Two component Bose-Hubbard model with higher angular momentum states

M.Brewczyk - UwB
M. Gajda, **J. Pietraszewicz**, T. Sowiński - IF PAN
Jakub Zakrzewski – UJ
Maciej Lewenstein – ICFO, ICREA

Kraków 2011

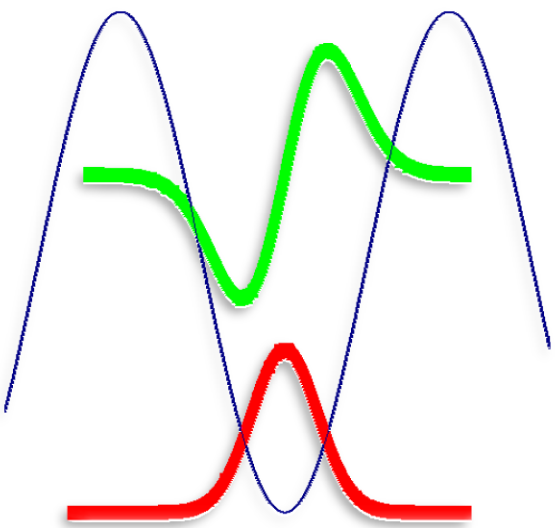
Introduction

- We combine three areas of ultracold physics :
  - ultracold dipolar gases
  - spinor gases in a lattice (in the context of MI and SF transition)
  - orbital superfluid
- The main issue is to account for the spin degree of freedom as a dynamical variable in the lattice.
- When spin dynamics takes place it could lead to the appearance of an orbital $(P_x + i P_y)$ superfluid.
- Moreover it introduces an additional degree of control and leads to variety of different stable phases (PhD for small particle number).

Phase Diagram

Assumptions

- 2D square optical lattice with Cr atoms
- Limit basis to two states coupled by dipolar interaction at every lattice site



$$m_S = 2 \; ; \quad l = 1$$

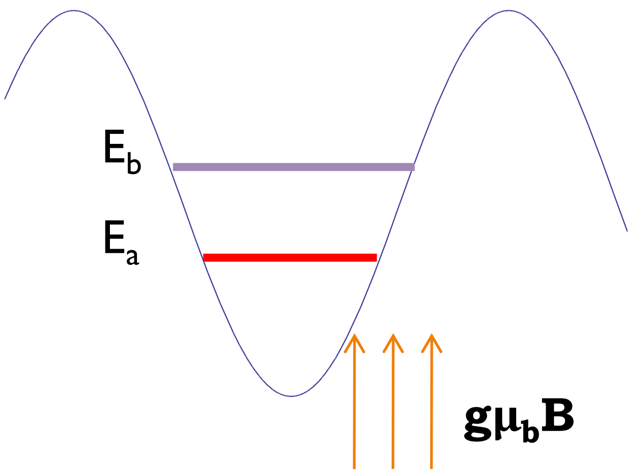
$$\Psi_b(\vec{r}) \;\sim\; \left(w_1(x)\; w_o(y) + i\; w_0(x)\, w_1(y) \right)\; e^{-\frac{\Omega\, z^2}{2}}$$

$$m_S = 3 \; ; \quad l = 0$$

$$\Psi_a(\vec{r}) \;\sim\; w_o(x)\; w_o(y)\; e^{-\frac{\Omega\, z^2}{2}}$$

- Limiting subspace of essential states is a cruical approximation and it is possible only due to a weakness of dipolar interactions.

What's the influence of weak dipolar interactions?



