Entanglement and sensitivity in precision measurements with a fluctuating number of particles

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Introduction

Aims of the project

Extend results on phase estimation with linear interferometers regarding

- Shot-noise limit vs. Spin-Squeezing and Entanglement
- Heisenberg limit

to probe states with a non-fixed number of particles *N*

Jessen (cold atoms) *PRL 2007*
Oberthaler (BEC) *Nature 2008 and 2010*
Polzik (cold atoms) *PNAS USA 2009*
Vuletic (cold atoms) *PRL 2010*
Treutlein (BEC on atom chip) *Nature 2010*
…

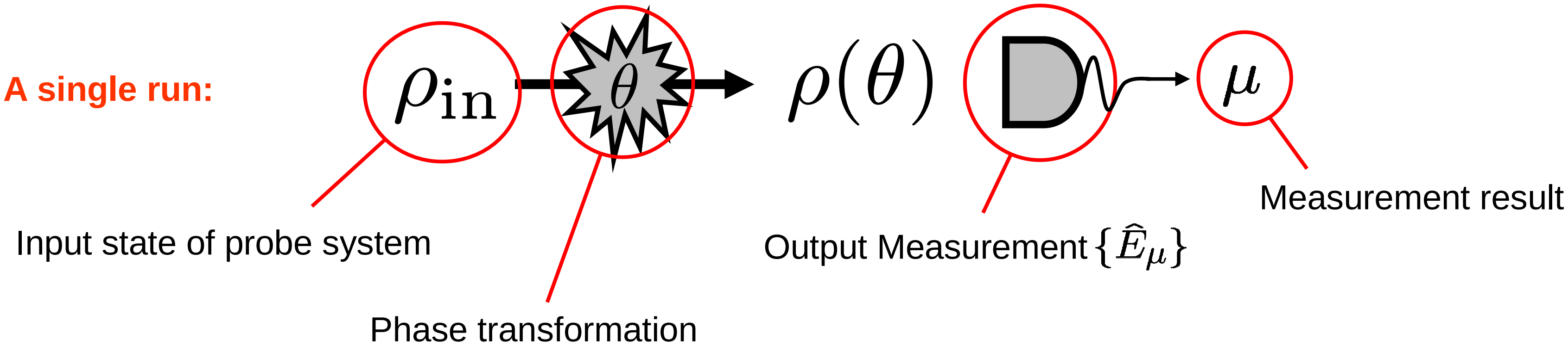
Motivation

- Fundamental extension of the theory
- Recent experimental works on Spin-Squeezing
  - work with a fluctuating number of atoms
  - use theory developed for fixed *N* ***Is that justified?***

Introduction

Phase Estimation

1) A single run:



2) m runs $\longrightarrow$ $\{\mu_i\}_{i=1}^{m}$ $\longrightarrow$ Phase estimator $\hat{\theta}(\{\mu_i\})$

3) Cramer-Rao bound

$$\boxed{\Delta\hat{\theta} \geq \frac{1}{\sqrt{mF}}}$$

Fisher information F $\left(\begin{array}{l}\text{input state } \rho_{\mathrm{in}} \\ \text{phase operation} \\ \text{output measurement}\end{array}\right)$

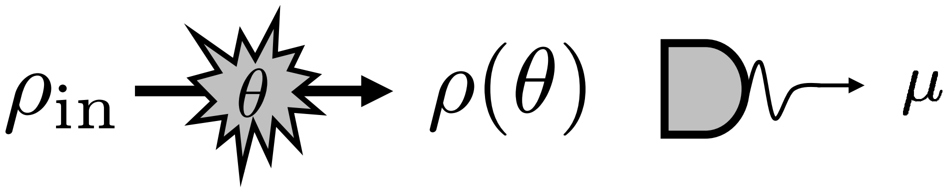
[For *unbiased* estimators $(\langle\hat{\theta}\rangle = \theta)$]

