Achromatic polarization retarder

(*Broadband polarization retarder*)

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QTC, Kraków, Poland 3 September 2011

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- Start from Maxwell’s equations

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• And show that polarization evolution equations are equivalent to Schrödinger equation for two state atom

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• And show that polarization evolution equations are equivalent to Schrödinger equation for two state atom

•Apply the adiabatic solution for the polarization evolution using the analogy with two state atom

•Thus we get adiabatic ACHROMATIC retarder

We begin with Maxwell's equations

$$\nabla.\vec{D} = \rho \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla.\vec{B} = 0 \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

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Using $\nabla \times \nabla \times \vec{E} = \nabla\left(\nabla.\vec{E}\right) - \nabla^2 \vec{E}$ we get

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Assuming monochromatic plane wave propagating in z direction

$$\vec{E}=\begin{pmatrix} A_x(z) \\ A_y(z) \\ 0 \end{pmatrix}\exp\left(ikz-i\omega t\right)$$

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$$\hat{\varepsilon}=\begin{pmatrix}\varepsilon_x & 0 & 0\\0 & \varepsilon_y & 0\\0 & 0 & \varepsilon_z\end{pmatrix}$$

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$$\hat{\varepsilon}=\begin{pmatrix}\varepsilon_x&0&0\\0&\varepsilon_y&0\\0&0&\varepsilon_z\end{pmatrix}=\begin{pmatrix}n_e^2&0&0\\0&n_o^2&0\\0&0&n_o^2\end{pmatrix}$$

$$(\mu\approx1)\Rightarrow n=\sqrt{\mu\varepsilon}\approx\sqrt{\varepsilon}$$

$$\hat{\varepsilon} = \begin{pmatrix} n_e^2 & iG & 0 \\ -iG & n_o^2 & 0 \\ 0 & 0 & n_o^2 \end{pmatrix}$$

G is responsible for the optical activity

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Now we write the wave equation as two scalar equations

$$\frac{\partial^2 A_x}{\partial z^2} + 2ik \frac{dA_x}{dz} - k^2 A_x = -\frac{\omega^2}{c^2}\left(n_e^2 A_x + iGA_y \right)$$

$$\frac{\partial^2 A_y}{\partial z^2} + 2ik \frac{dA_y}{dz} - k^2 A_y = \frac{\omega^2}{c^2}\left(iGA_x - n_o^2 A_y \right)$$

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slowly varying amplitude approximation $\left|\frac{\partial^2 A_{x,y}}{\partial z^2}\right| \ll \left|2k\frac{\partial A_{x,y}}{\partial z}\right| \Rightarrow$

$$i\frac{d}{dz}\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \frac{\omega}{2cn_o}\begin{pmatrix} n_e^2 - n_o^2 & -iG \\ iG & 0 \end{pmatrix}\begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

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Thus we rewrite the last equation as

$$i\frac{d}{dz}\begin{pmatrix}A_x\\A_y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}-\Delta & -i\Omega\\ i\Omega & \Delta\end{pmatrix}\begin{pmatrix}A_x\\A_y\end{pmatrix}$$

where

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Hayao Kubo and Ryo Nagata

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A. A. Rangelov, Opt. Lett. **36**, 2716 (2011).

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The connection between the Jones vector J in the original basis and the Jones vector J^a in the adiabatic basis is given by

$$\begin{bmatrix}A_x(z)\\A_y(z)\end{bmatrix}=\begin{bmatrix}\cos(2\varphi) & i\sin(2\varphi)\\ -i\sin(2\varphi) & \cos(2\varphi)\end{bmatrix}\begin{bmatrix}A^a{}_x(z)\\A^a{}_y(z)\end{bmatrix}$$

with φ defined by the equation

$$\tan(2\varphi)=\frac{\Omega}{\Delta}$$

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$$i\frac{d}{dz}\begin{bmatrix} A^a{}_x(z) \\ A^a{}_y(z) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\sqrt{\Omega^2+\Delta^2} & \frac{d\varphi}{dz} \\ \frac{d\varphi}{dz} & \frac{1}{2}\sqrt{\Omega^2+\Delta^2} \end{bmatrix} \begin{bmatrix} A^a{}_x(z) \\ A^a{}_y(z) \end{bmatrix}$$

