Disordered spin-1 Bose-Hubbard model

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Outline

- Disorder in ultracold bosonic gases.
- Spin-1 interacting bosons in a lattice in the strong interaction regime. Bose-Hubbard model.
- Effects of the spin interaction:
    - Stabilization of MI phase
    - Singlet BG
    - Possible occurrence of direct SF-MI transitions for disordered interactions

Strongly correlated bosons

Ultracold atoms in optical lattices:

- Control over the periodic crystal potential and particles' interactions
- Described by simple and controllable Hamiltonians
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In optical potentials spin degrees of freedom influence the interactions.

Spin-1 atoms (T=0): ^{23}Na, ^{39}K, ^{87}Rb $\cdots$

Model: Spin-1 Bose-Hubbard Hamiltonian

- MF/Gutzwiller [Imambekov et al., Phys. Rev. A **68**, 63602 (2003)]
  [Kimura et al., Phys. Rev. Lett. **88**, 110403 (2005)]
  [Pai et al., Phys. Rev. B **77**, 14503 (2008)]
- QMC [G. Batrouni et al., Phys. Rev. Lett. **102**, 140402 (2009)]
- DMRG [M. Rizzi et al., Phys. Rev. Lett. **95**, 240404 (2005)]

Realization of disorder in ultracold atomic gases

Disorder can be produced and controlled:

- Adding a disorder potential created by a speckle radiation pattern to the main potential

  [Horak et al., Phys. Rev. A **58**, 3953 (1998)]

  [Boiron et al., Eur. Phys. J. D **7**, 373 (1999)]

  [Billy et al., Nature **453**, 891 (2008)]

- Bicromatic lattices (quasi random)

  [Fallani et al., Phys. Rev. Lett. **98**, 130404 (2007)]

  [Roati et al., Nature **453**, 895 (2008)]

  [Damski et al., Phys. Rev. Lett. **91**, 080403 (2003)]

  [Diener et al., Phys. Rev. A. **64**, 033416 (2001)]

- Using an admixture of different atomic species randomly trapped in sites of the sample and acting as impurities (Bernoulli potentials)

  [Gavish et al., Phys. Rev. Lett. **95**, 020401 (2005)]

- Employing Feschbach resonances in random magnetic fields (Disorder in the interaction)

  [Gimperlein et al., Phys. Rev. Lett. **95**, 170401 (2005)]

Spin-1 Bose-Hubbard Model

- Alkali atoms in optical lattice.
- Spin degrees of freedom: manifold of Zeeman hyperfine energy levels.
- ^{23}Na ^{87}Rb: nuclear + electronic angular momentum $S = 3/2 \otimes 1/2 \to 1 \oplus 2$. We consider the case $S = 1$.
- Contact two-body interactions

$$\hat{H} = \underbrace{-t \sum_{\langle i,j \rangle, \sigma=0,\pm 1} \left(\hat{a}^{\dagger}_{i\sigma} \hat{a}_{j\sigma} + H.c. \right)}_{\hat{T}} + \underbrace{\sum_i \left[\frac{U_0}{2} \hat{n}_i (\hat{n}_i - 1) + \frac{U_2}{2} \left(\hat{\mathbf{S}}_i^2 - 2\hat{n}_i \right) - \mu \hat{n}_i \right]}_{\hat{H}^0 = \sum_i \hat{H}_i^0}$$

On each site:
$\hat{n} = \sum_\sigma \hat{n}_\sigma$
Bose statistics: $S + n$ even [Wo et al., Phys. Rev. A **54**, 4534 (1996)]

$$\begin{aligned} U_0 &= a_0 + 2a_2 \\ U_2 &= a_2 - a_0 \end{aligned}$$

a_S: proportional to s-wave scattering length corresponding to the channel with total spin S

Diagonal disorder: $\sum_i \hat{H}_{dis}(\epsilon_i)$ uniform distribution with $-\Delta \le \epsilon_i \le \Delta$.
ϵ_i sums to μ, U_2, U_0, a_0 or a_2

Mean-Field approach

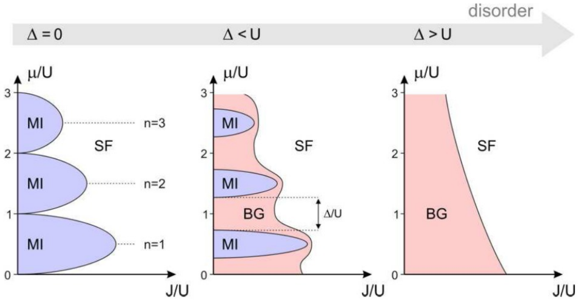
Nonperturbative Gutzwiller ansatz (GA)

$$|\psi\rangle = \prod_{i=1}^{M} |\phi_i\rangle$$

- The coefficients of $|\phi_i\rangle_i$ are the variational coefficients to be determined by minimizing the BH Hamiltonian.
- Corresponds to the Mean Field approximation but takes into account inhomogeneous lattices.
- Becomes exact for infinite dimension. We consider the 2D case.
- In the dirty case,translational invariance is broken
- Study of averaged density fluctuations and condensate fraction to reconstruct the phase diagram

