# Line-Centered Square Optical Lattices: Many Body Effects

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Quantum Technologies Conference II, August 31, 2011

# Plan

- Optical Lattices
- Line-Centered Square Optical Lattice
- Dispersion bands
- Localized states
- Many Body Effects

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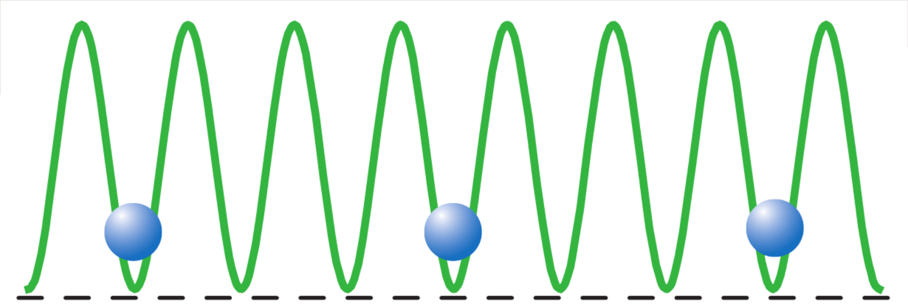
# Introduction: Optical Lattices

- By applying the laser standing wave fields to cold atoms, we can form optical lattices.
- Cold atoms in optical lattices allow us to simulate interesting quantum systems.
- Various lattices can be created: triangle, hexagonal, line-centered square, . . .



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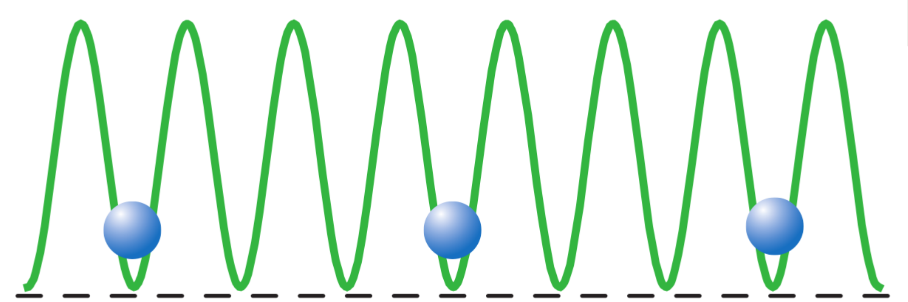

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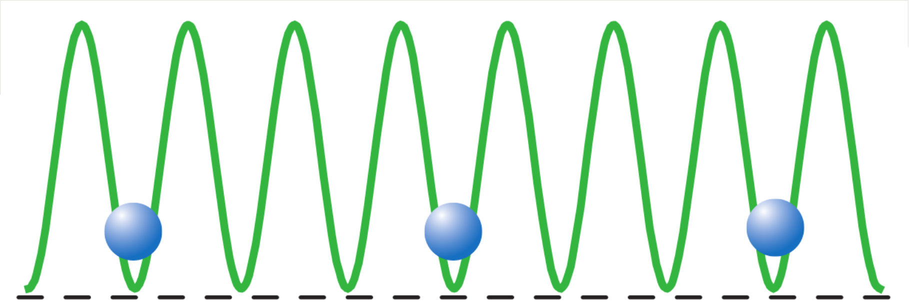

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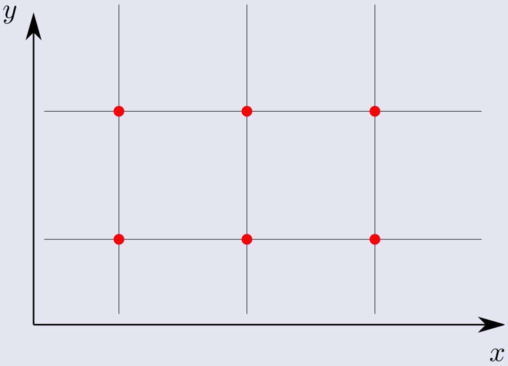

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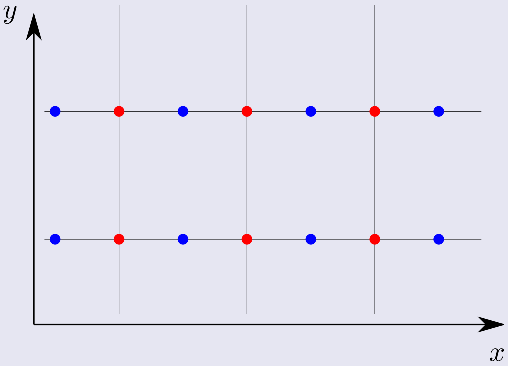