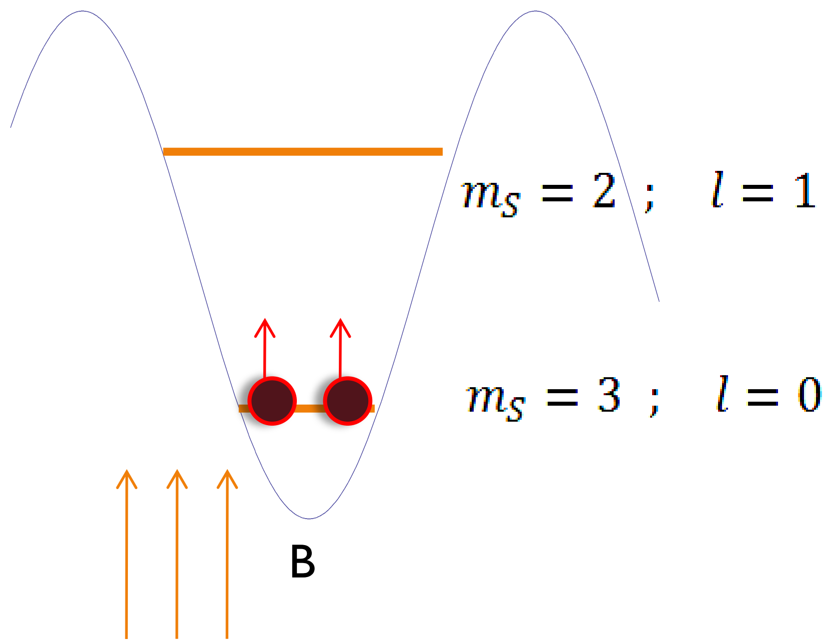
- Equlibration of the energy difference

$$E_b - E_a \approx E_{dip}$$

Spin dynamic triggered by dipolar interactions

- There are several channels of dipolar collisions for two atoms. Fortunately we can choose the desired channel by a proper adjustment of the resonant external magnetic field.

$$|2,0\rangle \overset{D}{\Rightarrow} |0,2\rangle$$



Hamiltonian of the system

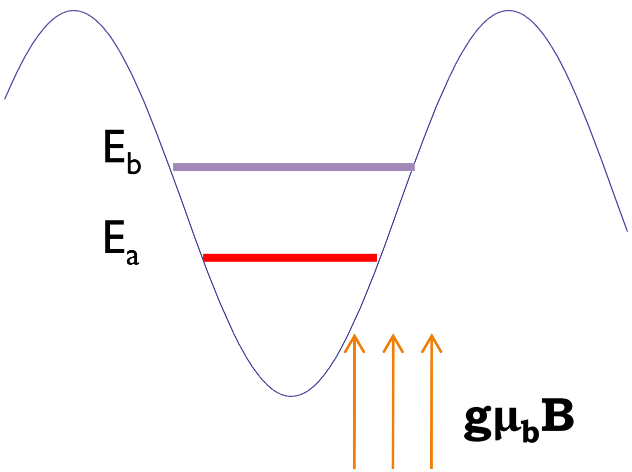
$$H = \sum_{i} (E_a + B)\ a_i^+ a_i + E_b\ b_i^+ b_i +$$

$$+ U_a\ a_i^+ a_i^+ a_i a_i + U_b\ b_i^+ b_i^+ b_i b_i + U_{ab}\ a_i^+ a_i^+ b_i b_i$$

$$+ D\ (\ a_i^+ a_i^+ b_i b_i + b_i^+ b_i^+ a_i a_i) + H_1(J_a, J_b)$$

$$H_1 = -J_a \sum_{\langle i,j \rangle} a_i^+ a_j - J_b \sum_{\langle i,j \rangle} b_i^+ b_j$$

What's the influence of weak dipolar interactions on 1 particle state per site?



- Equlibration of the energy difference

$$E_b - E_a \approx E_{dip}$$

- For 1 particle states average per site

$$|1,0\rangle \Leftrightarrow |0,1\rangle$$

- The lowest order process which contributes to the transfer between these state is a sequence of three events :

$$|1,0\rangle \overset{J_a}{\Rightarrow} |2,0\rangle$$

$$|2,0\rangle \overset{D}{\Rightarrow} |0,2\rangle$$

$$|0,2\rangle \overset{J_b}{\Rightarrow} |0,1\rangle$$

Fisher method to find thermodynamically stable phases of the system in a choosen subspace

