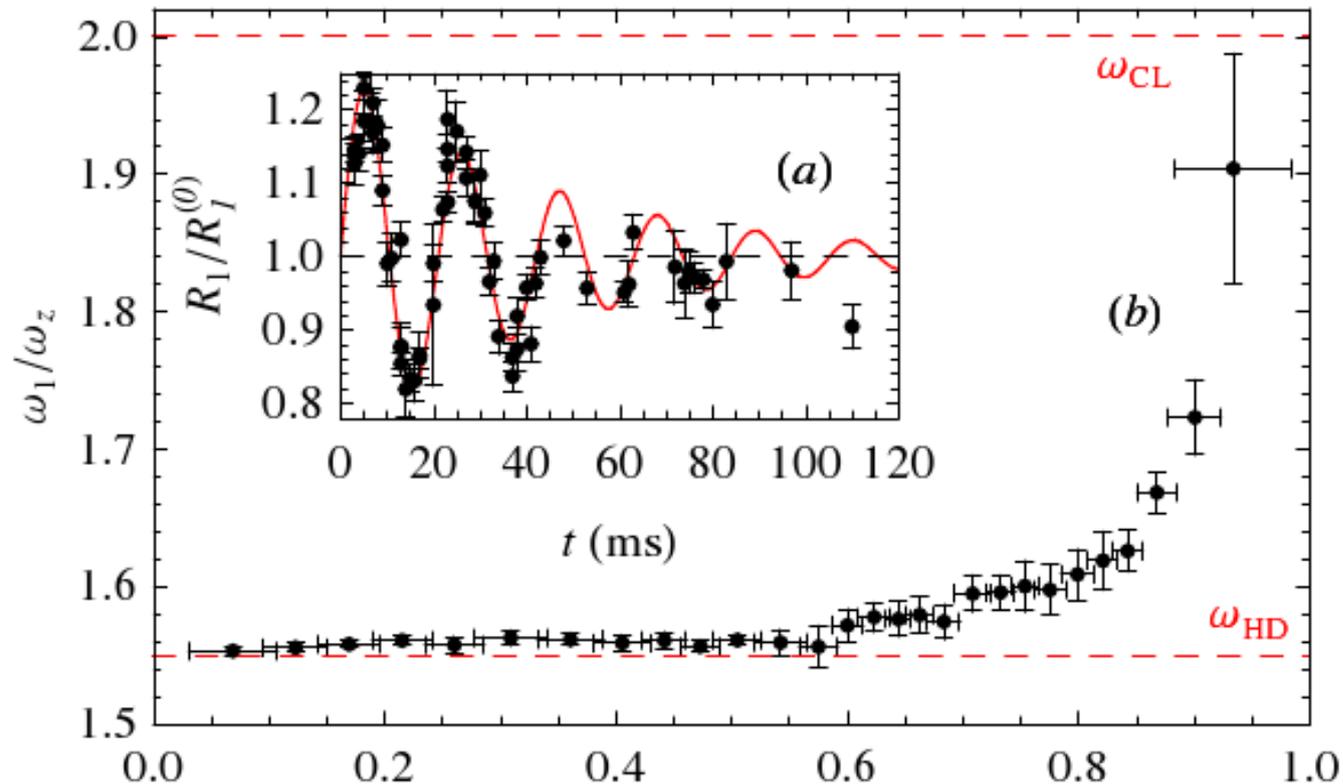
# Finite $P$ (many-Polaron) modes frequencies



Possible issue with the comparison:  
- Theory result for the **collisionless** regime.

- In the experiment **collision very effective**:  
Difficult to see minority oscillation

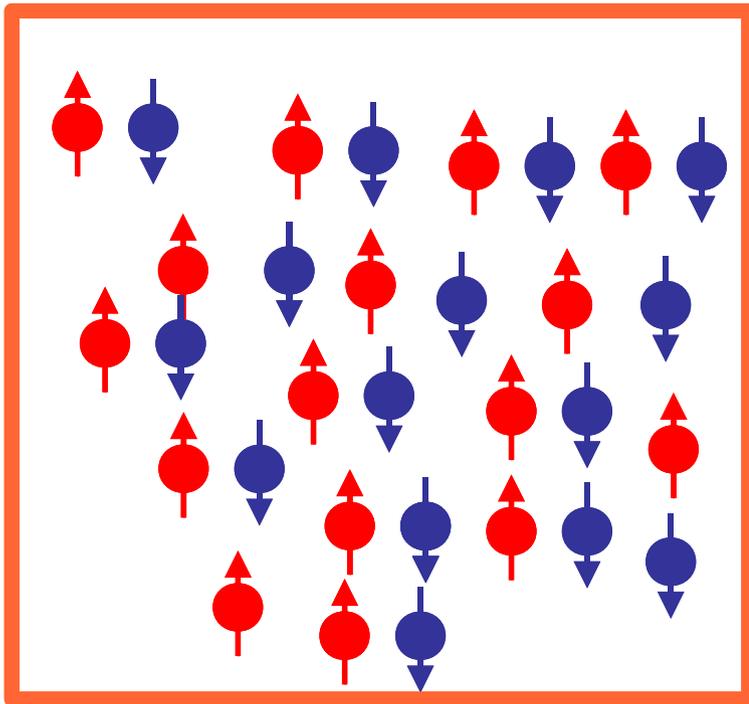
Majority component oscillation  
not in the collisionless regime



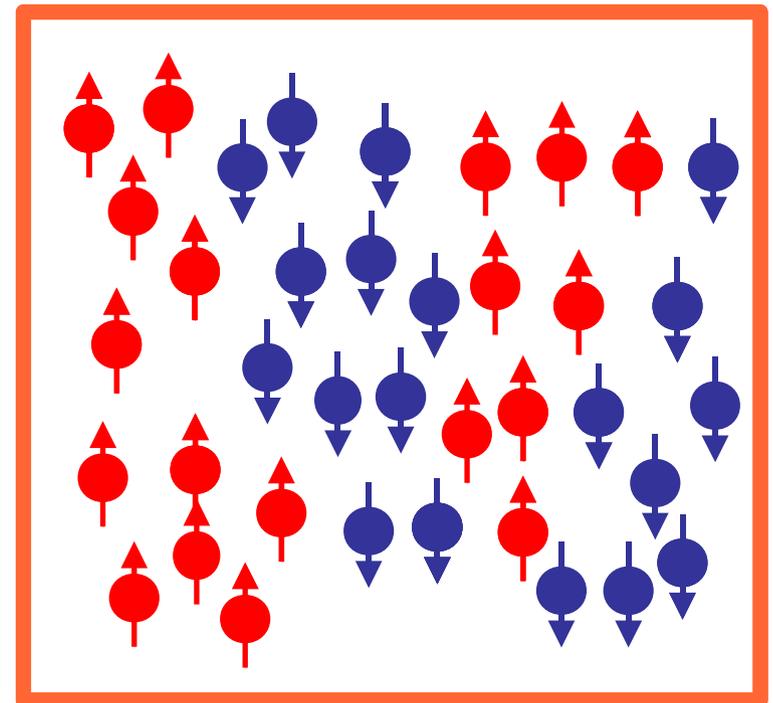
# Repulsive Fermi gas vs. Itinerant Ferromagnetism