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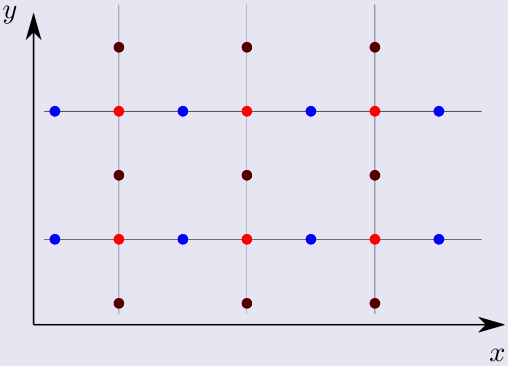

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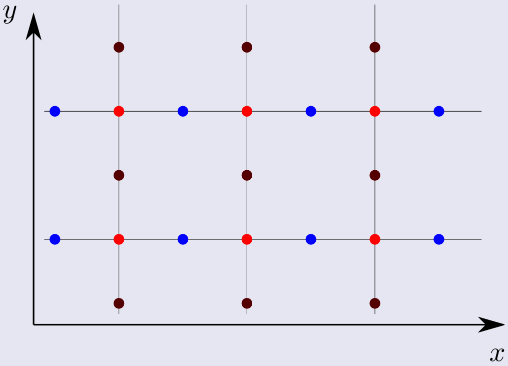
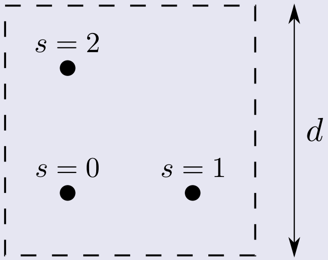



## Elementary Cell



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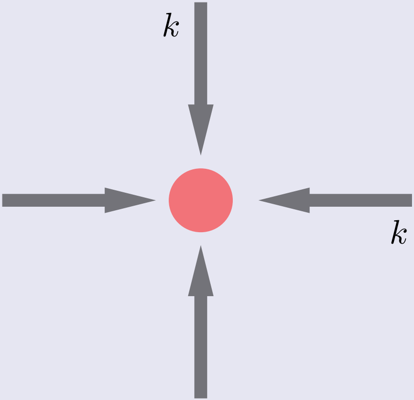
## Lasers

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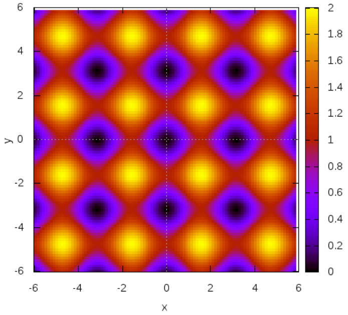

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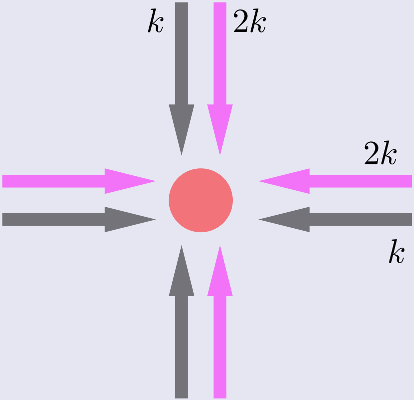


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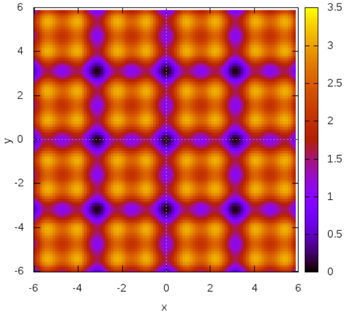

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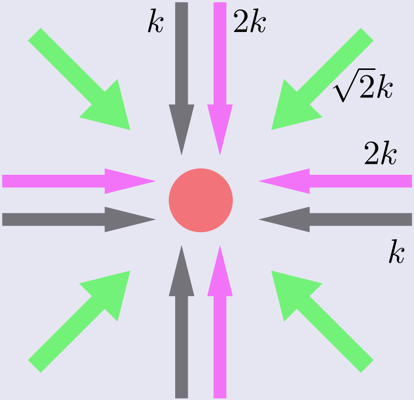