- In the Fisher method we assume for all sites that : $\langle a_i \rangle = \phi_{a_i}$

$$\langle b_i \rangle = \phi_{b_i}$$

$$H_1 = -J_a \sum_{\langle i,j \rangle} a_i^+ a_j - J_b \sum_{\langle i,j \rangle} b_i^+ b_j \longrightarrow H_1 = -J_a \sum_i (a_i^+ \phi_{a_i} + a_i \phi_{a_i}^*) - J_b \sum_i (b_i^+ \phi_{b_i} + b_i \phi_{b_i}^*)$$

$$H_o \to H_o - \mu \sum_i (a_i^+ a_i + b_i^+ b_i)$$

- Boundaries between MI and SF are obtained from :

$$\phi_{(a)} = \lim_{\beta \to \infty} \frac{Tr\left[\hat{a}\, e^{-\beta (H_o + H_1)} \right]}{Z(\beta)}$$

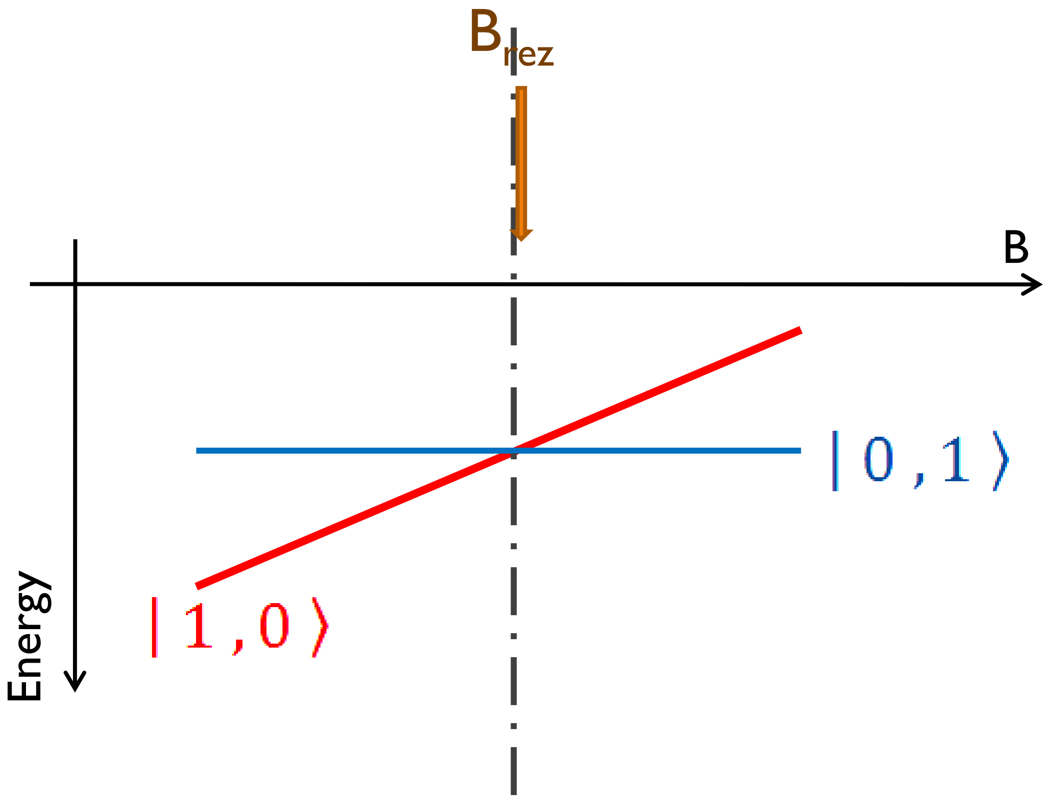
Z is the grand canonical partition function which reduces to a single lowest energy state contribution.

- To lowest order we get a linear and homogenous set of equations.