For  $a > 0$  and  $k_F a$  small: **Fermi liquid**

When the interaction increases one can hope to reach the so-called **Itinerant Ferromagnetic Phase**



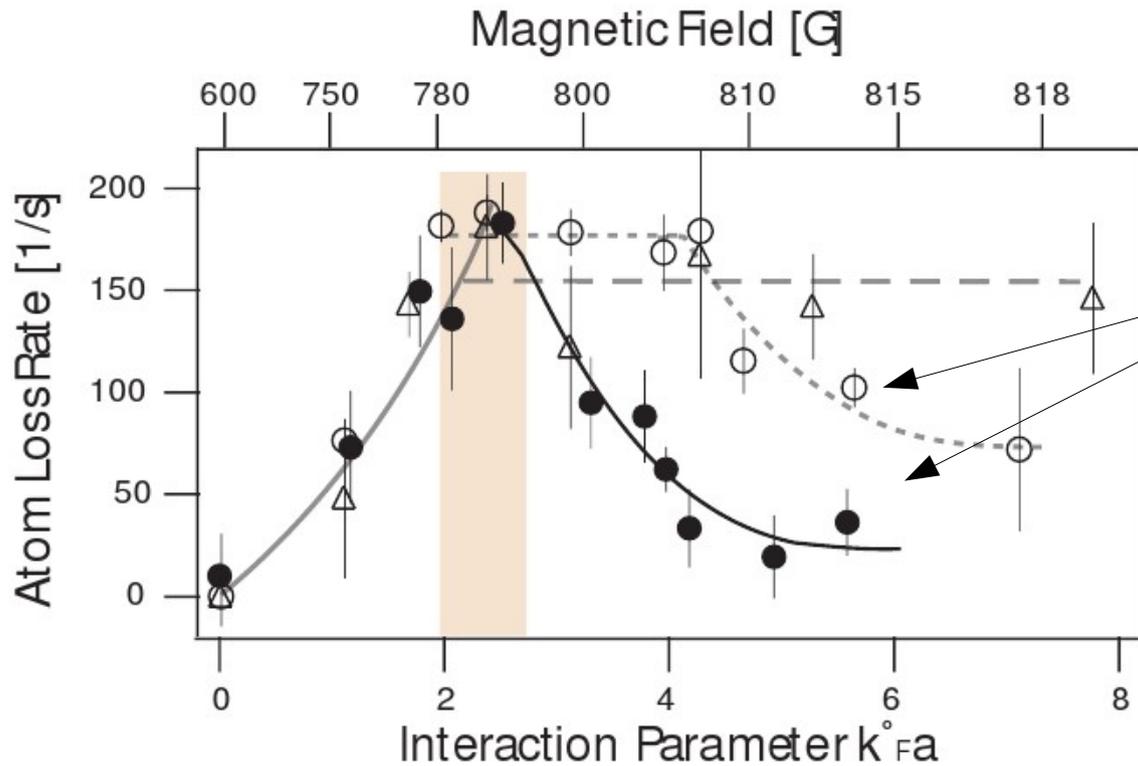
“Fermi liquid”



“Itinerant ferromagnetism”

# Repulsive Fermi gas vs. Itinerant Ferromagnetism

Recent experiment at MIT [G.-B. Jo et al., Science **325**, 1521 (2009)]



Decrease due probably to (local) polarization of the gas - first in the center

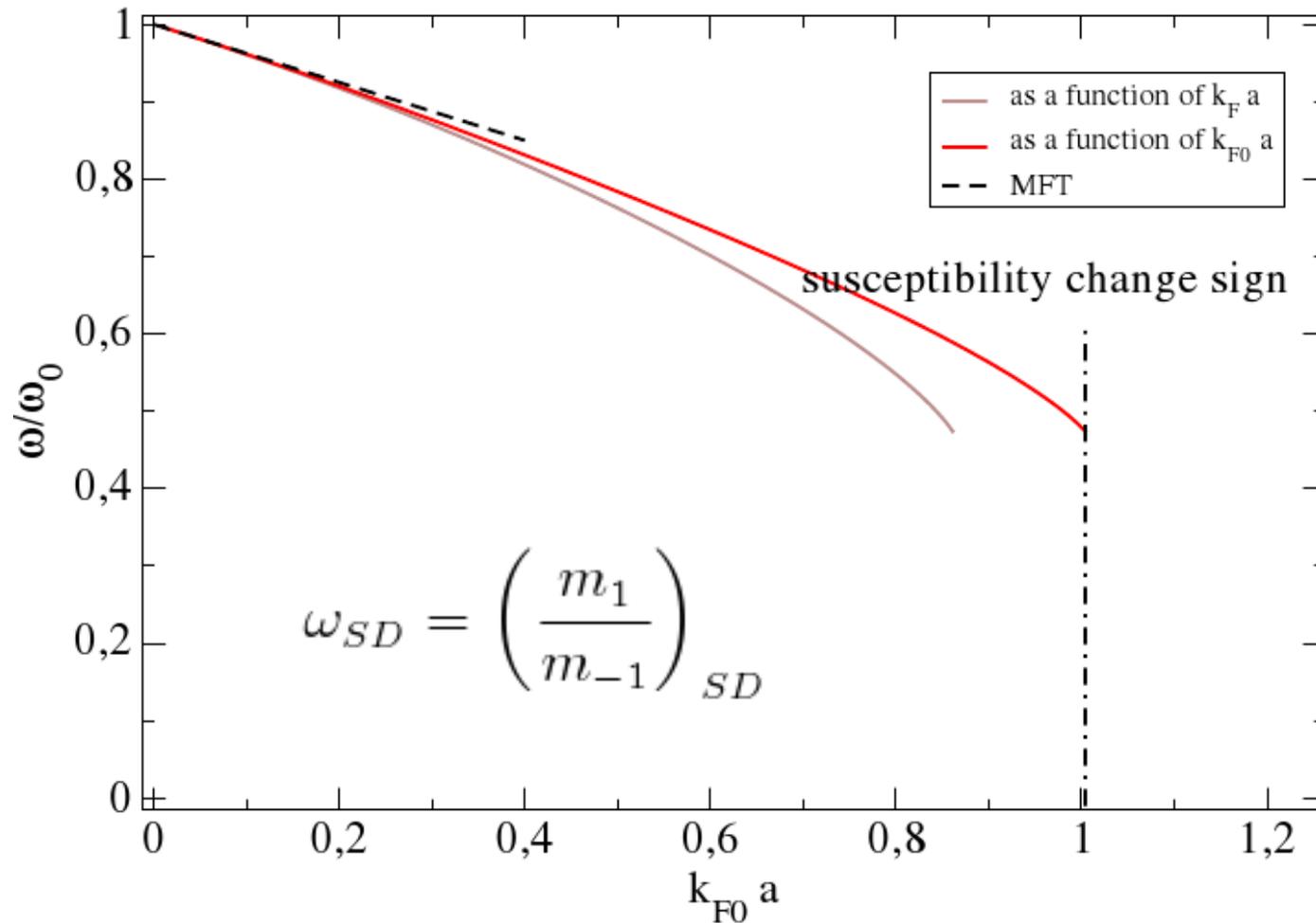
Note that is not a direct measurement of any magnetic property.  
How can we get more insight?