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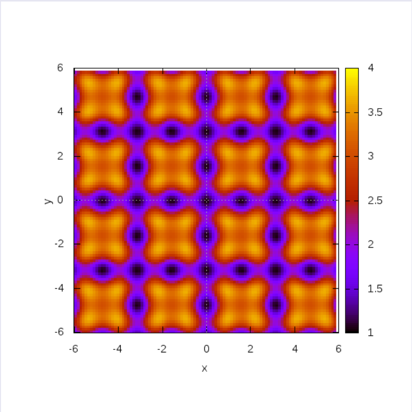

# Line-Centered Square Optical Lattice

## Tight-binding Hamiltonian

The tight-binding Hamiltonian:

$$H = -t \sum_{m,n,\alpha} \left\{ \left( a^{\dagger}_{m,n,1,\alpha} + a^{\dagger}_{m,n,2,\alpha} + a^{\dagger}_{m-1,n,1,\alpha} + a^{\dagger}_{m,n-1,2,\alpha} \right) a_{m,n,0,\alpha} + \mathrm{H.c.} \right\},$$

$(m, n)$ numbers the elementary cell, $\alpha = A, B$ - two internal states,

$a^{\dagger}_{m,n,s,\alpha}$, $a_{m,n,s,\alpha}$ - creation and annihilation operators:

$$a^{\dagger}_{m,n,s,\alpha}|0\rangle = |m, n, s, \alpha\rangle,$$
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# Dispersion

## Hamiltonian

After the Fourier transformation the Hamiltonian becomes block-diagonal:

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with

$$H_{\kappa,\xi,\alpha} = b_1^\dagger \Omega\left(\kappa\right) b_0 + b_2^\dagger \Omega\left(\xi\right) b_0 + \mathrm{H.c.}$$

Wave vector:

$$\mathbf{k} = \frac{\pi}{d}\left(\kappa \mathbf{e}_x + \xi \mathbf{e}_y\right).$$

Function $\Omega\left(x\right)$:

$$\Omega\left(x\right) = \cos\left(\frac{\pi}{2}x\right).$$

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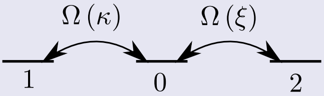
One block describes an effective three level system.

## Three-level system

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The eigenvalue equation

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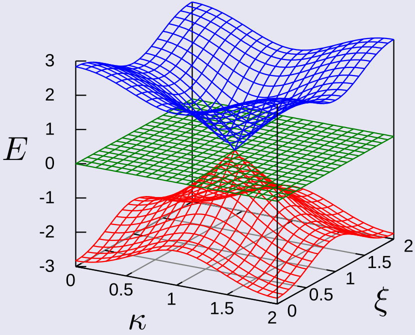

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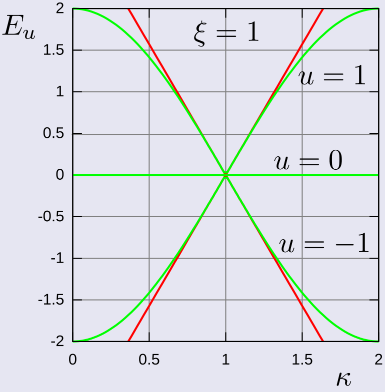

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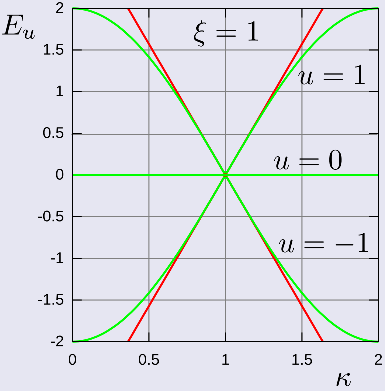
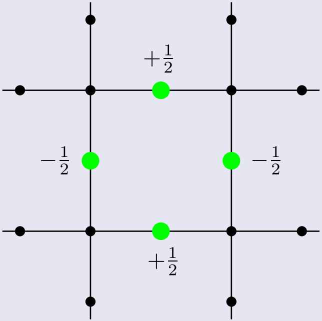
- Other two bands touches the flat one and form Dirac cone.
- They are approximately linear near the zero energy.