Disordered ultracold bosonic gases: strong interaction

- Mott Insulator (MI): incompressible $\kappa = \frac{\partial \rho}{\partial \mu} = 0$, zero condensate fraction ρ_C
- Superfluid (SF): $\kappa \neq 0$, $\rho_C \neq 0$
- Bose Glass (BG): $\kappa \neq 0$, $\rho_C = 0$ (degeneracy between states with different densities)



μ: chemical potential
U: interaction
J: hopping
disorder in the local potential with fluctuation Δ

Taken from [Fallani et al., arXiv , 0804.2888 (2008)]

Dirty boson problem

Is the direct transition between the MI and SF phases possible in the presence of disorder? [M. P. A. Fisher et al. , Phys. Rev. B **40**, 546 (1989)]

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- Exception: Griffiths transitions. Δ_c does not depend on details of the disorder distribution. MI-BG trasition given by a concentration of rare regions where disorder mimics a homogeneous changing in the clean Hamiltonian. Gapless (SF) domains are distant (no phase coherence).

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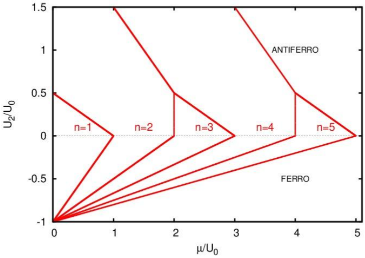
- If $\Delta_c \propto E_{gap}$, near the clean MI-SF boundaries BG phase appears for disorder of any strength $\rightarrow$ no direct SF-MI transition.
- What happens if Δ_c remains finite even for vanishing E_{gap}?

Atomic limit $(t \to 0)$

$$E_0 = -\mu n + \frac{1}{2} U_0 n(n-1) + \frac{1}{2} U_2 \left[S(S+1) - 2n \right]$$

- Single site problem $\to$ many-body, commensurate lattice.
- Boundaries $\to$ $E_0(n) = E_0(n+1)$
- Antiferromagnetic case $U_2 > 0$ (Ex. ^{23}Na, ^{85}Rb)
  - Smaller possible value of S:
    $S = 0$ for even lobes
    $S = 1$ for odd lobes
  - Odd lobes shrink and for $U_2/U_0 > 0.5$ they disappear.
  - Even lobes enlarge and translate for $U_2 > 0.5$.
- Ferromagnetic case $U_2 < 0$ (Ex. ^{87}Rb)
  - Larger possible value of S: $S = n$
  - All lobes shrink. Similar to the scalar case (Renormalized U_0)

Ground state density phase diagram in the atomic limit



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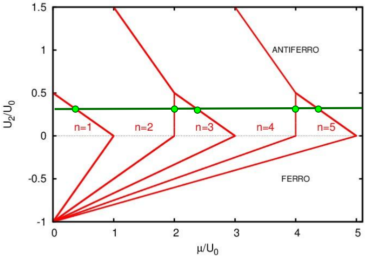

- Tracing horizontal lines in the diagram one obtains the "basis" of the MI lobes in the $\mu/U_0 - t/U_0$ phase diagram with U_2 fixed

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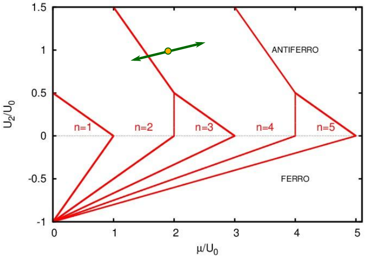
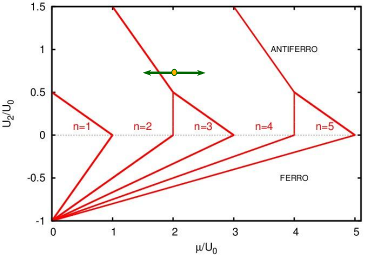


- Disorder: fluctuations along some direction

Disorder in μ

ρ_C



- Horizontal fluctuations in the $t = 0$ phase diagram.

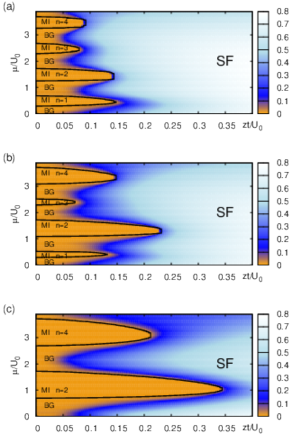


FIGURE *1:* $\Delta/U_0 = 0.3$.
(a)$U_2/U_0 = 0.02$,
(b)$U_2/U_0 = 0.1$
(c)$U_2/U_0 = 0.3$.