Can we get, e.g., any infos on the magnetic susceptibility of the system?

$$\chi = n^2 \left( \frac{\partial^2 A/V}{\partial P^2} \right)^{-1}$$

# Spin-dipole mode of a repulsive Fermi gas

Using the previous energy functional to calculate the spin-dipole frequency in a trap:



$$\omega_{SD} = \left( \frac{m_1}{m_{-1}} \right)_{SD}$$

$$m_{-1} = \int z^2 n^2 \left( \frac{\partial^2 \epsilon}{\partial P^2} \right)^{-1}$$

# Conclusions

Normal phase of polarized Fermi gas at Unitarity  
[A.Recati, S.Stringari, PRA82, 013635 (2010)]

An elastic theory (collisionless) of the mode of a Fermi gas at Unitarity as a function of its polarization gives a behaviour understandable in terms of the size of the minority cloud, BUT in disagreement with recent experiments.

*Are collisions responsible for?*

*Measuring transverse modes could give a partial answer and is a very important test for the theory of the normal phase when applied to dynamics*

Normal phase of repulsive Fermi gas and itinerant ferromagnetism  
[A.Recati, S.Stringari, arXiv:1007.4504]

*The spin dipole mode frequency – and the spin fluctuations - of a repulsive Fermi gas represent a direct measurement of magnetic properties of such a system.*

We calculate it within MF to  $O(k_F a)$  and to all orders using a functional theory for  $P \sim 0$  built using the available MC data.