## Dispersion bands



# Localized States

## One localized state



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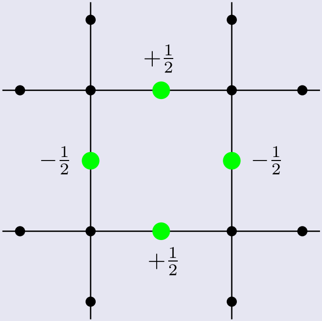


## Creation operator

A localized state is created with the operator

$$R^{\dagger}_{m,n,\alpha} = \frac{1}{2}\left(a^{\dagger}_{m,n,1,\alpha} - a^{\dagger}_{m,n,2,\alpha} + a^{\dagger}_{m,n+1,1,\alpha} - a^{\dagger}_{m+1,n,2,\alpha}\right).$$

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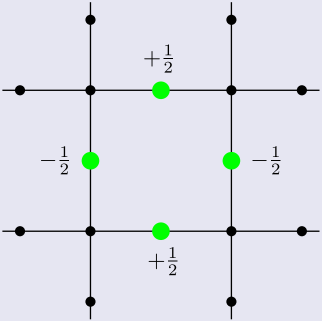


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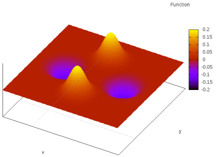


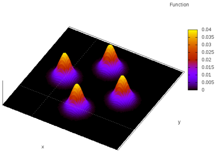

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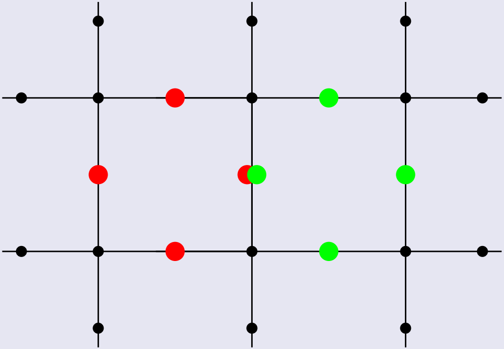

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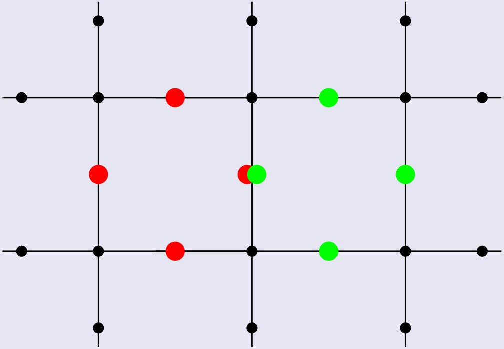

## Inner product

$$\langle 0 | R_{m,n,\alpha} R^{\dagger}_{m\pm1,n,\alpha} | 0 \rangle \neq 0$$

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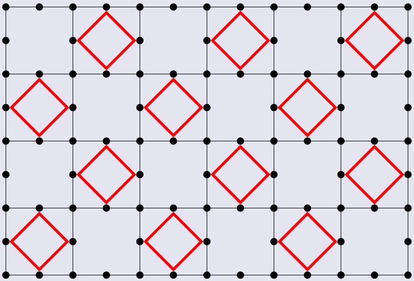
We can divide the localized states into two subsets:

**$m + n$ is even (Even states)**



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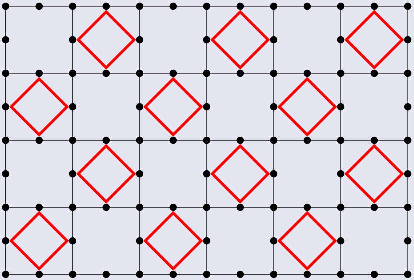

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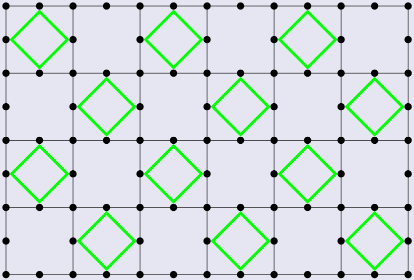

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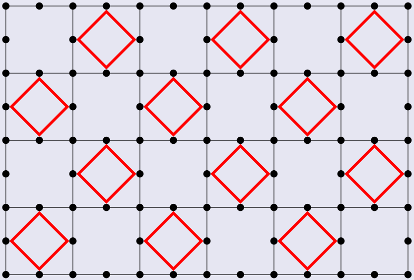

