Beyond standard two-mode dynamics in bosonic Josephson junctions

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Quantum Technologies Conference II: Manipulating photons, atoms and molecules

Krakow 2011

In collaboration with
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and A. Polls

Phys. Rev. A 82 063626 (2010)

Outlook

- Josephson effect
- Theoretical description:
  - Mean-field
  - Two-mode approximation
  - Beyond two-mode: effective potential
- Summary and Conclusions

Josephson effect

- The Josephson effect was first predicted in superconductors

- In cold atoms, the first experimental realization of the Josephson effect was done in 2005 by the Heidelberg group.
M. Albiez et al. PRL **95**, 010402 (2005)

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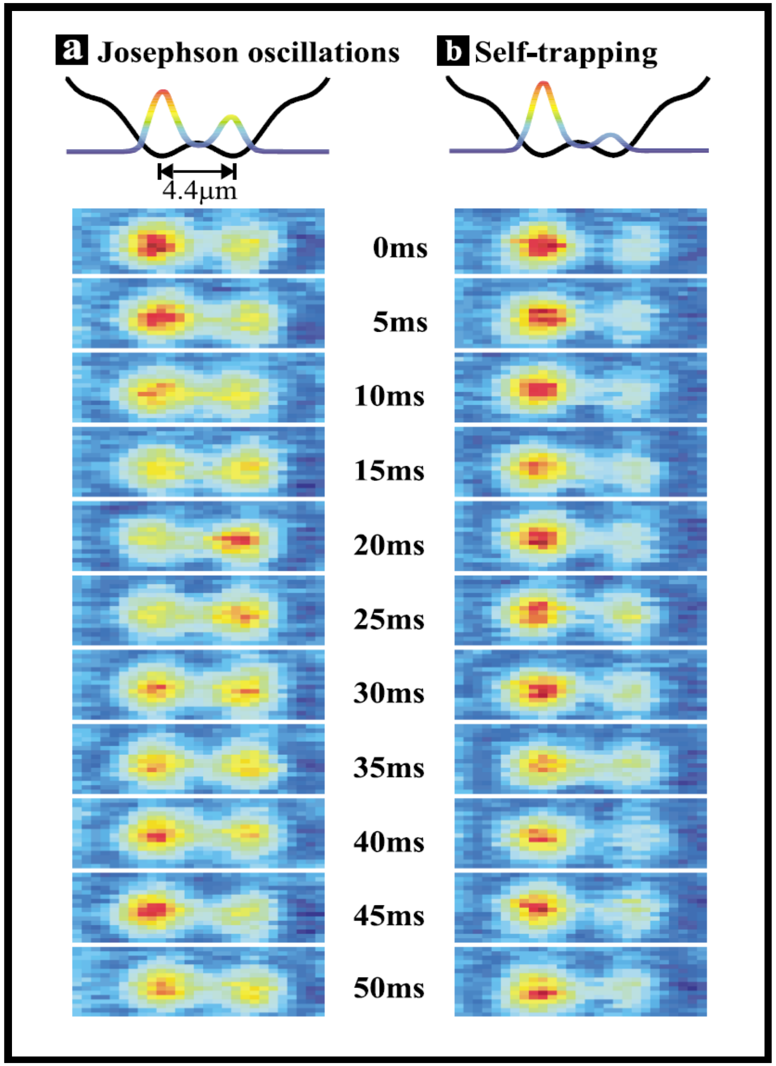
• Two Bose-Einstein condensates weakly linked, using a double-well potential

• It is the coherent tunneling of the atoms across the potential barrier

• For population imbalances very large, where most of the particles are on one well, one can also have self-trapping



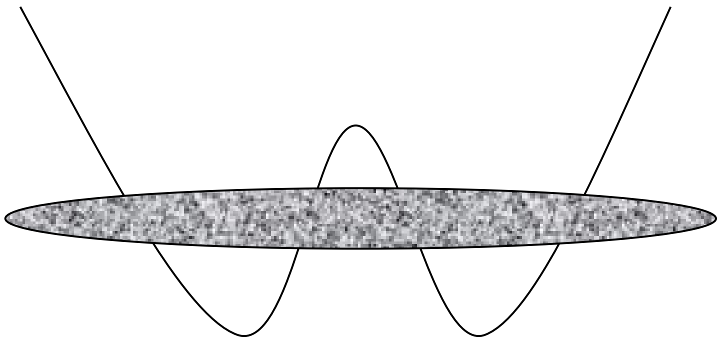
Mean-field

Dilute gas of N atoms at T=0

Cigar shaped

Double-well in the long axis

Contact interaction



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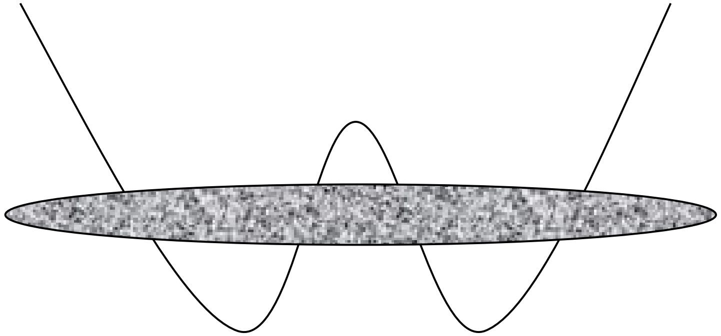
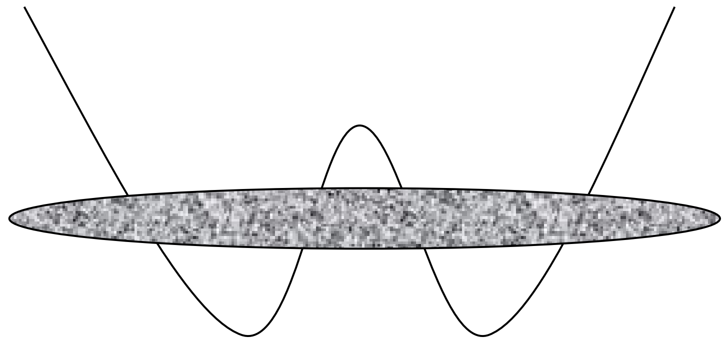