*Is the lifetime of the system long enough? Probably.*

**Post-Doc position  
available!**

**A new experimental activity on ultracold atoms is starting at the  
CNR- BEC Centre, University of Trento**

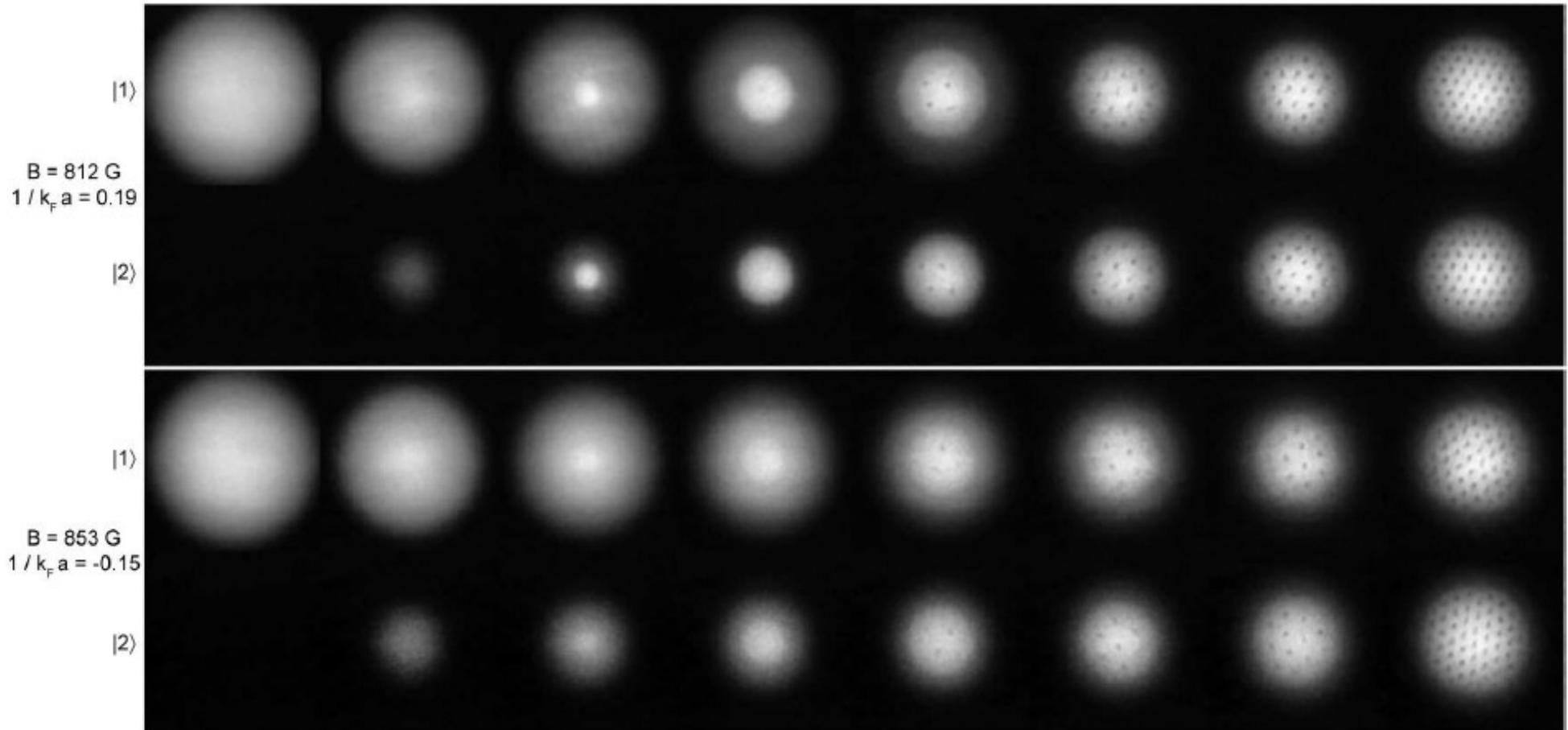
**Focus will be on:**

- 1) pure and mixed quantum gases (Fermi - Bose, Bose – Bose)**
- 2) Fermionic superfluidity**
- 3) transport phenomena**



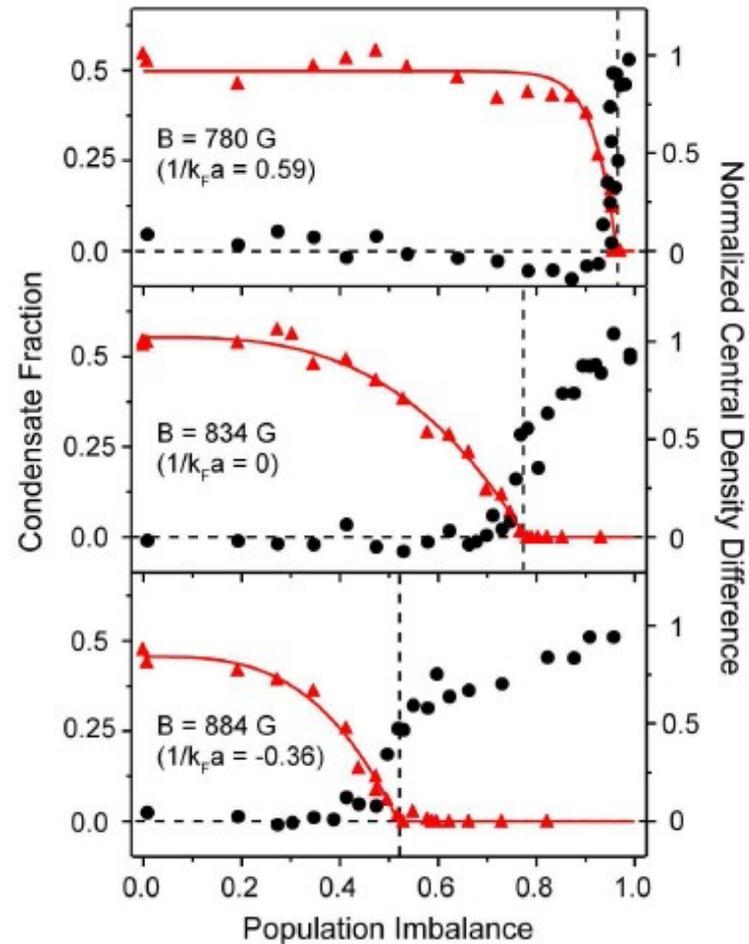
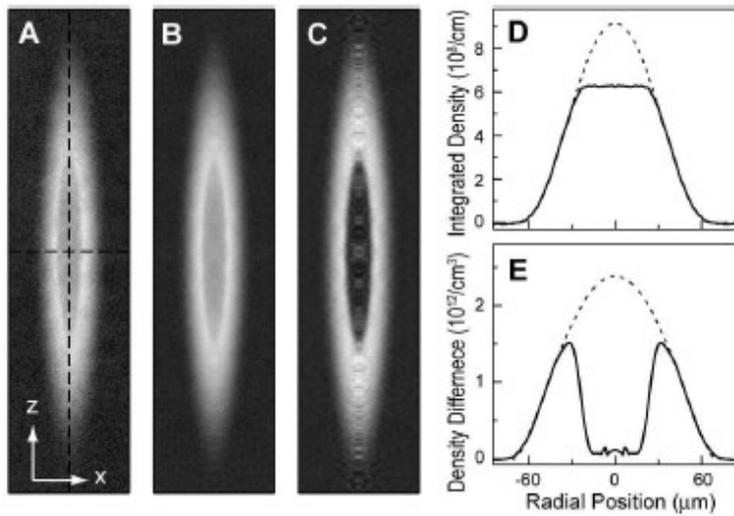
**Contact: GABRIELE FERRARI [ferrari@lens.unifi.it](mailto:ferrari@lens.unifi.it)**

# Recent Experiments on imbalanced Fermi gases at unitarity



MIT, Science **311**, 492 (2006)

# Recent Experiments on imbalanced Fermi gases at unitarity



BEC

Unitarity

BCS

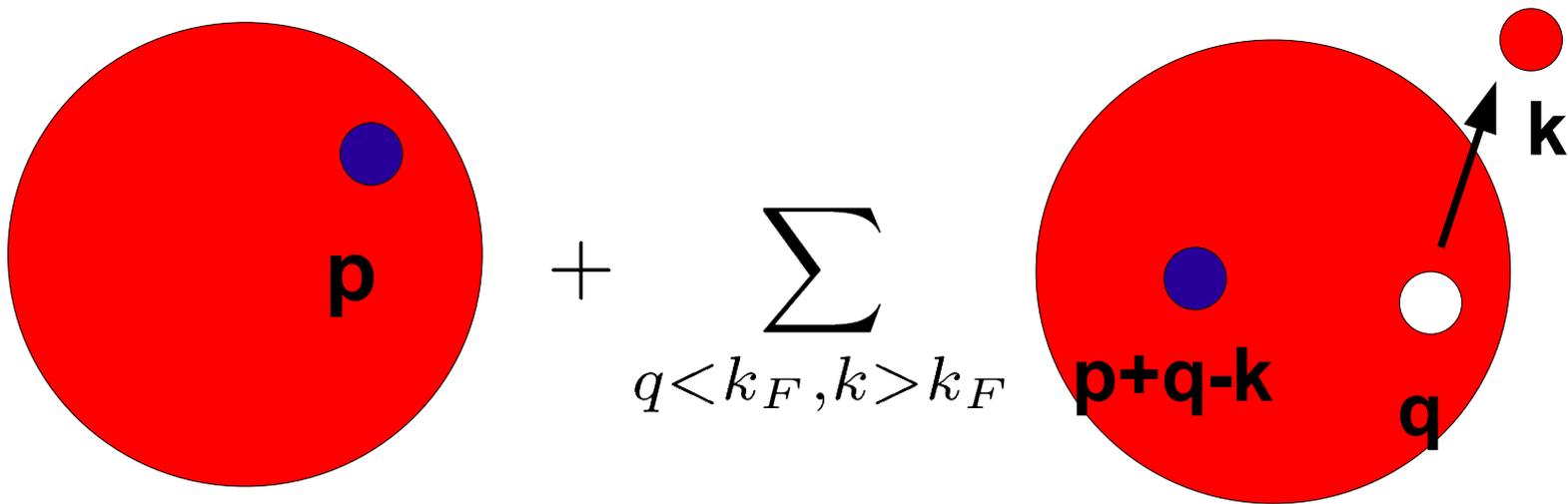
[MIT, Phys. Rev. Lett. **97**, 030401 (2006)]