Single particle states in magnetic field

$$\mu < 0 \quad \rightarrow \quad |\,0,0\,\rangle$$

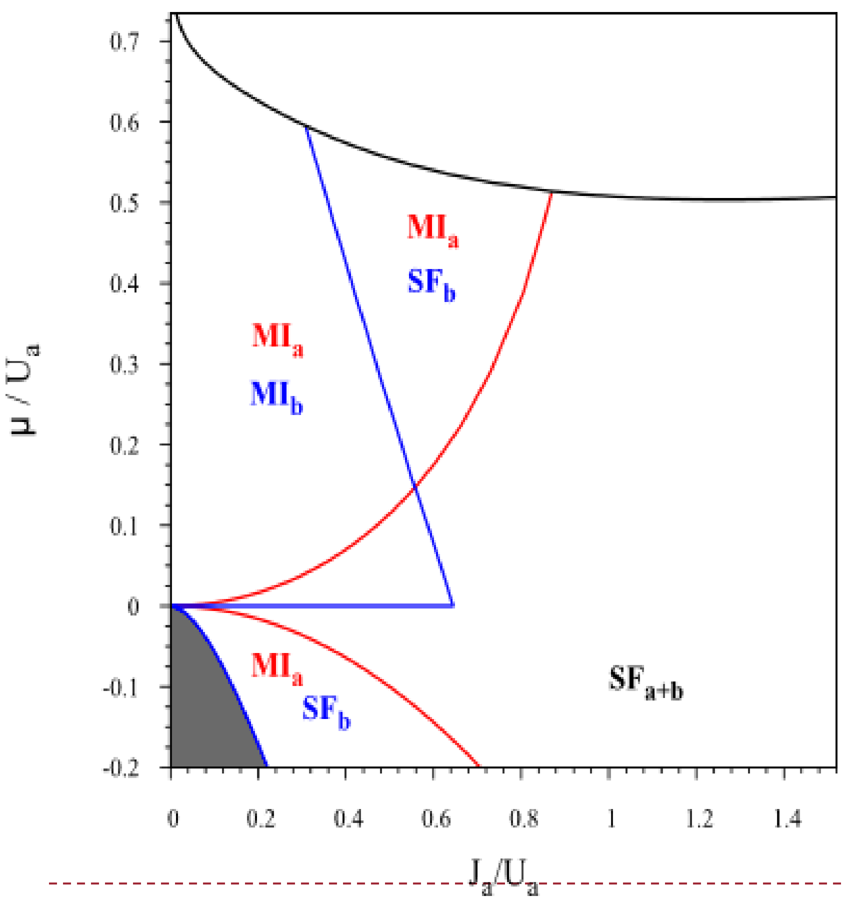
$$\mu > 0 \quad \rightarrow \quad |\,1,0\,\rangle \quad or \quad |\,0,1\,\rangle$$