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For adiabatic evolution

$$i\frac{d}{dz}\begin{bmatrix} A^a{}_x(z) \\ A^a{}_y(z) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\sqrt{\Omega^2+\Delta^2} & 0 \\ 0 & \frac{1}{2}\sqrt{\Omega^2+\Delta^2} \end{bmatrix} \begin{bmatrix} A^a{}_x(z) \\ A^a{}_y(z) \end{bmatrix}$$

which holds when

$$\left|\frac{d\varphi}{dz}\right| << \left|\sqrt{\Omega^2+\Delta^2}\right|$$

Thus the adiabatic propagator is

$$U^{A}(L,0)=\begin{bmatrix}\exp[i\eta] & 0\\ 0 & \exp[-i\eta]\end{bmatrix}$$

where

$$\eta=\int_{0}^{L}\frac{1}{2}\sqrt{\Omega^{2}+\Delta^{2}}\,dz$$

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The propagator in the original basis is

$$U(L,0) = R(L)U^A(L,0)R^\dagger(0)$$

or explicitly:

$$U(L,0)$$
$$||$$
$$\begin{bmatrix} e^{i\eta}\cos\varphi(0)\cos\varphi(L) + e^{-i\eta}\sin\varphi(0)\sin\varphi(L) & ie^{i\eta}\sin\varphi(0)\cos\varphi(L) - ie^{-i\eta}\cos\varphi(0)\sin\varphi(L) \\ ie^{-i\eta}\sin\varphi(0)\cos\varphi(L) - ie^{i\eta}\cos\varphi(0)\sin\varphi(L) & e^{-i\eta}\cos\varphi(0)\cos\varphi(L) + e^{i\eta}\sin\varphi(0)\sin\varphi(L) \end{bmatrix}$$

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If initially the light is linearly polarized in horizontal direction $J(0) = (A_x, A_y) = (1, 0)$

and if we ensure the initial condition $\varphi(0) = \pi / 2$

$$\begin{bmatrix} A_x(L) \\ A_y(L) \end{bmatrix} = \begin{bmatrix} e^{-i\eta} \sin\varphi(L) \\ ie^{-i\eta} \cos\varphi(L) \end{bmatrix}$$

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For such achromatic conversion we can end up with left circularly polarized light:

$$J(L) = \left(\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}} \right)$$

if we set the final angle $\varphi(L) = \pi/4$

And the process is reversible…

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For such achromatic conversion we can end up with right circularly polarized light:

$$J(L) = \left(\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}} \right)$$

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If initially the light is linearly polarized in horizontal direction $J(0)=\left(A_x, A_y\right)=(1,0)$ and if we ensure the initial condition $\varphi(0)=\pi/2$

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such polarization conversion will be frequency independent $\tan(2\varphi)=\frac{\Omega}{\Delta}$

For such achromatic conversion we can end up with vertical linearly polarized light:

$$J(L)=(0,1)$$

if we set the final angle $\varphi(L)=0$

And the process is reversible…

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ACHROMATIC

Polarization manipulation presented here depends only from angle φ

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where $\Omega = \frac{\omega G}{c n_o}$ $\Delta = \frac{\omega\left(n_0^2 - n_e^2\right)}{2 c n_0}$

Thus $\tan(2\varphi) = \frac{\Omega}{\Delta} = \frac{2 c n_0 \omega G}{c n_o \omega\left(n_0^2 - n_e^2\right)} = \frac{2G}{\left(n_0^2 - n_e^2\right)}$