# Normal phase of polarized Fermi gas at unitarity

Consider a SINGLE down atom interacting with an ideal Fermi gas (up-atoms).

Variational Ansatz (single particle hole excitations):

$$|\psi\rangle = \phi_0 |\mathbf{p}\rangle_{\downarrow} |0\rangle_{\uparrow} + \sum_{\substack{k > k_F \\ q < k_F}} \phi_{\mathbf{qk}} |\mathbf{p} + \mathbf{q} - \mathbf{k}\rangle_{\downarrow} c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}\uparrow} |0\rangle_{\uparrow}$$

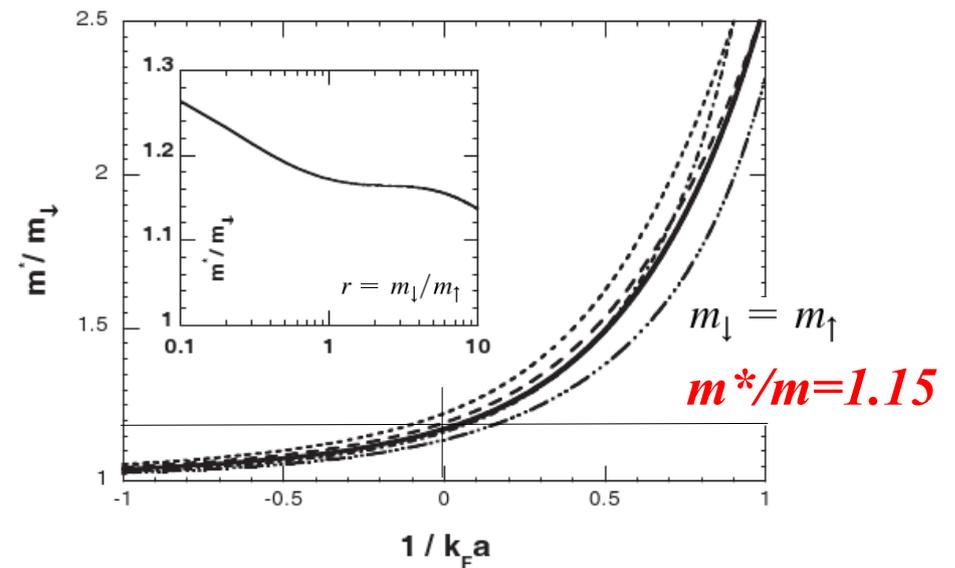
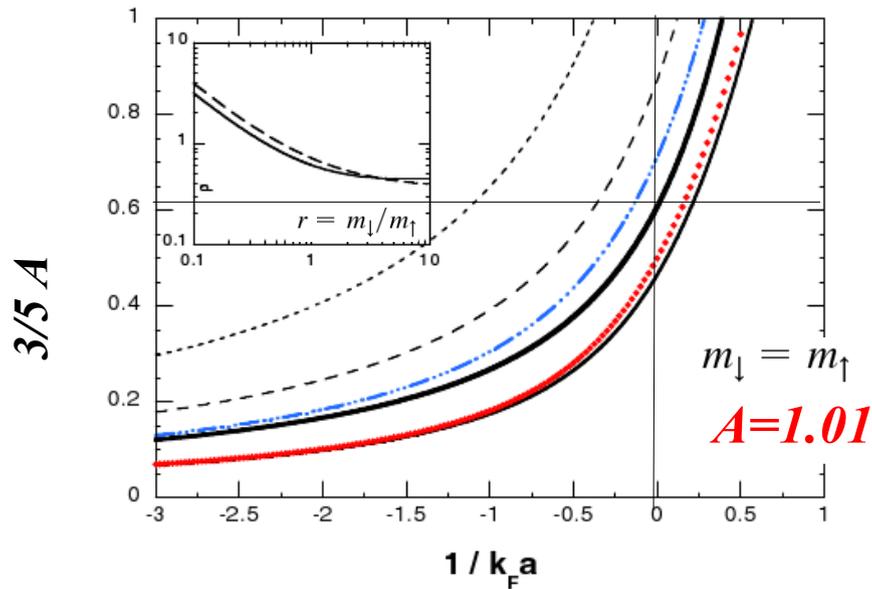


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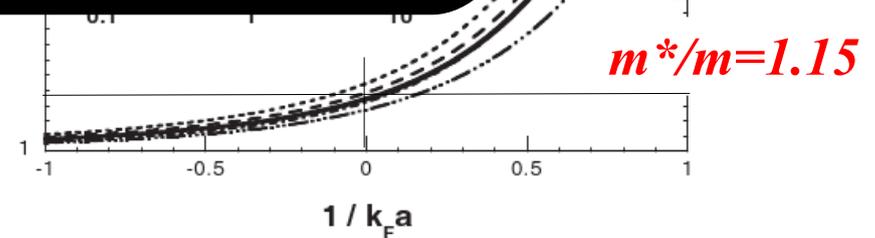
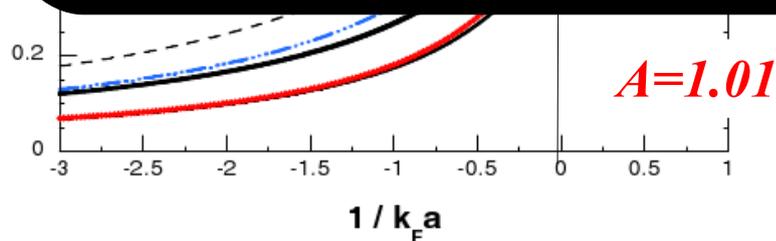
$$|\psi\rangle = \phi_0 |\mathbf{p}\rangle_{\downarrow} |0\rangle_{\uparrow} + \sum_{k > k_F} \phi_{\mathbf{qk}} |\mathbf{p} + \mathbf{q} - \mathbf{k}\rangle_{\downarrow} c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}\uparrow} |0\rangle_{\uparrow}$$

