Coherent transport of waves in nonlinear disordered media

T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner (U Freiburg)

B. Grémaud (NUS Singapore/LKB Paris)

Kraków, 01.09.2011

Outline

1.) Introduction: coherent transport

2.) Nonlinear coherent backscattering

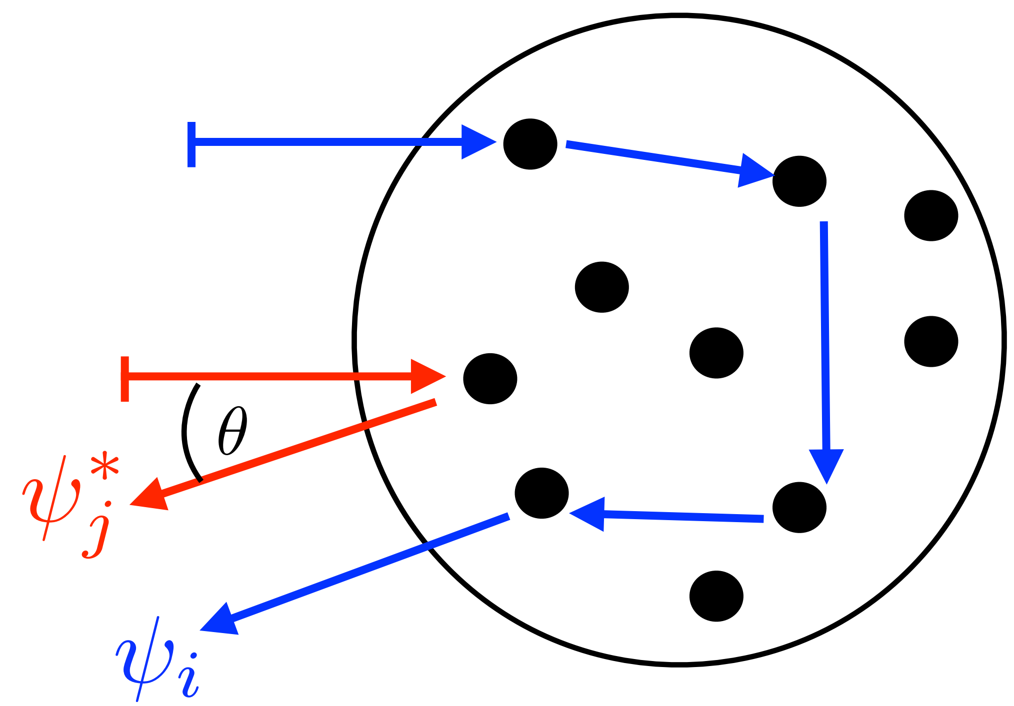
3.) Multiple scattering of intense laser light by cold atoms