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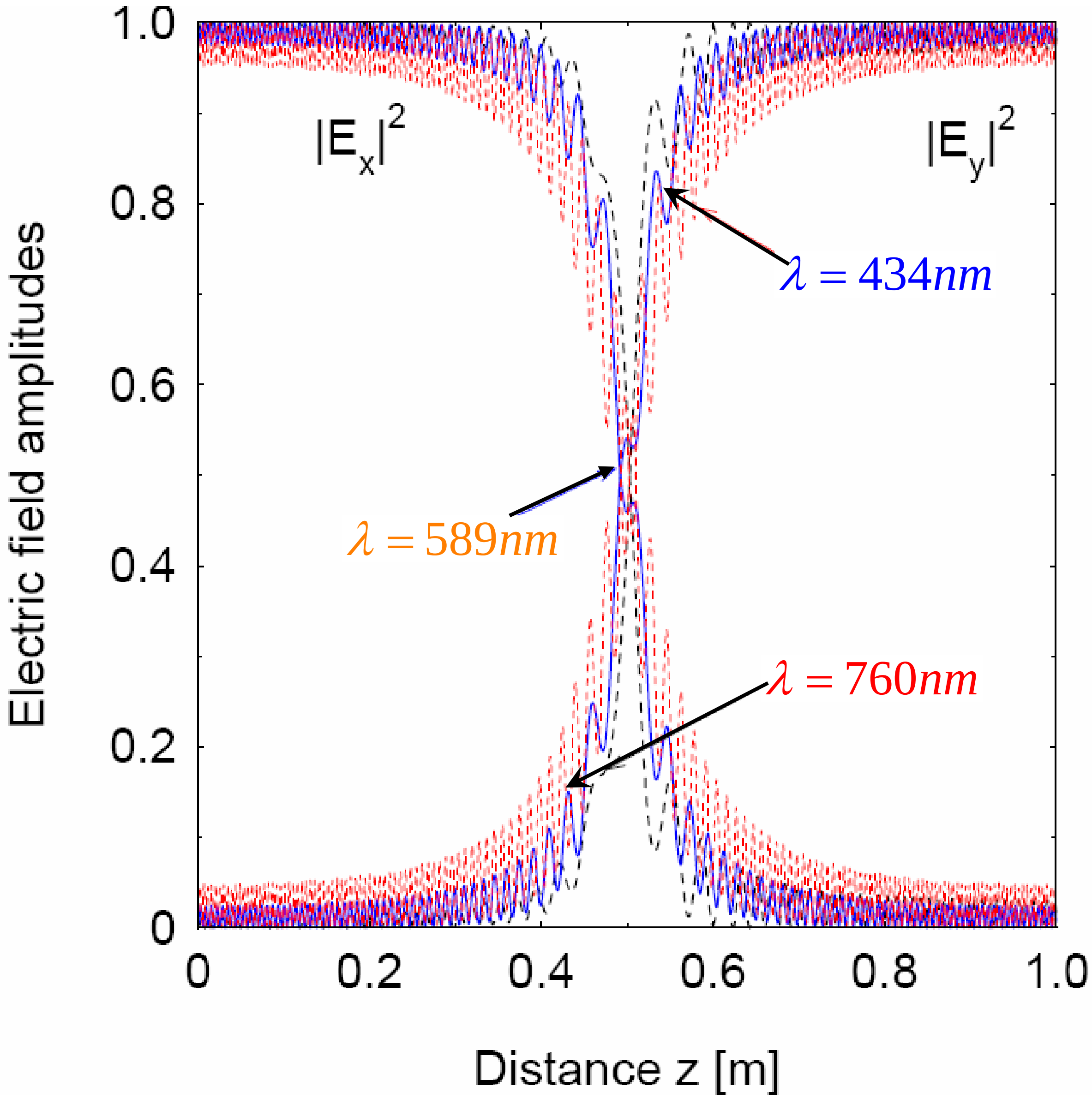
From the definition of the angle φ we see that the needed values of $0, \pm\pi/4, \pi/2$ would be achieved when:

$$\varphi \xrightarrow[G/\left(n_0^2 - n_e^2\right) \to 0+]{} 0 \qquad \varphi \xrightarrow[G/\left(n_0^2 - n_e^2\right) \to \pm\infty]{} \pm\pi/4 \qquad \varphi \xrightarrow[G/\left(n_0^2 - n_e^2\right) \to 0-]{} \pi/2$$



$$\Omega(z) = \Omega_0, \quad \Delta(z) = \Delta_0 \sin[\pi(z/L - 1/2)].$$

Summary

- Start from Maxwell's equations

- And show that polarization evolution equations are equivalent to Schrödinger equation for two state atom

- Apply the adiabatic solution for the polarization evolution using the analogy with two state atom

- Thus we get adiabatic ACHROMATIC retarder

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A. A. Rangelov “*Achromatic polarization retarder realized with slowly varying linear and circular birefringence*” Opt. Lett. **36**, 2716 (2011).

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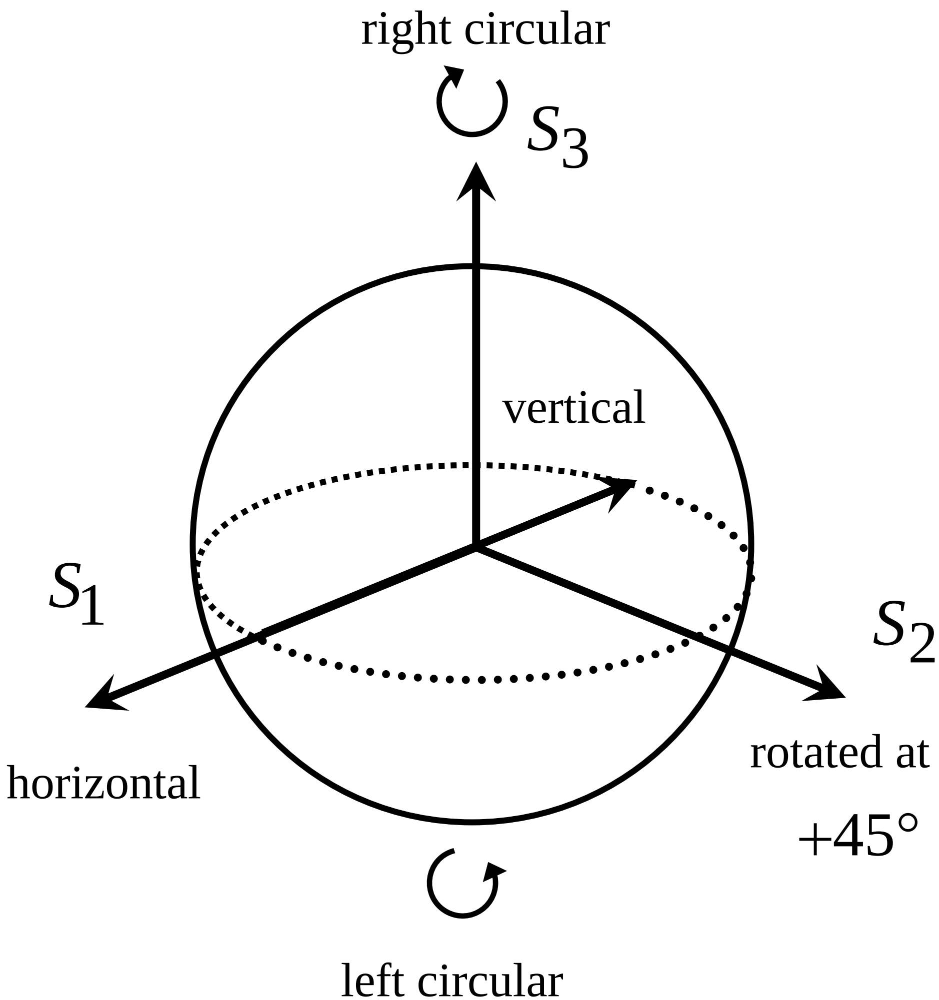
arXiv:1105.0316