For large enough number of atoms, N>1000, the mean-field Gross-Pitaevskii equation provides a good description of the system:

$\hbar = m = 1$

$$i\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x) + \lambda_0 N|\psi(x,t)|^2\right]\psi(x,t)$$

Mean-field



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$g_{1D} \equiv \lambda_0 N$ defines the dynamics

Mean-field

Simulation of the Gross-Pitaevskii equation using 1150 atoms of ^{87}Rb

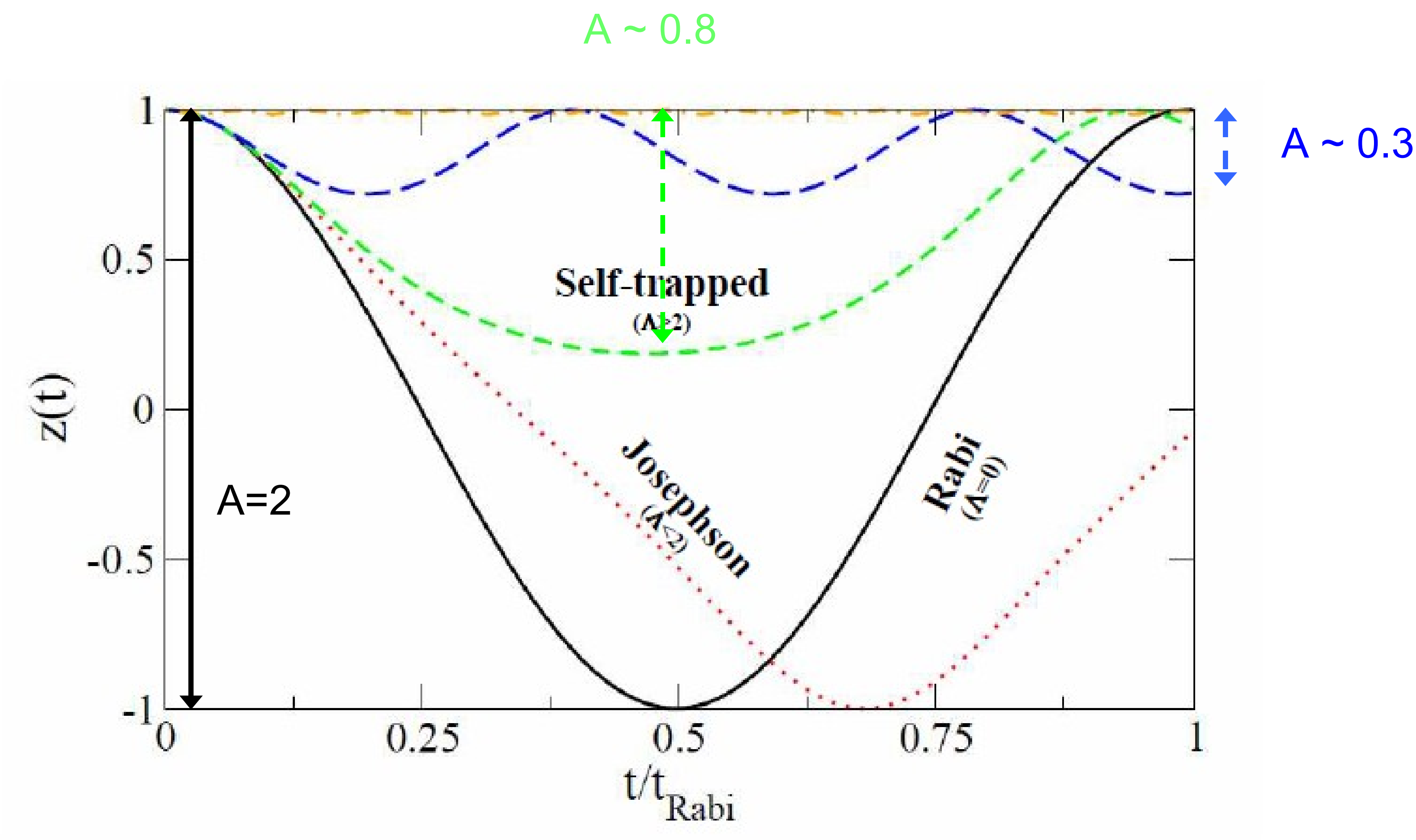
- Phase almost constant at each side
- Density profile almost bi-modal