In the following slides

fixed N

Introduction

Linear Interferometers



two-mode approximation interferometer acts only in a two-level subspace $\longrightarrow$ *"spins"*

unitary linear interferometer $\rho(\theta) = e^{-i\hat{H}\theta}\rho_{\text{in}}e^{i\hat{H}\theta}, \quad \hat{H} = \frac{1}{2}\sum_{i=1}^{N}\hat{\sigma}_{\vec{n}_i}$

Shot-Noise Limit and Entanglement

For separable input states $\rho_{\text{sep}} = \sum_k p_k |\psi_{\text{sep}}^{(k)}\rangle\langle\psi_{\text{sep}}^{(k)}|$

$$|\psi_{\text{sep}}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots |\psi_N\rangle$$

state of particle 1
with two degrees of freedom

and linear interferometers: $\boxed{F \leq N}$ *Pezze and Smerzi, PRL 09*

$\Rightarrow$ $\Delta\hat{\theta} \geq \frac{1}{\sqrt{mF}} \geq \frac{1}{\sqrt{mN}}$ **Entanglement is necessary for Sub Shot-Noise Interferometry**

Introduction

BEC

Spin Squeezing

Kitagawa and Ueda, PRA 1993; Wineland et al., PRA 1994

Collective Spin Operators $\hat{J}_k = \frac{1}{2}\sum_{l=1}^{N} \hat{\sigma}_k^{(l)}, \quad k = x, y, z, \quad [\hat{J}_k, \hat{J}_j] = i\epsilon_{kjr}\hat{J}_r$

Spin Squeezing Parameter $\boxed{\xi = \frac{N\Delta\hat{J}_z}{|\langle\hat{J}_x\rangle|}}$

if $\xi < 1$

- state useful for Sub Shot-Noise Interferometry
- state of the atoms is entangled *Soerensen et al., Nature 2001*

Fisher vs Spin-Squeezing: $\chi^2 \equiv \frac{N}{F} \le \xi^2 \longrightarrow$ Spin-Squeezing implies $F > N$

Pezze and Smerzi, PRL 09

Heisenberg limit

$\boxed{F \le N^2} \longrightarrow \Delta\hat{\theta} \ge \frac{1}{\sqrt{mN^2}}$ *Giovannetti et al., PRL 06*

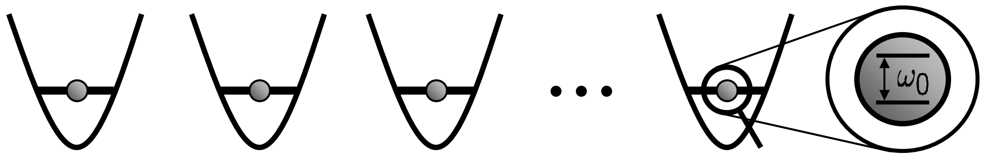
$F = N^2$ saturated by Cat/NOON/GHZ state

Introduction

Illustration: Ramsey spectroscopy in an ion trap

Wineland et al., PRA 1994

System: *N* locally addressable ions
with two internal states $|z\pm\rangle$



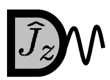
Aim: Measure transition frequency ω_0 between internal states

Tools:
- Manipulate internal states with classical radiation
- Measurement of populations in the states $|z\pm\rangle$

(Modified) **Ramsey scheme:**

1) Initialization: Prepare all atoms in $|z-\rangle$

2) Sequence: $\rho_{\text{in}} \xrightarrow{e^{-i\hat{J}_y\omega_0 T}} \rho(\omega_0 T)$



Linear Interferometer with
$\hat{H} = \hat{J}_y$ **and** $\theta = \omega_0 T$
and addressable particles

Separable input state
$\longrightarrow$ **Shot Noise Limited**

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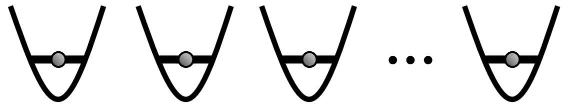
Motivation

- Fundamental extension of the theory
- Recent experimental works on Spin-Squeezing
  - work with a fluctuating number of atoms
  - use simple extension of the theory developed for fixed *N*

Is that justified?