Thank you for your attention



Poincaré sphere representation

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

$$2ik\frac{dA_x}{dz} = \left(k^2 - \frac{\omega^2}{c^2}\varepsilon_{xx}\right)A_x - \frac{\omega^2}{c^2}\varepsilon_{xy}A_y$$

$$2ik\frac{dA_y}{dz} = -\frac{\omega^2}{c^2}\varepsilon_{xy}A_x + \left(k^2 - \frac{\omega^2}{c^2}\varepsilon_{yy}\right)A_y$$

taking into account that $k^2 = \frac{\omega^2}{c^2}\varepsilon_z$ we will have

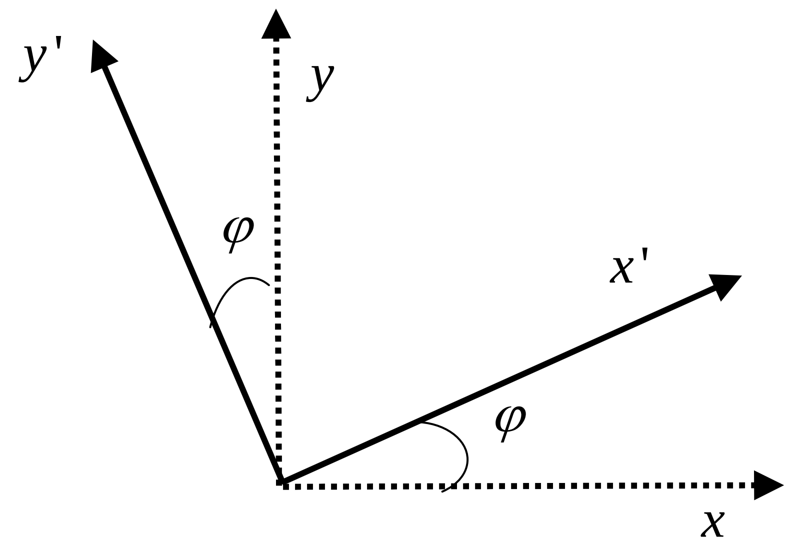
$$i\frac{d}{dz}\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \frac{\omega}{2c\sqrt{\varepsilon_z}}\begin{pmatrix} \varepsilon_z - \varepsilon_{xx} & -\varepsilon_{xy} \\ -\varepsilon_{xy} & \varepsilon_z - \varepsilon_{yy} \end{pmatrix}\begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$i\frac{d}{dz}\begin{pmatrix}A_x\\A_y\end{pmatrix}=\frac{\omega}{2c\sqrt{\varepsilon_z}}\begin{pmatrix}\varepsilon_z-\varepsilon_{xx} & -\varepsilon_{xy}\\-\varepsilon_{xy} & \varepsilon_z-\varepsilon_{yy}\end{pmatrix}\begin{pmatrix}A_x\\A_y\end{pmatrix}$$

$$\varepsilon_{xx}=\frac{\varepsilon_x-\varepsilon_y}{2}\cos(2\varphi)$$

$$\varepsilon_{yy}=\frac{\varepsilon_y-\varepsilon_x}{2}\cos(2\varphi)$$

$$\varepsilon_{xy}=\frac{\varepsilon_y-\varepsilon_x}{2}\sin(2\varphi)$$



Thus we rewrite the last equation as

$$i\frac{d}{dz}\begin{pmatrix}A_x\\A_y\end{pmatrix}=\frac{1}{2}\begin{pmatrix}-\Delta & \Omega\\\Omega & \Delta\end{pmatrix}\begin{pmatrix}A_x\\A_y\end{pmatrix}$$

where

$$\Omega=\mu\sin(2\varphi)$$

$$\Delta=\mu\cos(2\varphi)$$

$$\mu=\frac{\omega(\varepsilon_x-\varepsilon_y)}{2c\sqrt{\varepsilon_z}}$$