Population imbalance: $z(t) = \dfrac{N_L(t) - N_R(t)}{N}$

Phase difference: $\delta\phi(t) = \phi_R(t) - \phi_L(t)$

Mean-field

Initial state: all the atoms on the left



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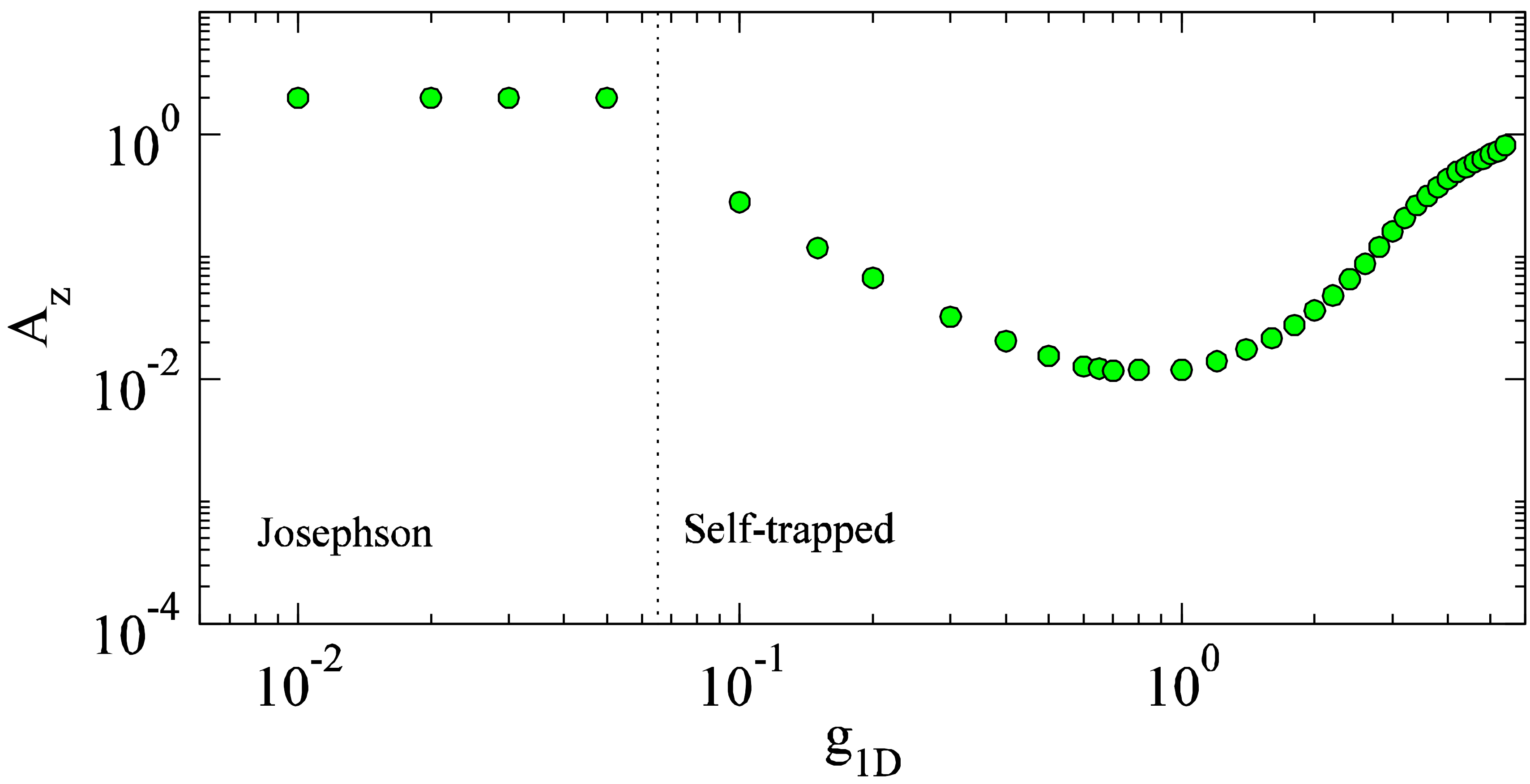
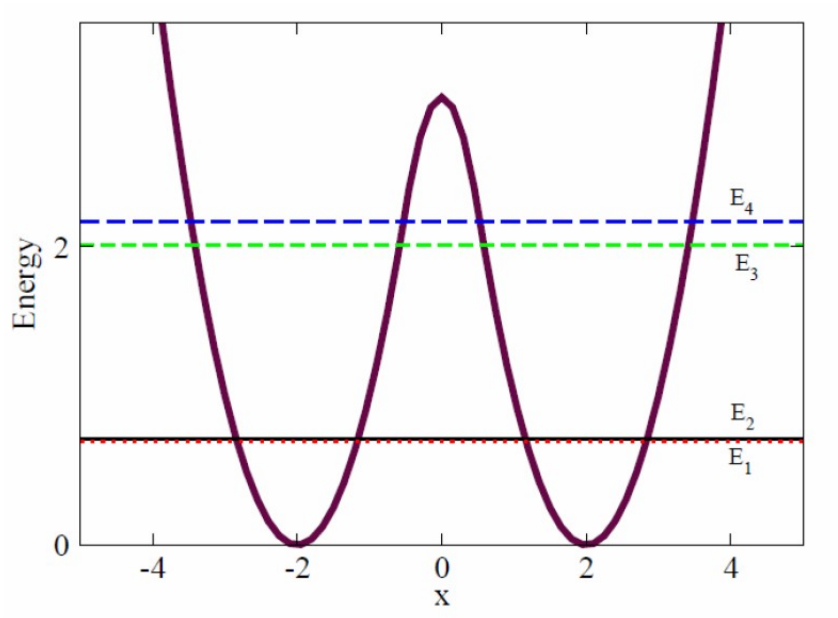
GP equation