First measurements of the coefficient  $A$  reported by

A. Schirotzek, C. Wu, A. Sommer, and M. W. Zwierlein  
arXiv:0902.3021

$$A = 1.06(7)$$

$3/5 A$



**Note:** it is equivalent to a T-matrix approach

$$\omega - \epsilon_{\downarrow, k} + \mu_{\downarrow} - \Sigma(k, \omega) = 0 \quad \longrightarrow$$

$$\mu_{\downarrow} = \Sigma(0, 0) \quad \& \quad \frac{m^*}{m_{\downarrow}} = \frac{1 - \frac{\partial \Sigma}{\partial \omega}}{1 - 2m_{\downarrow} \frac{\partial \Sigma}{\partial k^2}}$$

# Finite P (many-Polaron) modes frequencies

Method:

Collective oscillation via variational principle  $\delta S = 0$  applied to the action

$$S = \int dt \langle \Psi | H - i\hbar \partial_t | \Psi \rangle = \int dt (E - \langle \Psi | i\hbar \partial_t | \Psi \rangle)$$

We write the energy functional as:

Equilibrium:  
Normal Phase Energy Functional

$$E = \sum_{\sigma} \int d\mathbf{x} \left( \frac{\tau_{\sigma}}{2m} + \frac{m}{2} (\omega_{\perp}^2 r^2 + \omega_z^2 z^2) n_{\sigma} \right) + \frac{3}{5} A \frac{\hbar^2 (6\pi^2)^{2/3}}{2m} \int d\mathbf{x} n_{\downarrow} n_{\uparrow}^{2/3} + a \int d\mathbf{x} \left( \frac{\tau_{\downarrow}}{2m} - \frac{n_{\downarrow} i_{\uparrow}^2}{2m n_{\uparrow}^2} \right)$$

$\tau_{\sigma} = \hbar^2 (6\pi^2 n_{\sigma})^{2/3} n_{\sigma}$

counter current term  
(Galilean Invariance)

## Decaying time of the collective modes

We consider the *momentum relaxation* of an homogeneous highly polarized Fermi gas.

$$\frac{d\mathbf{P}_{\downarrow}}{dt} = -\frac{\mathbf{P}_{\downarrow}}{\tau_{\mathbf{P}}}$$

The minority component have a mean momentum  $\mathbf{k}$  with respect to the majority one:  
total momentum per unit volume  $\mathbf{P}_{\downarrow} = n_{\downarrow} \mathbf{k}$

$$\frac{d\mathbf{P}_{\downarrow}}{dt} = -2\pi \frac{|U|^2}{V^3} \sum_{\mathbf{p}, \mathbf{p}', \mathbf{q}} \mathbf{p} [n_{\mathbf{p}} n_{\mathbf{p}'} (1 - n_{\mathbf{p}-\mathbf{q}}) (1 - n_{\mathbf{p}'+\mathbf{q}}) - n_{\mathbf{p}-\mathbf{q}} n_{\mathbf{p}'+\mathbf{q}} (1 - n_{\mathbf{p}}) (1 - n_{\mathbf{p}'})] \delta(\epsilon_{\mathbf{p}} + \epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}-\mathbf{q}} - \epsilon_{\mathbf{p}'+\mathbf{q}})$$

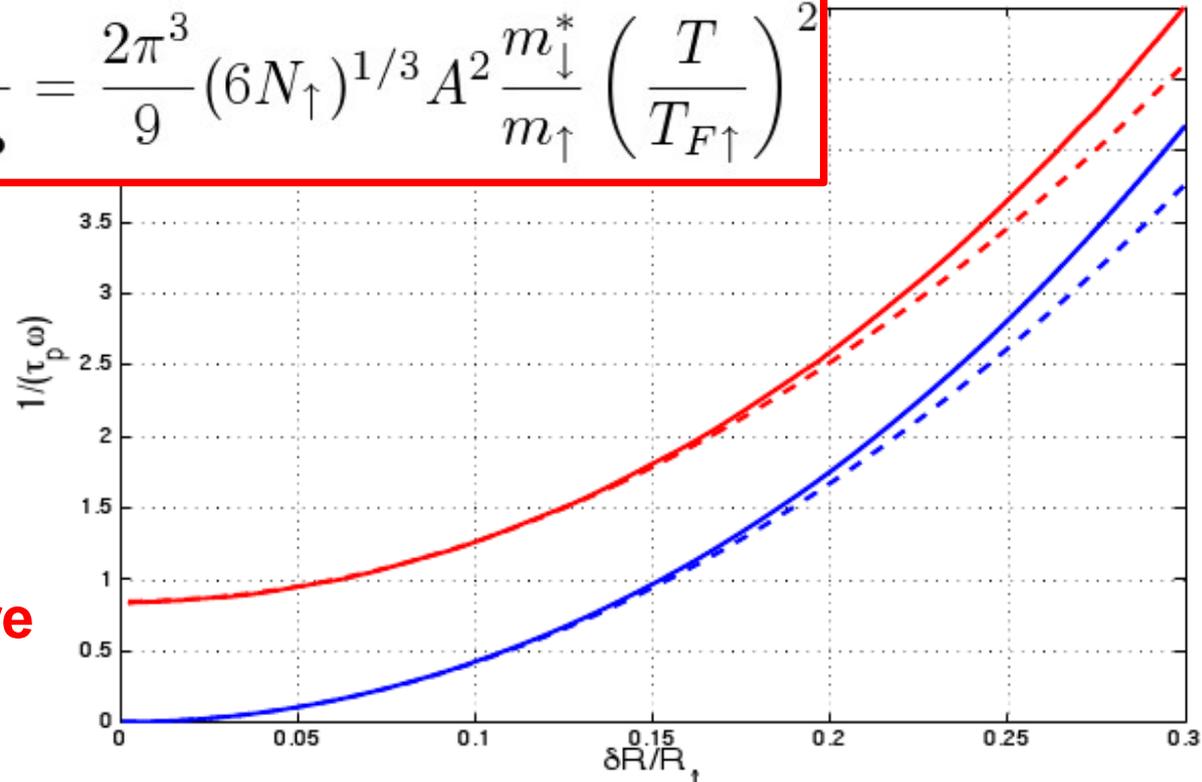
$$\begin{array}{ccc}
 & n_{\mathbf{p}_{\downarrow}} = f[\beta(\epsilon_{\mathbf{p}_{\downarrow}} - \mathbf{p} \cdot \mathbf{v} - \mu_{\downarrow})] & \\
 \epsilon_{\mathbf{p}_{\downarrow}} = p^2/2m_{\downarrow}^* & \mathbf{p}_{\downarrow} & \mathbf{p} - \mathbf{q}_{\downarrow} \\
 & \searrow & \nearrow \\
 \epsilon_{\mathbf{p}'_{\uparrow}} = p'^2/2m_{\uparrow} & \mathbf{p}'_{\uparrow} & \mathbf{p}' + \mathbf{q}_{\uparrow} \\
 & \nearrow & \searrow \\
 & n_{\mathbf{p}'_{\uparrow}} = f[\beta(\epsilon_{\mathbf{p}'_{\uparrow}} - \mu_{\uparrow})] & 
 \end{array}$$