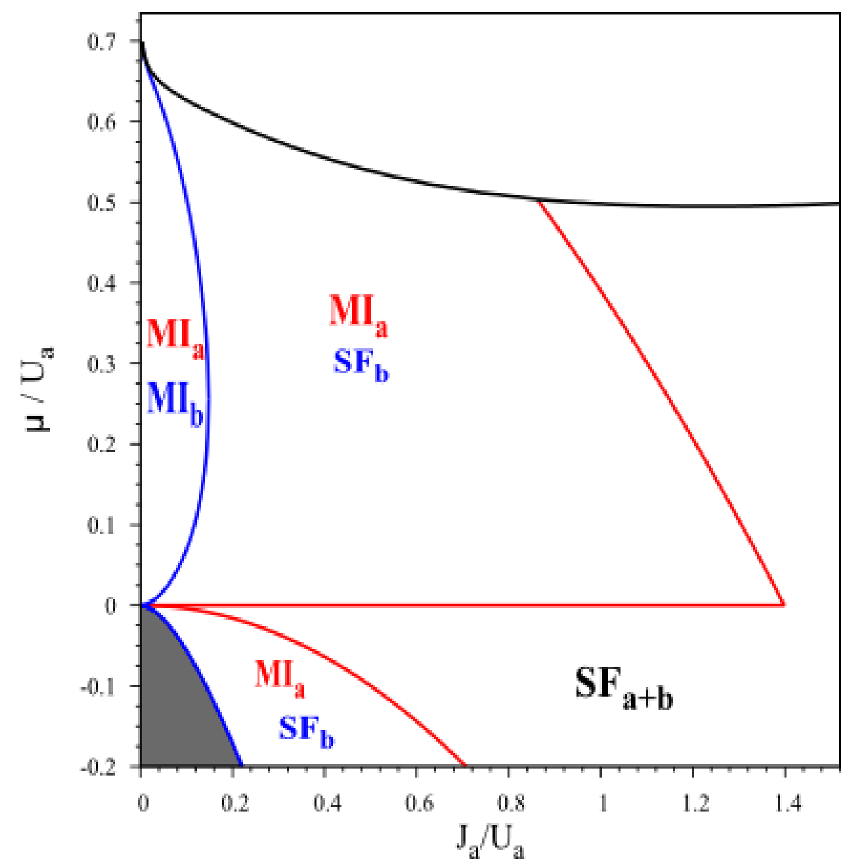
Phase Diagram –

regions of stability of different posible phases of the system

$|1,0\rangle$



$|0,1\rangle$



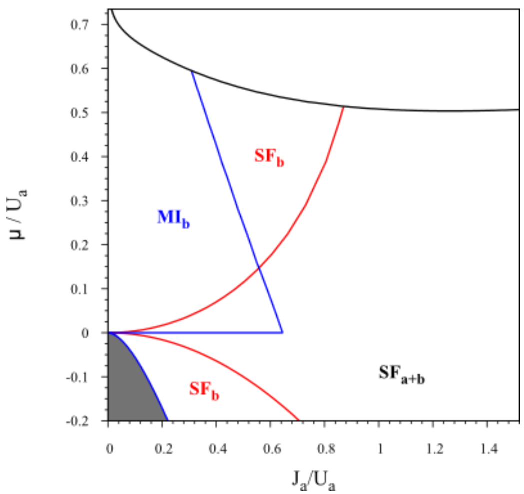
Final conclusions

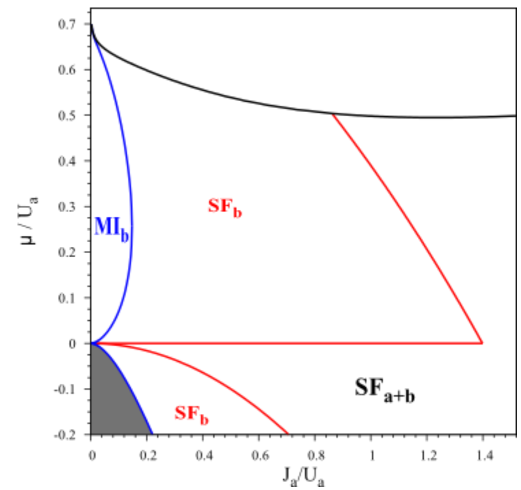
- Dipolar interactions can lead to novel phases, in particular to the appearance of orbital ($P_x + i P_y$) superfluids in the b - component.

- The experiments with ultra weak magnetic fields with Cr atoms in the lattice are under extensive studies of B. Laburthe-Tolra group in Paris (PRA **81**, 042716 (2010)) .