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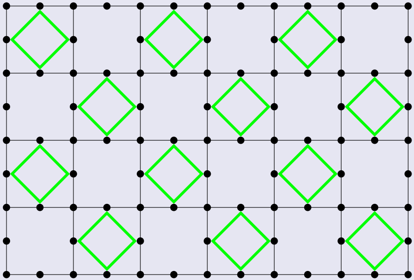

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# Localized States

- We would like to construct orthogonal basis.
- It is hard to do orthogonalization in current basis.
- Before orthogonalization we perform Fourier transformation for each subset of localized states:

$$|R_{m,n,\alpha,\mathrm{even}}\rangle \Rightarrow |F_{K_x,K_y,\alpha}\rangle\,, \qquad |R_{m,n,\alpha,\mathrm{odd}}\rangle \Rightarrow |G_{K_x,K_y,\alpha}\rangle$$

- and then do the orthogonalization procedure: $|G\rangle \Rightarrow |\tilde{G}\rangle$
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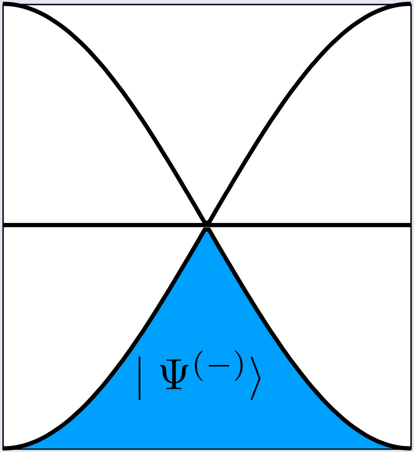
## Filing



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Let's fill all negative-energy band states with both types of atoms. $\Rightarrow$

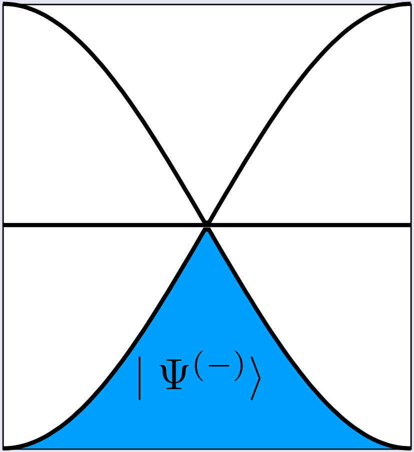
The lattice sites $s = 1$ and $s = 2$ will be filled equally.

This will be the new Fock vacuum: $|0'\rangle = |\Psi^{(-)}\rangle$

We shift energy to zero level...

Now we will work in the flat zero-energy band.

## Filing



# Many Body Effects

Consider the Hubbard-type interaction. We add the following two-body operator to the Hamiltonian:

$$V_H = U \sum_{m,n,s} n_{m,n,s,A} n_{m,n,s,B}$$

The interaction is small and repulsive ($U > 0$).

Example: two atoms ($A$ and $B$) in the same lattice site.

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**With interaction**

Energy is bigger with interaction: $E = E_1 + E_2 + U$.

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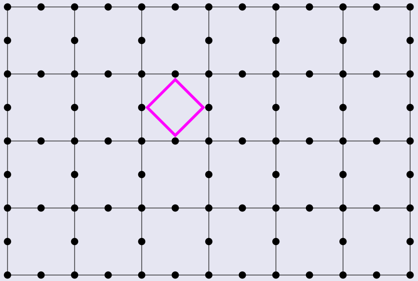

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- Put one A atom to the system.
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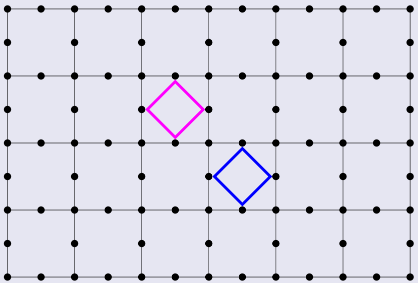

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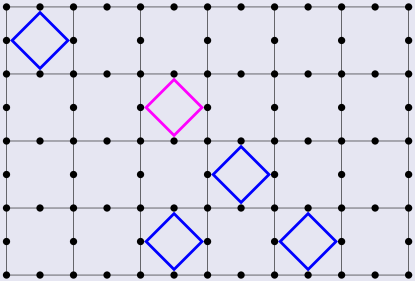

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- Put more A atoms.
- They can go to superpositions of localized states.