## Decaying time of the collective modes

- $\left\{ \begin{array}{l} \omega_D \tau_P \gg 1 \\ \omega_D \tau_P \ll 1 \end{array} \right.$  **Collisionless regime: possible to see the dipole mode**
- Hydrodynamic regime: the dipole mode overdamped**

$$\delta R/R_{\downarrow} \ll T/T_{F\downarrow} : \frac{1}{\omega \tau_P} = \frac{2\pi^3}{9} (6N_{\uparrow})^{1/3} A^2 \frac{m_{\downarrow}^*}{m_{\uparrow}} \left( \frac{T}{T_{F\uparrow}} \right)^2$$

**MIT lowest temperature**



$$T = 0 : \frac{1}{\omega \tau_P} = \frac{8\pi}{25} (6N_{\uparrow})^{1/3} A^2 \frac{m_{\downarrow}^*}{m_{\uparrow}} \left( \frac{T_{F\downarrow}}{T_{F\uparrow}} \right)^2 \left( \frac{\delta R}{R_{\uparrow}} \right)^2$$

# Spin-dipole mode of a balanced Fermi gas

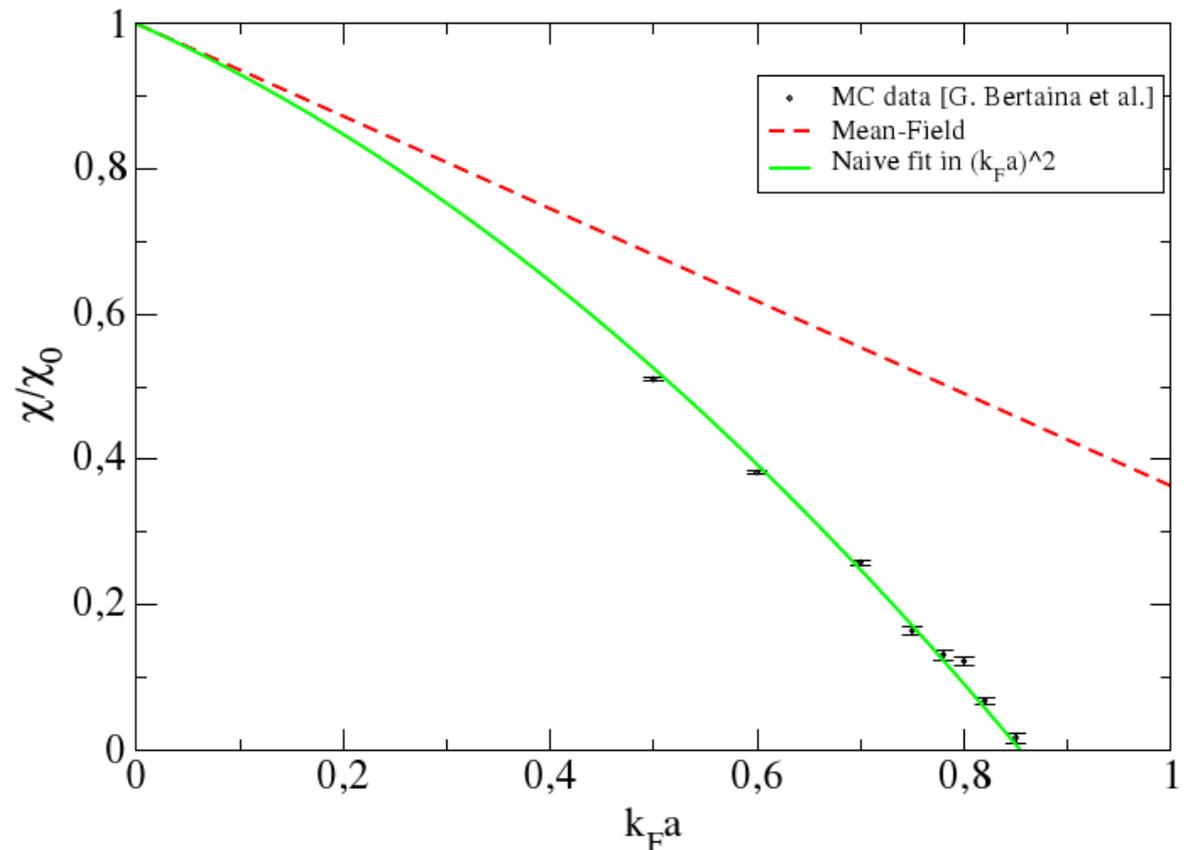
Interestingly with the same parameter and a dimensional argument we can fit pretty well also the spin susceptibility!