THX

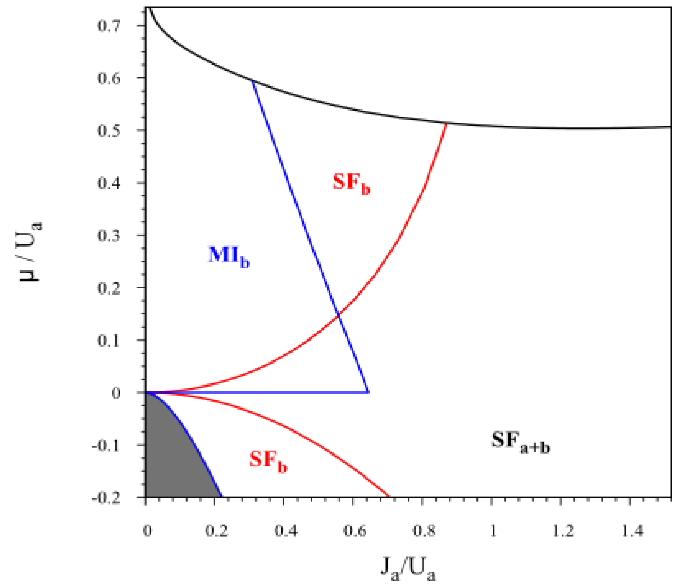
Region $\frac{\mu}{U_a} < 0$



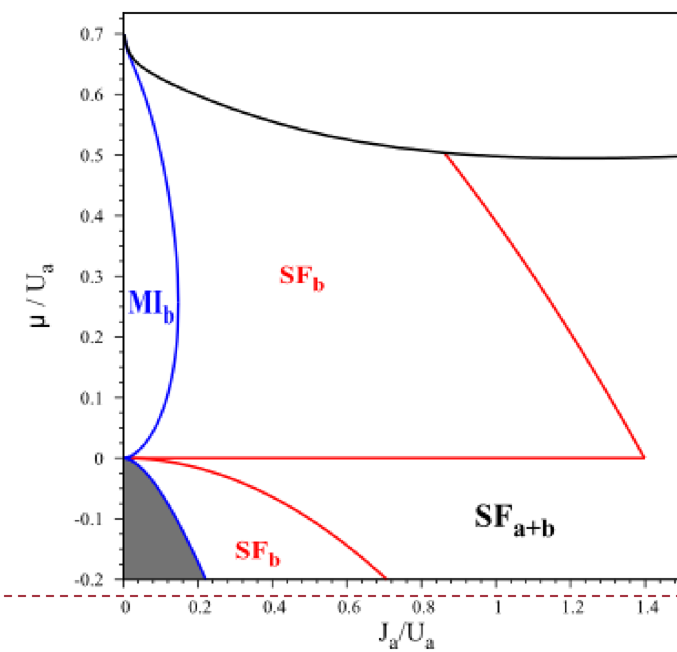


- System is in superfluid phase (the mean occupation is fractional)
- Large tunneling supports the 'standard' SF_a and orbital $(P_x + i\,P_y)$ SF_b .
- When decreasing tunneling - particles enter SF_b .
- The grey area corresponds to the 'stable vacuum'.

Region $0 < \frac{\mu}{U_a} < \frac{2\,U_b}{U_a}$



- Large tunneling - SF_{a+b}
- Lower tunneling – MI_a ($n_a = 1$) , SF_b
- There is an additional stable phase – small region of the Mott insulator in the vortex component MI_b ($n_b = 0$).