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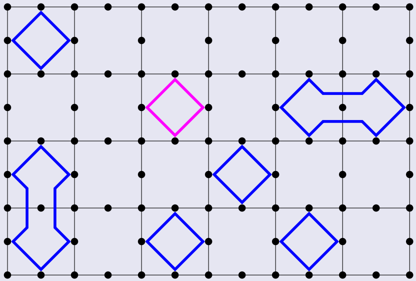

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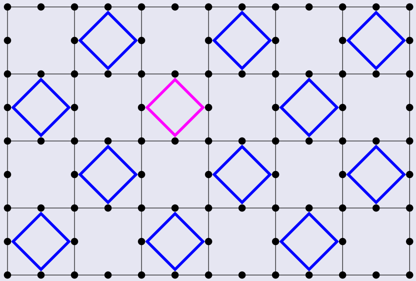
- Put one A atom to the system.
- Both atoms will go to localized states.
- Put more A atoms.
- They can go to superpositions of localized states.
- At the filling $\frac{1}{2}$ all atoms will be in localized states.

## Picture



# Many Body Effects

- By putting more $A$ atoms to the system, they will go to orthogonalized states $|\tilde{G}\rangle$.
- The orthogonalized states $|\tilde{G}\rangle$ have distrubuted probability apmplitudes over all lattice sites $s = 1$ and $s = 2$.
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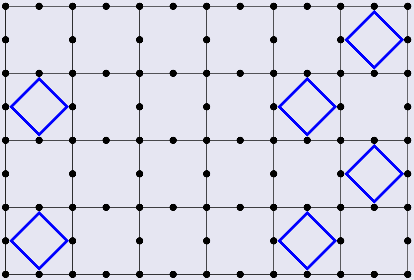

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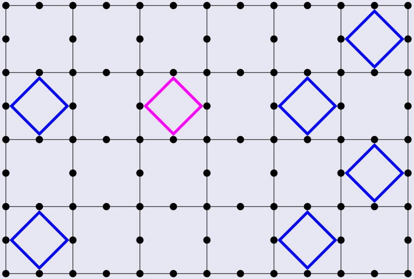

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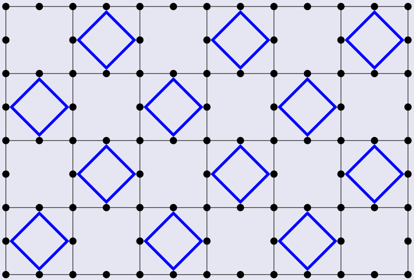

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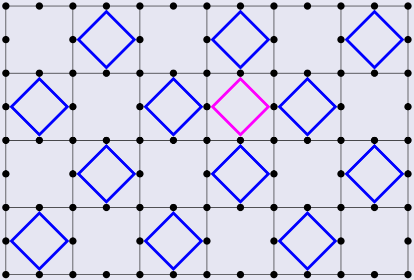

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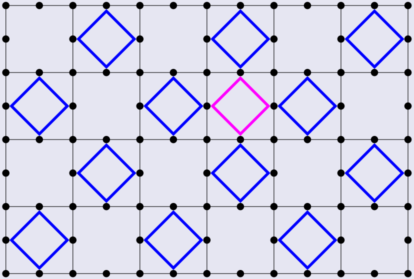
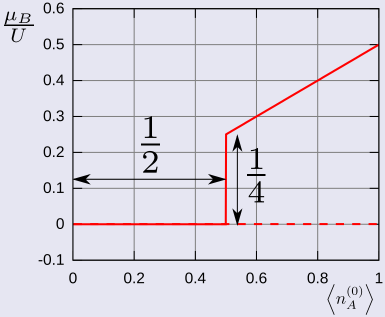


## Chemical potential



# Conclusions

- Dispersion of the LCS lattice has flat band.
- Two other bands touches the first one forming the Dirac cone.
- The localized states can be formed in the flat band.
- Atoms go to these states when on-site interaction is presented.
- Chemical potential of the atoms jumps when the filling reaches $\frac{1}{2}$.
- In low temperatures with the filling $< \frac{1}{2}$ the many-body state of atoms is ferromagnetic.

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Thank you