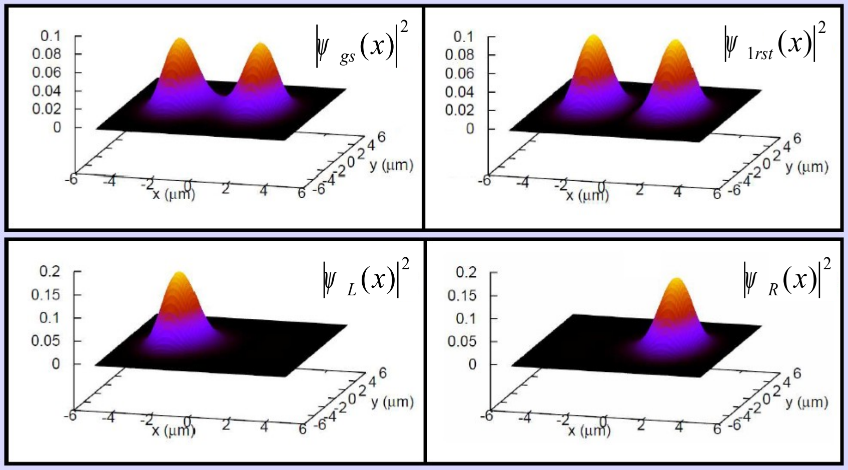


Mean-field

Single particle Hamiltonian with a quasi-degenerate doublet E1~E2<<E3



Two mode approximation



$$\psi_L(x) = \frac{\psi_{gs}(x) + \psi_{1rst}(x)}{\sqrt{2}}$$

$$\psi_R(x) = \frac{\psi_{gs}(x) - \psi_{1rst}(x)}{\sqrt{2}}$$

$$\psi(x,t) = \Psi_L(x)\sqrt{N_L(t)}e^{i\phi_L(t)} + \Psi_R(x)\sqrt{N_R(t)}e^{i\phi_R(t)}$$

Smerzi et al. (1997)
Raghavan et al. (1998)
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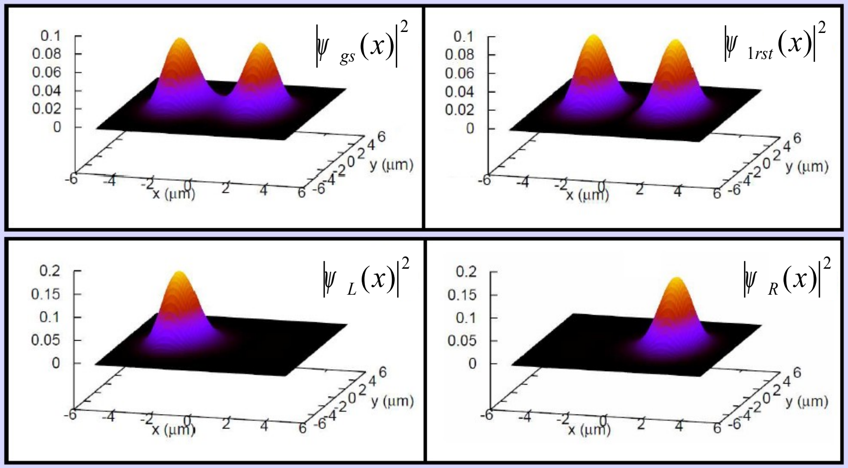
Review Leggett (2001)

Two mode approximation



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$$\dot{z}(t) = -2K\sqrt{1 - z^2(t)}\sin\delta\phi(t)$$

$$\delta\dot{\phi}(t) = NUz(t) + 2K\frac{z(t)}{\sqrt{1 - z^2(t)}}\cos\delta\phi(t)$$

$$z(t) = \frac{N_L(t) - N_R(t)}{N}$$

$$\delta\phi(t) = \phi_R(t) - \phi_L(t)$$

$$U = \lambda_0 \int \Phi_L^4(x)\,dx \qquad K = -\int\left[\frac{1}{2}\partial_x\Phi_L(x)\partial_x\Phi_R(x) + \Phi_L(x)V(x)\Phi_R(x)\right]dx$$