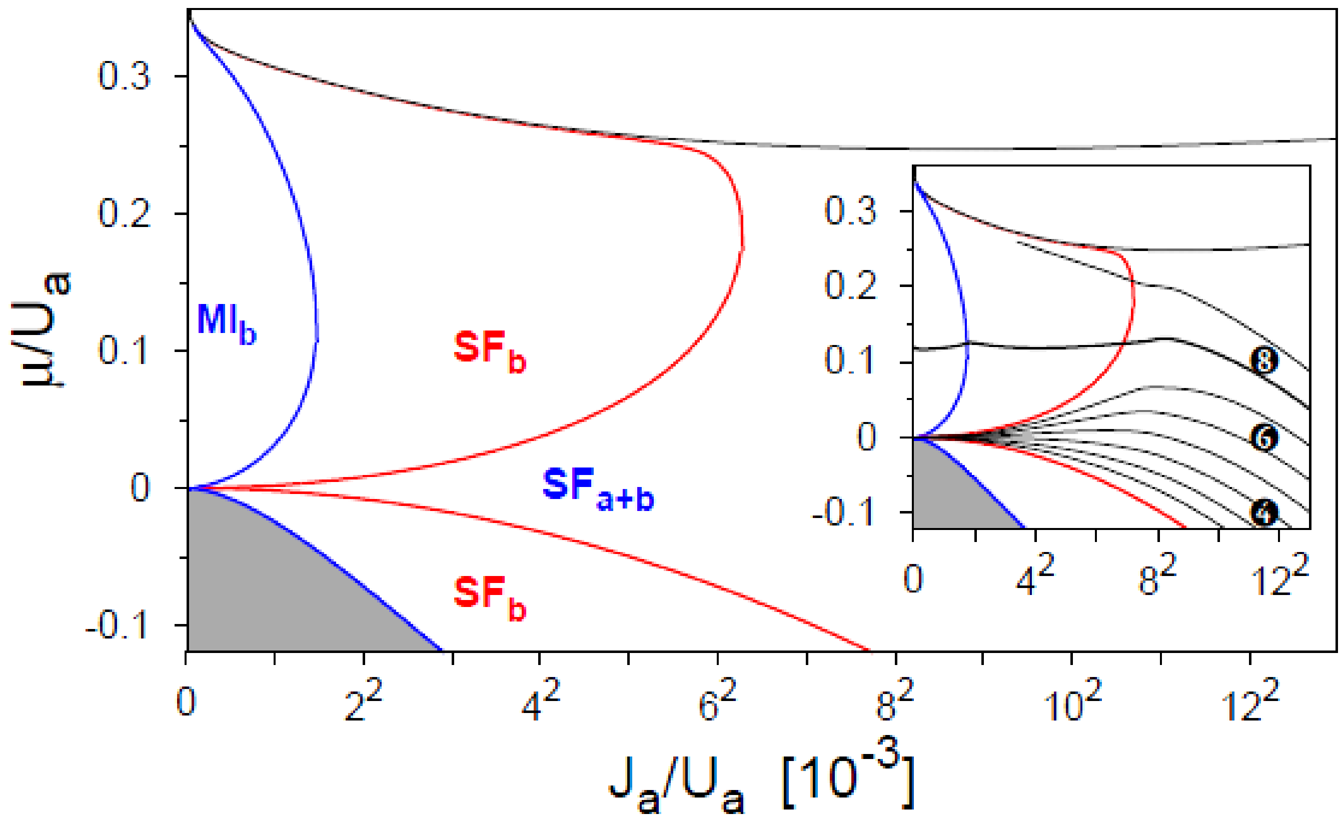
- Large tunneling - SF_{a+b}
- Lower tunneling – MI_a ($n_a = 0$) , SF_b
- There is an additional stable phase – small region of the Mott insulator in the vortex component MI_b ($n_b = 1$).