$$\frac{E}{V} = \frac{E_{MFT}}{V} + \frac{3}{5} N \epsilon_F C_E (k_F a)^2 (1 - P)^{7/6} (1 + P)^{7/6}$$



$$\frac{\chi}{\chi_0} = 1 - \frac{2}{\pi} k_F a - C_\chi (k_F a)^2$$

where  $C_\chi = \frac{10}{21} C_E$



# Spin-dipole mode of a balanced Fermi gas

Theoretically we expect that the spin susceptibility goes to infinity



the spin-dipole mode (could) become soft

At the MF level:

$$\frac{E_{MFT}}{V} = \frac{3}{5} N \epsilon_F \left( \frac{1}{2} (1+P)^{5/3} + \frac{1}{2} (1-P)^{5/3} \right) + \frac{10}{9\pi} k_F a (1-P)(1+P)$$

Inverse Susceptibility:  $\frac{\partial^2 E}{\partial P^2} \propto 1 - \frac{2}{\pi} k_F a$    $(k_F a)_{MF} \sim \pi/2$

Let us consider the axial (along  $z$ ) spin-dipole mode

$$\omega_{SD} = \left( \frac{m_1}{m_{-1}} \right)_{SD}$$

and  $\epsilon[n, P]$  be the energy functional of the system, we have:

$$m_{-1} = \int z^2 n^2 \left( \frac{\partial^2 \epsilon}{\partial P^2} \right)^{-1}$$

i.e., it depends on an integrated spin-susceptibility

# Spin-dipole mode of a balanced Fermi gas

Mean-field

$$\frac{E_{MFT}}{V} = \frac{3}{5} N \epsilon_F \left( \frac{1}{2} (1+P)^{5/3} + \frac{1}{2} (1-P)^{5/3} \right) + \frac{10}{9\pi} k_F a (1-P)(1+P)$$

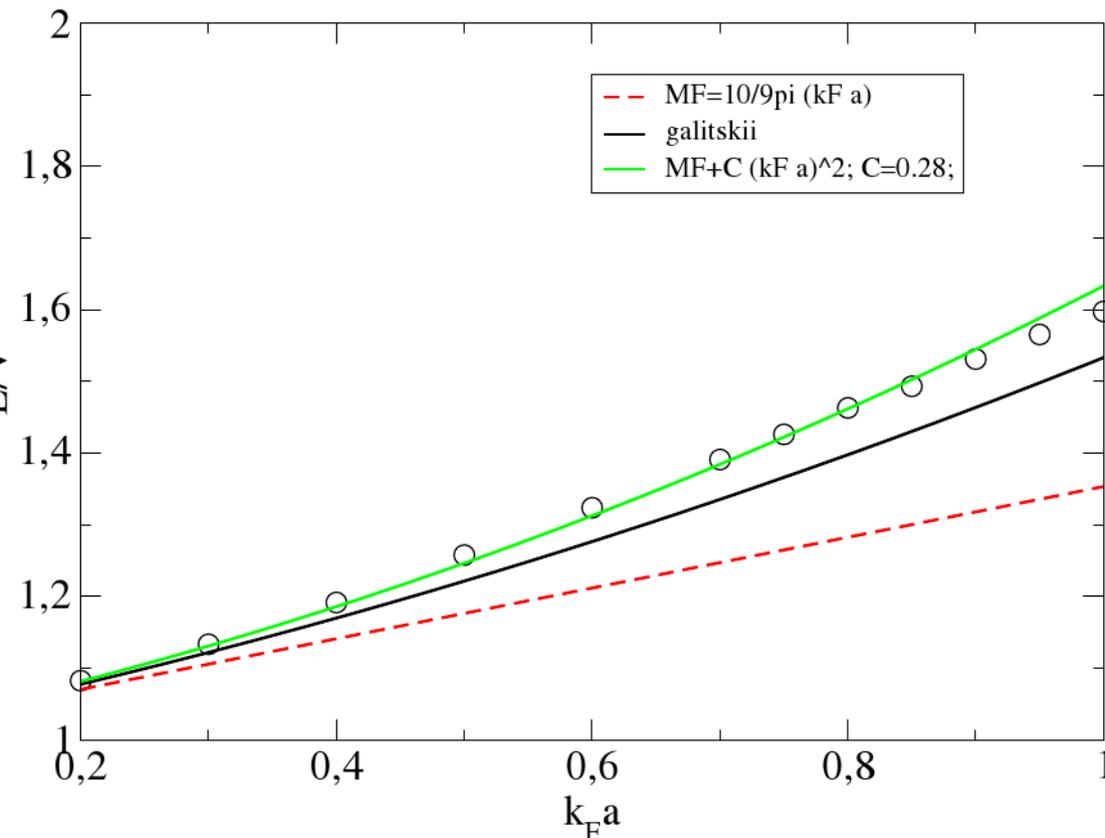
$$\omega_{SD,MF} = \omega_{ho} \left( 1 - \frac{2^7}{35\pi^2} k_{F,0} a \right) = \omega_z \left( 1 - \frac{128\sqrt{2}(3N)^{1/6}}{35\pi^2} \frac{a}{a_{ho}} \right)$$

Building a simple energy functional from MC recent data:

[G. Bertaina et al., arXiv:1004.1169 ]

$$\frac{E}{V} = \frac{3}{5} N \epsilon_F \left( 1 + \frac{10}{9\pi} k_F a + C_E (k_F a)^2 \right)$$

& the same for the susceptibility..



# Spin-fluctuations in a repulsive Fermi gas

Let us consider a part of the sample of which we measure the relative number fluctuations:

$$\frac{\Delta(N_{\uparrow} - N_{\downarrow})^2}{N} = k_B T \frac{\chi(T)}{n}$$

**At low-T and close to Stoner instability they diverge since  $\chi$  has a pole.**

*But what about the quantum fluctuations, which could dominate at very low-T and small N?*

Quantum fluctuations scales differently with  $N$ :

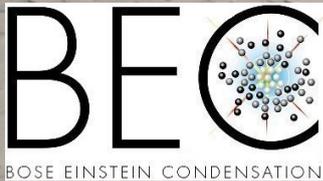
$$\frac{\Delta(N_{\uparrow} - N_{\downarrow})^2}{N} = 2\alpha \left( \frac{12}{\pi^4 N} \right)^{1/3} \ln(C N^{1/3})$$

Where  $\alpha$  is the low-q behaviour of the static structure factor.

**We find, within Landau theory that it also diverges at the Stoner instability but only **logarithmically****

# Post-Doc positions available!!

A new experimental activity on ultracold atoms is starting at *INO-BEC CNR Centre, Trento Univ.*



Focus will be on:

- i) pure and mixed quantum gases (Fermi - Bose, Bose -Bose)
- ii) Fermionic superfluidity
- iii) transport phenomena

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