Two-mode approximation

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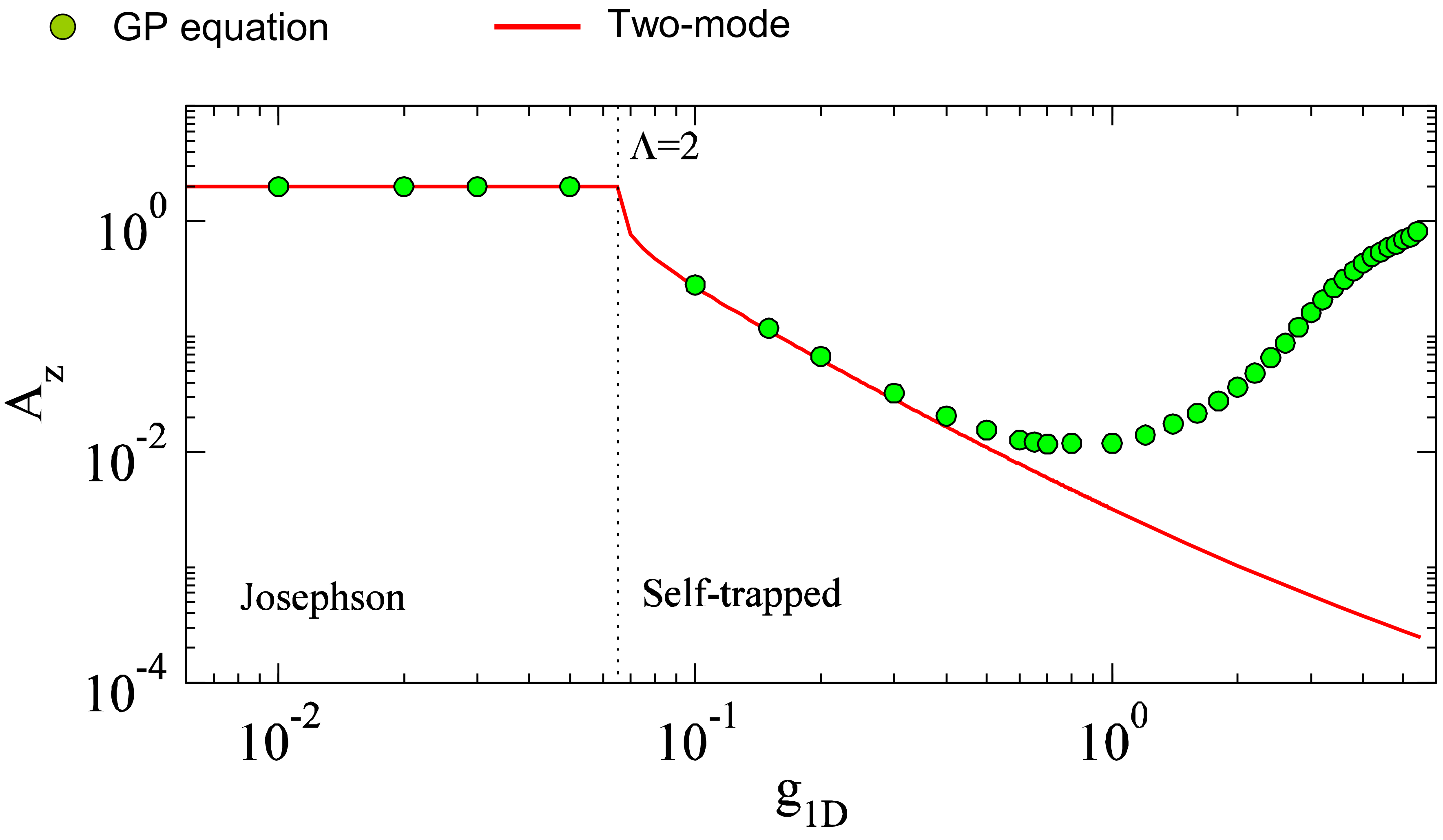
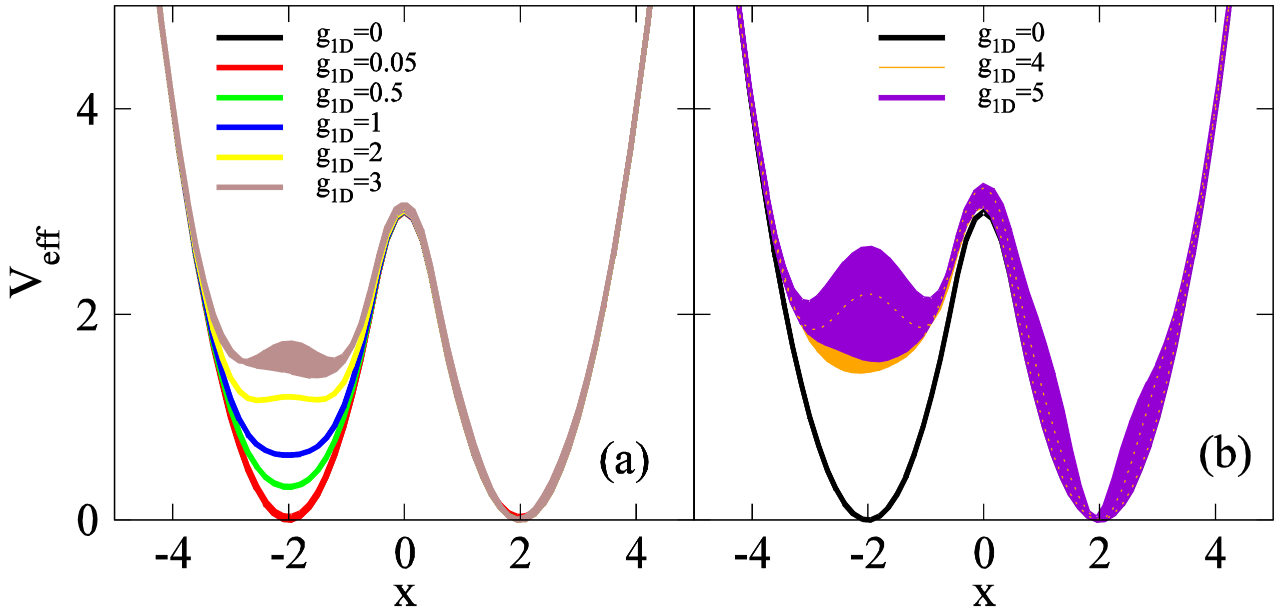


Effective potential

Analysis of the effective potential:



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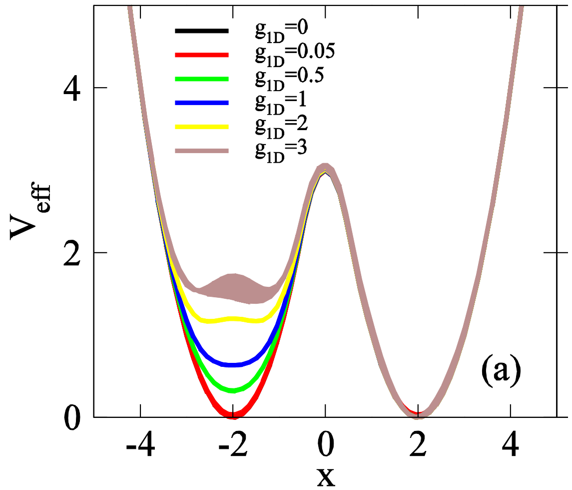
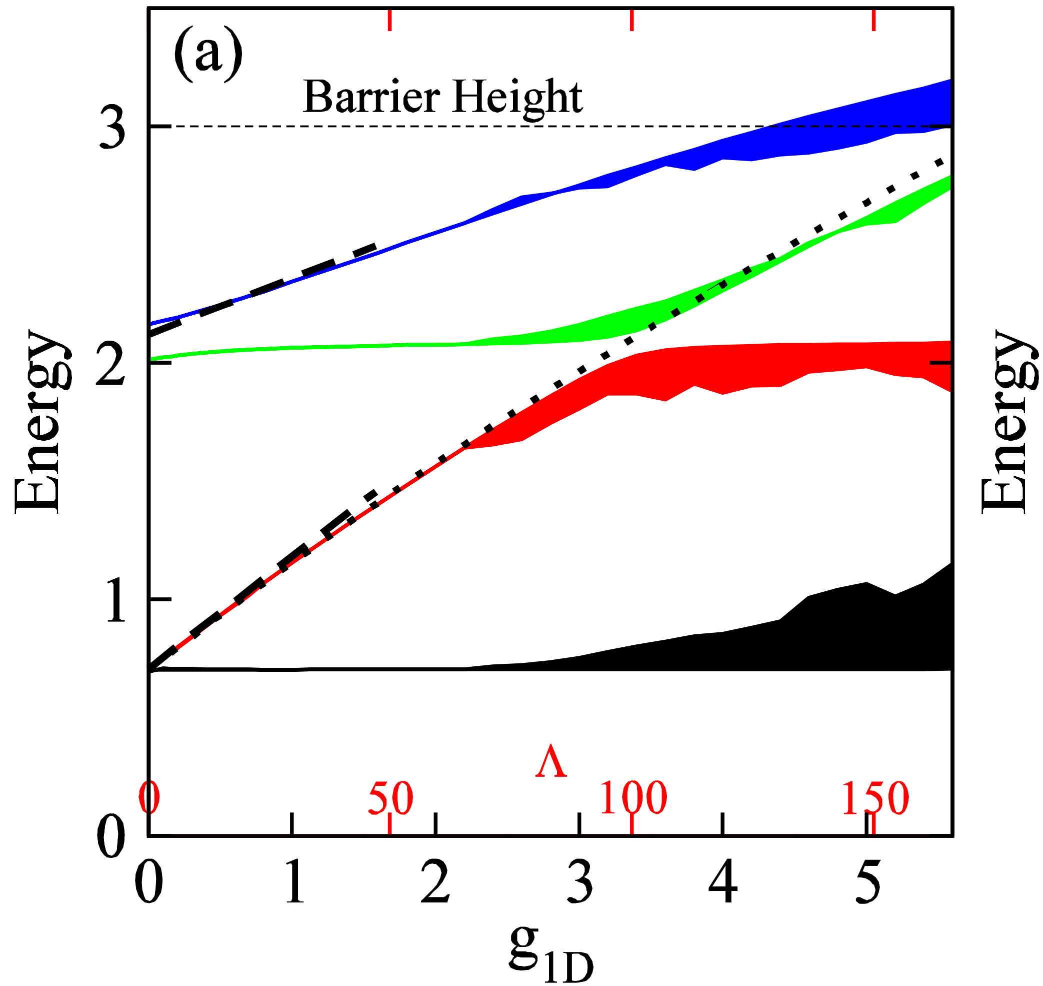
Effective potential



Eigenvalues of the effective potential:

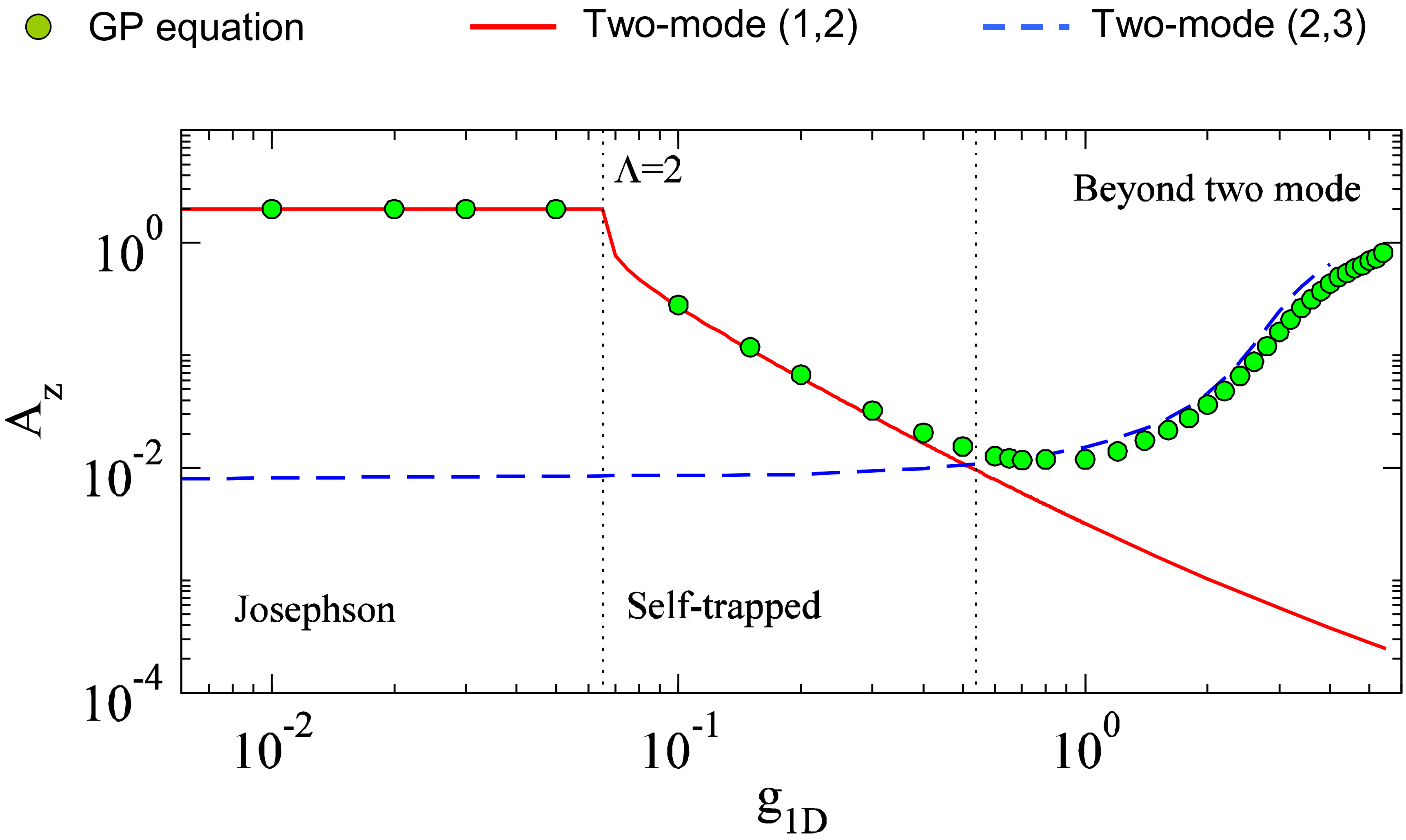
- (1) and (3) remain almost independent of the interaction