Final conclusions

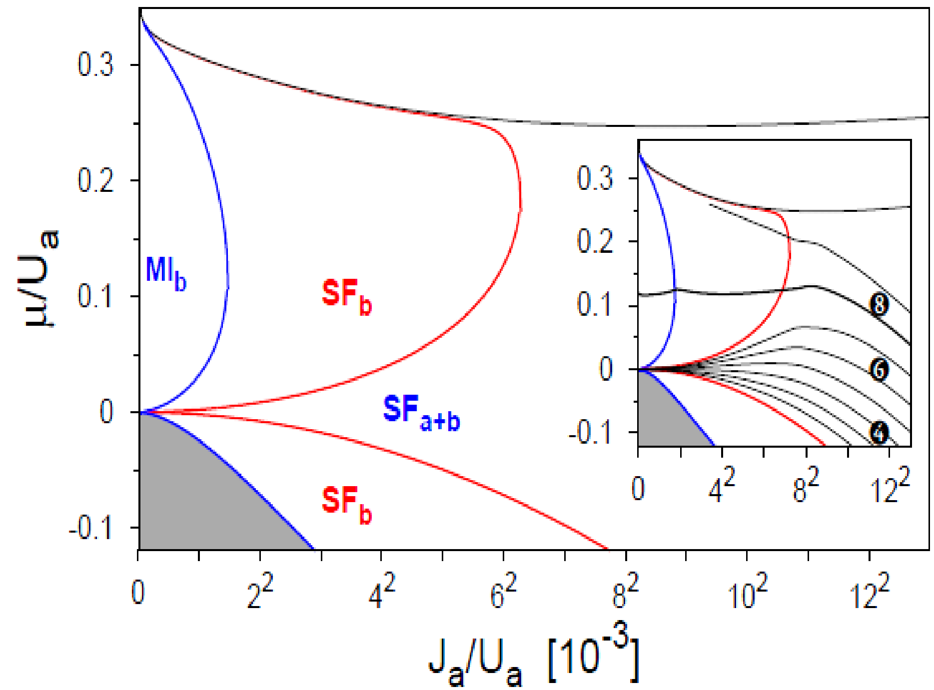
- Even the case of one particle on average per site can introduce various novel phases to the system (especially the orbital superfluids in the excited energy state).
- Weak dipolar interaction can be resonantly tuned to couple the ground Wannier state to the excited one with orbital angular momentum.
- In future …
- When two particles occupy the same site, it is more favorable for dipolar transfer.

Phase Diagram –

regions of stability of different posible phases of the system

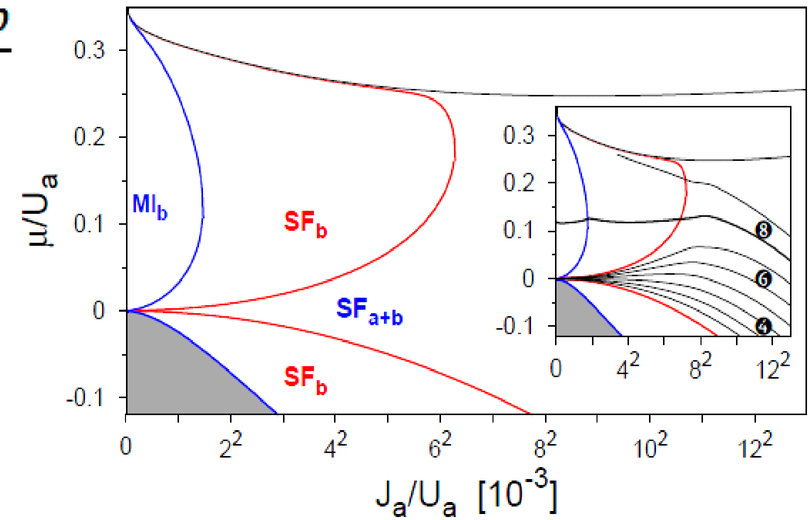


Region $\frac{\mu}{U_a} < 0$



- System is in superfluid phase (the mean occupation is fractional)
- Large tunneling supports the 'standard' SF_a and orbital $(P_x + i\,P_y)$ SF_b .
- When decreasing tunneling - particles enter SF_b .
- The grey area corresponds to the 'stable vacuum'.

Region $0 < \frac{\mu}{U_a} < \frac{2\,U_b}{U_a}$



- Large tunneling - SF_{a+b}
- Lower tunneling – SF_b
- There is an additional stable phase – small region of the Mott insulator in the vortex component MI_b (1 particle devide oneself like $n_b = 1$, $n_a = 0$) .

Why doesn't MI_a exist ?

Why do particles choose to be in the b component?

Athough the energies are equal at the resonance, larger tunneling in the vortex states favoure the b-component.

Final conclusions

- Even the case of one particle on average per site can introduce various novel phases to the system.
- Weak dipolar interaction can be resonantly tuned to couple the ground Wannier state to the excited one with orbital angular momentum.
- System realises the scenarios in which energy of the system is lower - favourable phases are the orbital ($P_x + i P_y$) superfluid or vortex Mott Insulator phases.
- In future …
- When two particles occupy the same site, it is more favorable for dipolar transfer.