Disorder in μ

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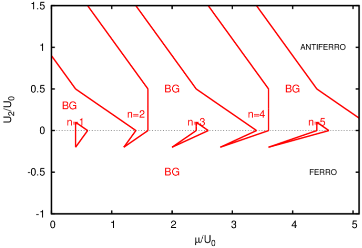

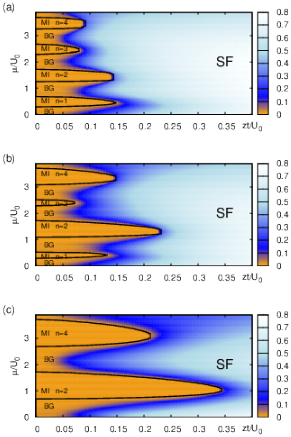

- Horizontal fluctuations in the $t = 0$ phase diagram.
- Disorder destroys odd lobes if $\Delta > \Delta_o = U_0/2 - U_2$
- Accuracy decreases near the tips. Because:
  - MF approximation [Bissbort, EPL **86**, 50007 (2009)]
  - Very thin BG region predicted [Gurarie et al., Phys. Rev. B **80**, 214519 (2009)]
- BG phase:
  Nematic for $U_2/U_0 < 0.5$
  Singlet for $U_2/U_0 > 0.5$

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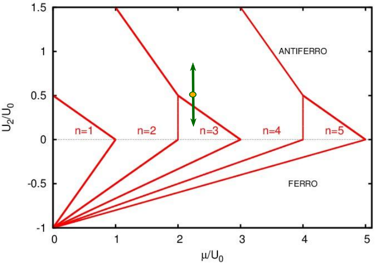
Disorder in U_2



- Vertical fluctuations in the atomic phase diagram
- $\Delta < |U_2|$: no mixing between ferro and antiferro regions
- Ferro: similar to the scalar case.
- Antiferro: the MI lobes become more unstable increasing n. , disappearing for for $n > (U_0 + U_2 + \Delta)/(2\Delta)$
- For $U_2/U_0 < 0.5$ A small disorder does not mix n and $n+1$ occupation lobes with n-odd. Finite Δ_c even for vanishing gap: direct MI-SF transition

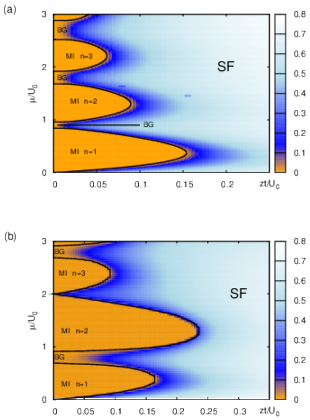
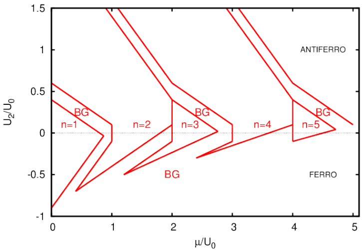


FIGURE 2: $\Delta/U_0 = 0.06$. $U_2 = \pm 0.1 U_0$.

Disorder in U_2



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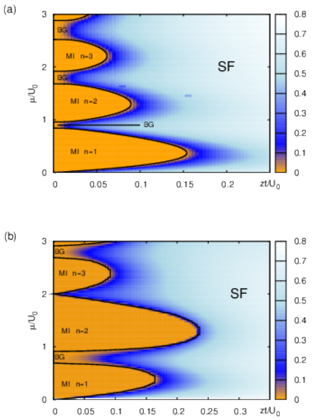

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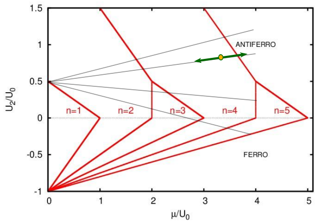


Boundaries of the first lobe lie on fluctuations' direction Direct MI-SF transition.BG near the tip.

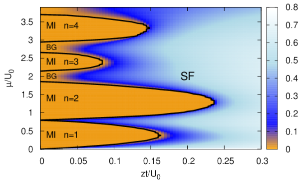
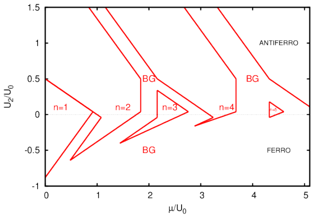


FIGURE *3:* $\Delta/U_0 = 0.04$, $U_2/U_0 = 0.1$.