- (2) and (4) grow linearly with the interaction



Beyond two-mode

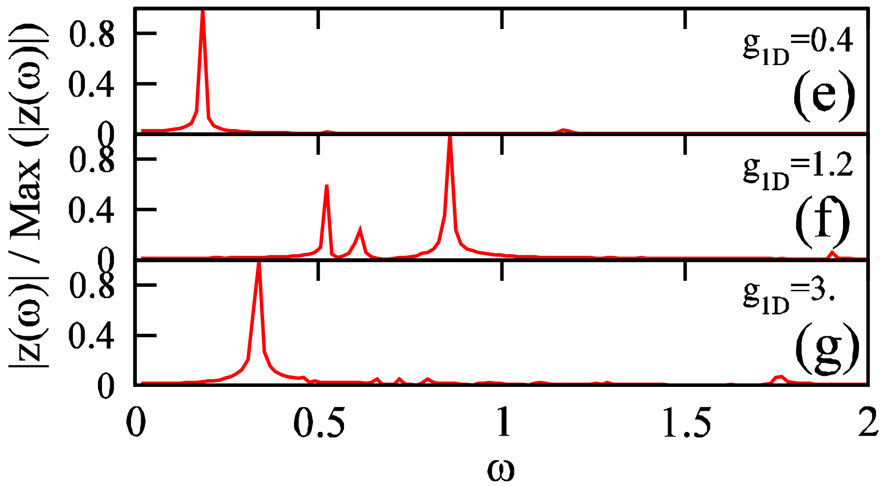
Initial state: all the atoms on the left



Beyond two-mode

In a two-level system, the frequency of the oscillations is related to the energy difference between the levels.

The frequency of the oscillations of the population imbalance can be found using the Fourier Transform.