Non-fixed N

Separability and Entanglement



What kind of states are possible?
- only incoherent mixtures $\rho_{\text{inc}} = \sum_N Q_N \rho^{(N)}$
- or states with coherences $|\psi_{\text{coh}}\rangle = \sum_N \sqrt{Q_N} |\psi^{(N)}\rangle$?

Super Selection Rules (SSRs) generally forbid the creation and detection of such coherences …
… but this may be possible if suitable *Reference Frames* can be established.

Wick et al., PR 52, *Moelmer, PRA 97*, *Bartlett et al., RMP 07* , …

1 SSR: $\boxed{\rho_{\text{sep}} = \sum_N Q_N \rho_{\text{sep}}^{(N)}}$

recall:
$\rho_{\text{sep}}^{(N)} = \sum_k p_k |\psi_{\text{sep}}^{(k,N)}\rangle\langle\psi_{\text{sep}}^{(k,N)}|,$
$|\psi_{\text{sep}}^{(N)}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots |\psi_N\rangle$

2 COH:

$$\rho_{\text{sep}} = \begin{pmatrix} \ddots & \vdots & \vdots & \iddots \\ \cdots & \boxed{\rho_{\text{sep}}^{(N)}} & \rho^{(N,N+1)} & \cdots \\ \cdots & \rho^{(N+1,N)} & \boxed{\rho_{\text{sep}}^{(N+1)}} & \cdots \\ \iddots & \vdots & \vdots & \ddots \end{pmatrix}$$

Non-separable states are **entangled**

Non-fixed N

Collective Spin Operators

$$\hat{J}_i \longrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \iddots \\ \cdots & \hat{J}_i^{(N)} & 0 & \cdots \\ \cdots & 0 & \hat{J}_i^{(N+1)} & \cdots \\ \iddots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \hat{J}_i^{(N)} = \frac{1}{2}\sum_{j=1}^{N} \hat{\sigma}_i^{(j)}$$

Non-fixed N - Results

	fixed-N	non-fixed-N: SSR	non-fixed-N: COH
Shot-Noise limit = Minimal $\Delta\theta$ with separable states	$\Delta\theta \geq \frac{1}{\sqrt{mN}}$	$\Delta\theta \geq \frac{1}{\sqrt{m\langle\hat{N}\rangle}}$	
Spin-Squeezing	$\chi^2 = \frac{N}{F}$ $\chi \leq \ \xi \equiv \frac{N\Delta\hat{J}_z}{\lvert\langle\hat{J}_x\rangle\rvert}$	$\chi^2 = \frac{\langle\hat{N}\rangle}{F}$ $\chi \leq \ \xi \equiv \frac{\langle\hat{N}\rangle\Delta\hat{J}_z}{\lvert\langle\hat{J}_x\rangle\rvert}$	
Heisenberg limit	$\Delta\theta \geq \frac{1}{\sqrt{mN^2}}$	$\Delta\theta \geq$ max $[\frac{1}{\sqrt{m\langle\hat{N}^2\rangle}}, \frac{1}{m\langle\hat{N}\rangle}]$	$\Delta\theta \geq \frac{1}{\sqrt{m\langle\hat{N}^2\rangle}}$ (but $\Delta\theta \geq \frac{1}{m\langle\hat{N}\rangle}$ in central limit)

Details in: **P. Hyllus, L. Pezze, and A. Smerzi, arXiv:1003.0649** (to appear in PRL)

Multiparticle entanglement, Spin Squeezing, and Phase Estimation → arXiv:1006.4366 and **arXiv:1006.4368**

Thank you very much for your attention!