Disorder in a_2



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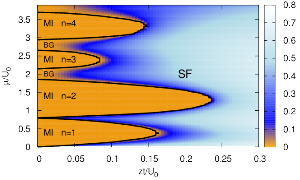
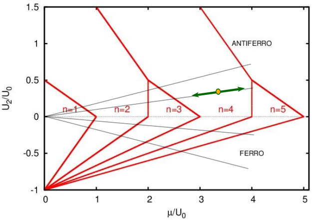


FIGURE *3:* $\Delta/U_0 = 0.04$, $U_2/U_0 = 0.1$.

Disorder in U_0

- Lobes with $n > (U_0 + \Delta)/2\Delta$ disappear while the first one remains stable
- Direct MI-SF transition before the first lobe.



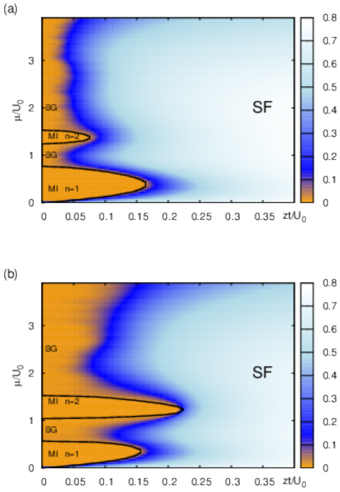


FIGURE *4:* $\Delta/U_0 = 0.25$.
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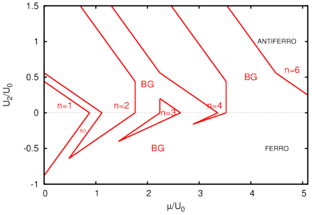

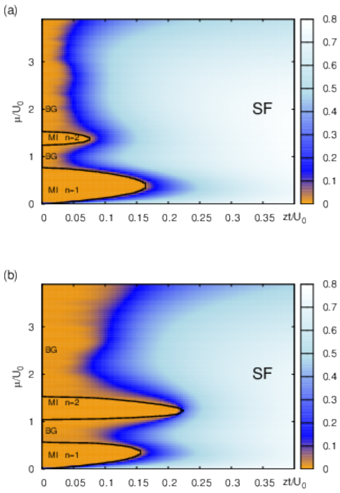

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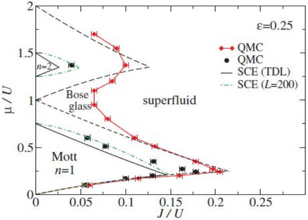
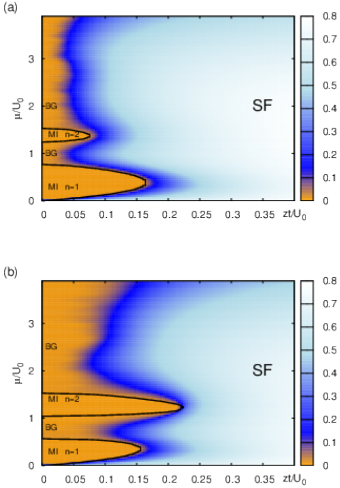
- Direct MI-SF transition before the first lobe.
- Also observed in the 1D scalar case by QMC and SCE [Gimperlein et a., Phys. Rev. Lett. **95**, 170401 (2005)]





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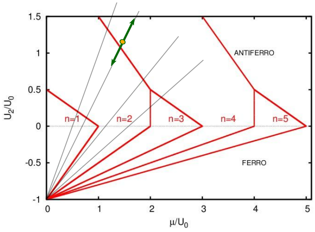

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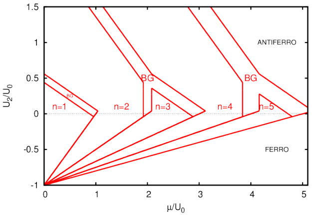
Ferromagnetic regime: all the boundaries lie on fluctuation directions and no BG appears. Distinction between scalar and spinor case with ferromagnetic spin correlations, where disorder in the $s = 0$ scattering channel only is not enough to produce BG.

Disorder in a_0



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Summary

Main results:

- Spin-1 2D BH model in presence of diagonal disorder has been studied within the Gutzwiller MF approximation and in the atomic limit
- Antiferromagnetic spin coupling gives a phase diagram qualitatively different from the scalar case
- Odd MI lobes disappear for sufficiently large spin coupling. In this case disorder produces BG of spin singlets
- Disorder can also destroy odd lobes, even if the spin coupling cannot in the clean case. In this case BG is nematic
- Direct SF-MI is observed in some cases (disorder in U_2, a_0, a_2)

Perspectives:

- Cluster MF
- Spin properties inside the MI phases. Frustration?

[M. Łącki, S. Paganelli, V. Ahufinger, A. Sanpera, J. Zakrzewski, Phys. Rev. A **83**, 013605 (2011)]

[S. Paganelli, M. Łącki, V. Ahufinger, J. Zakrzewski, A. Sanpera, arXiv , 1105.2746 (2011)]