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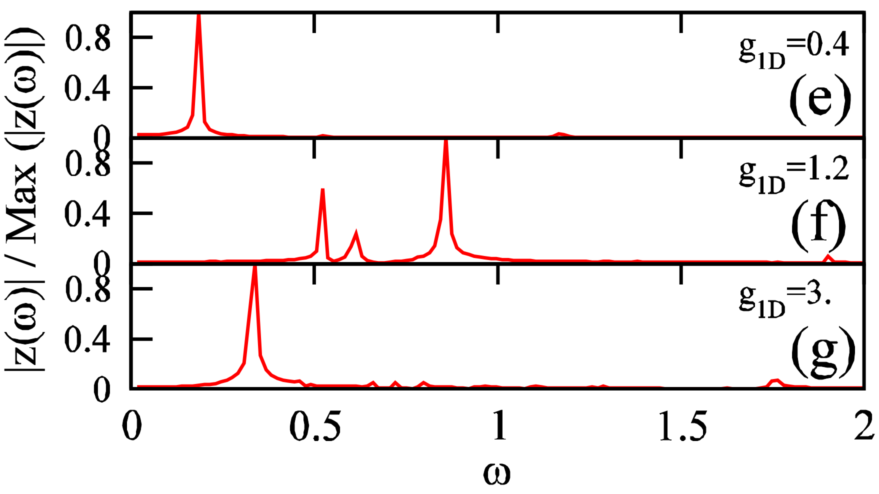
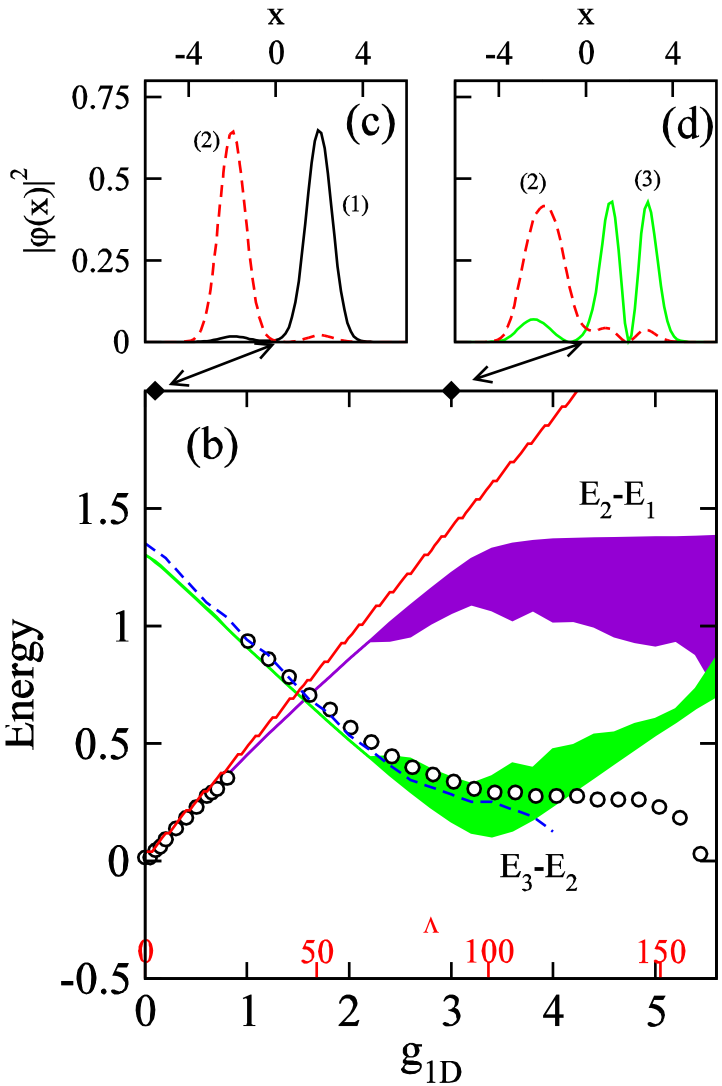
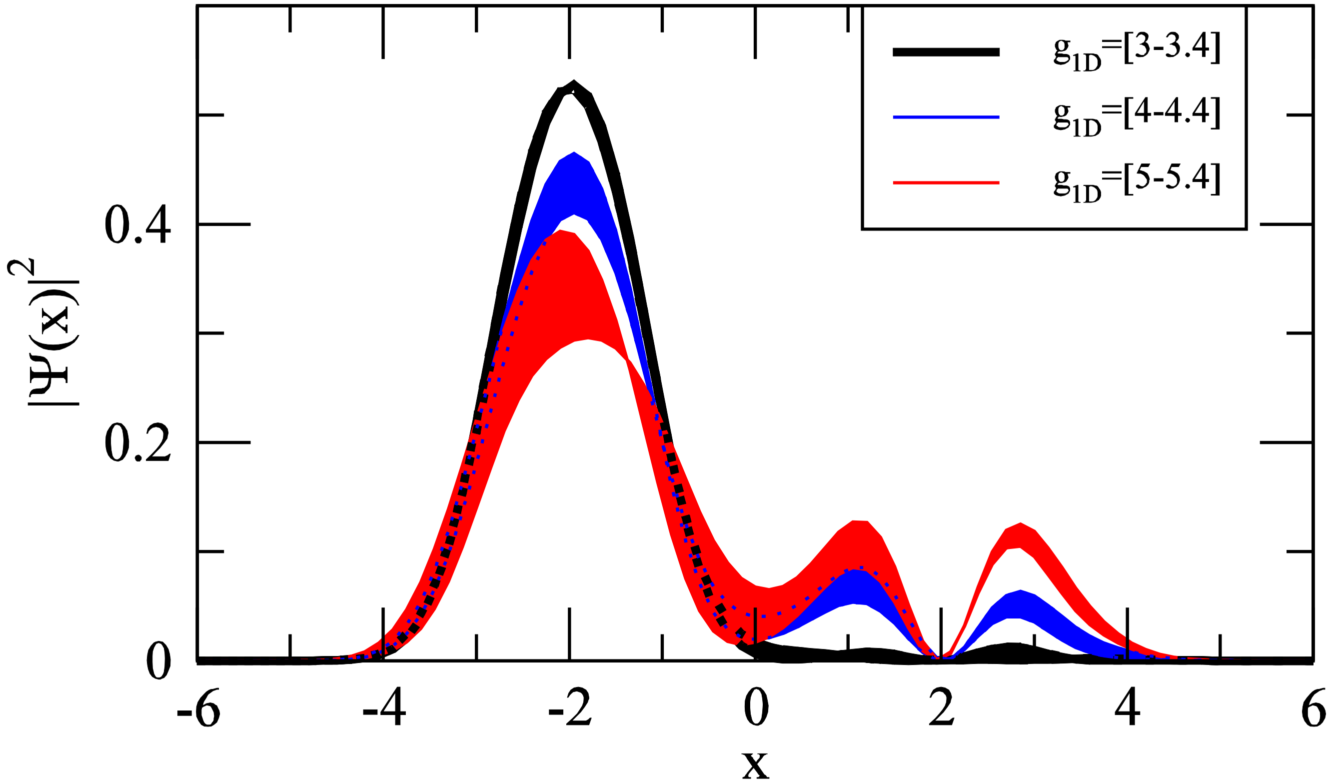




Beyond two-mode

An experimental trace of the coupling between the modes (2,3) is the appearence of a node in the density distribution, characteristic of the excited state of the right well.



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Summary and Conclusions

- The two-mode approximation reproduces the Gross-Pitaevskii results for low values of the interaction. It predicts the Josephson regime and the transition to the self-trapping regime.
- For higher interactions, it is useful to analyze the effective potential and its eigenvalues. It is then clear that one has to consider the coupling between the 2nd and the 3rd modes to understand the dynamics.
- This coupling it is not due to an external biased double-well, but due to the interactions.
- Experimentally, a nice way to see the coupling between these two modes will be the observation of a node on the density profile.

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