1.) Introduction: Coherent transport

Multiple scattering: Interference

$$\boxed{\psi = \sum_{\text{paths } i} \psi_i}$$

Born series

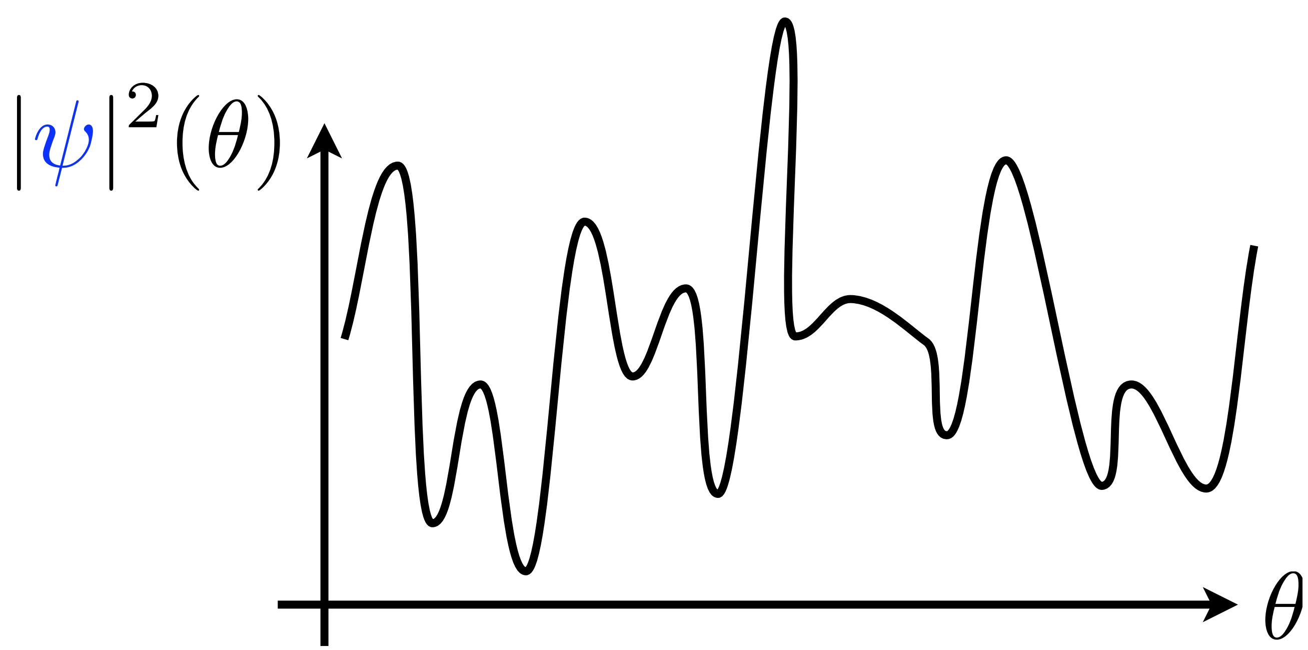


$$|\psi|^2 = \sum_{i,j} \psi_i \psi_j^*$$

$$= \sum_i |\psi_i|^2 + \sum_{i \neq j} \psi_i \psi_j^*$$

Interferences:

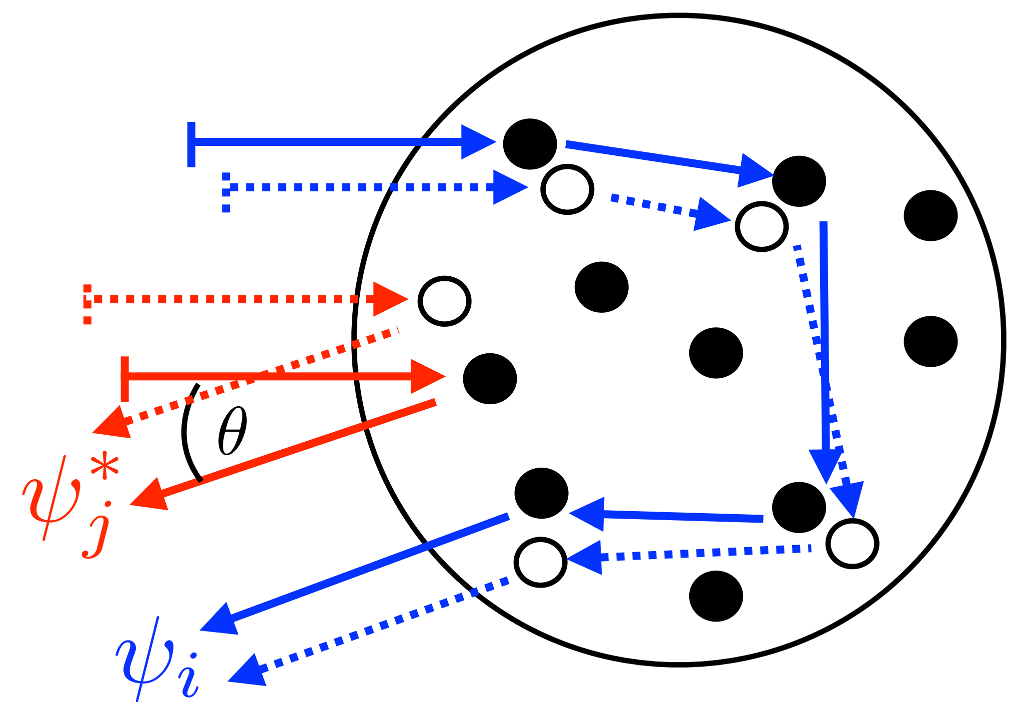
$\rightsquigarrow$ speckle



Multiple scattering: Interference

$$\psi = \sum_{\text{paths } i} \psi_i$$

Born series

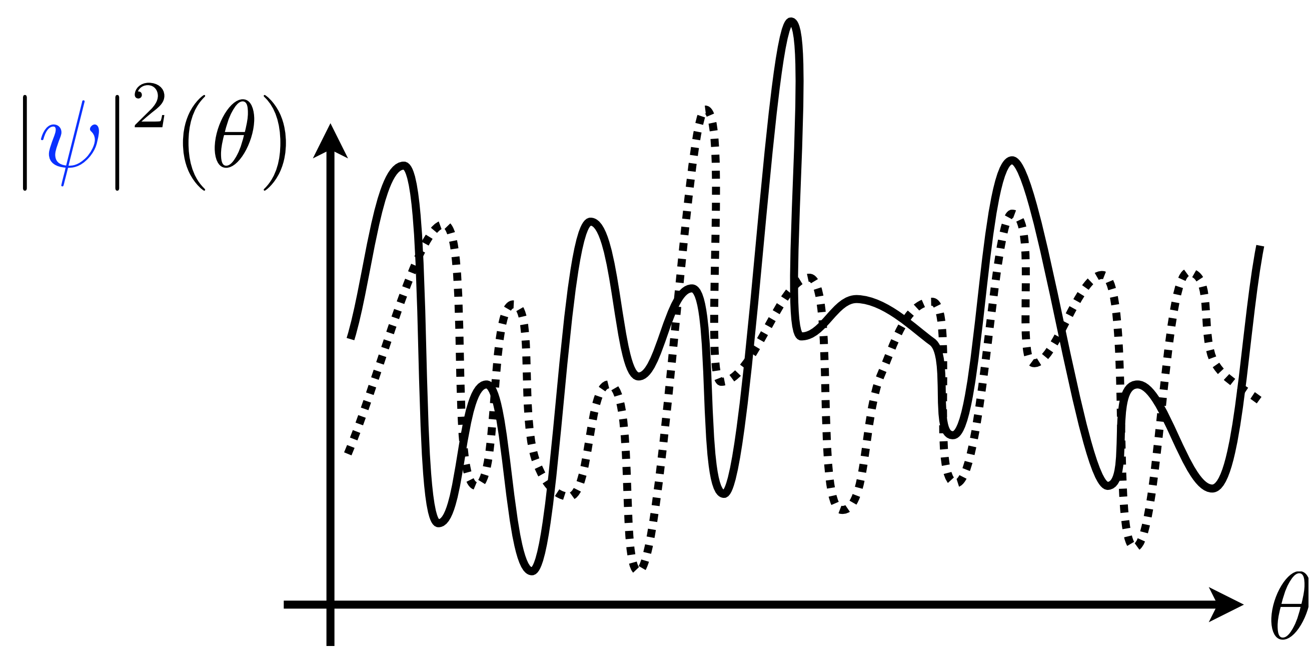


$$|\psi|^2 = \sum_{i,j} \psi_i \psi_j^*$$

$$= \sum_i |\psi_i|^2 + \sum_{i \neq j} \psi_i \psi_j^*$$

Interferences:

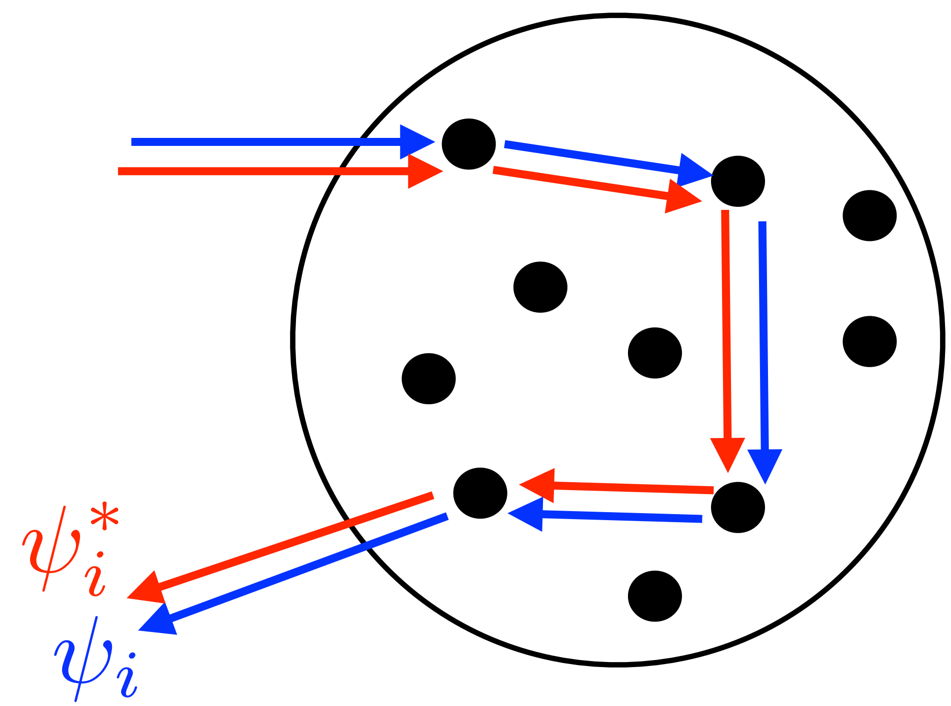
$\rightsquigarrow$ speckle



Multiple scattering: Interference

$$\psi = \sum_{\text{paths } i} \psi_i$$

Born series

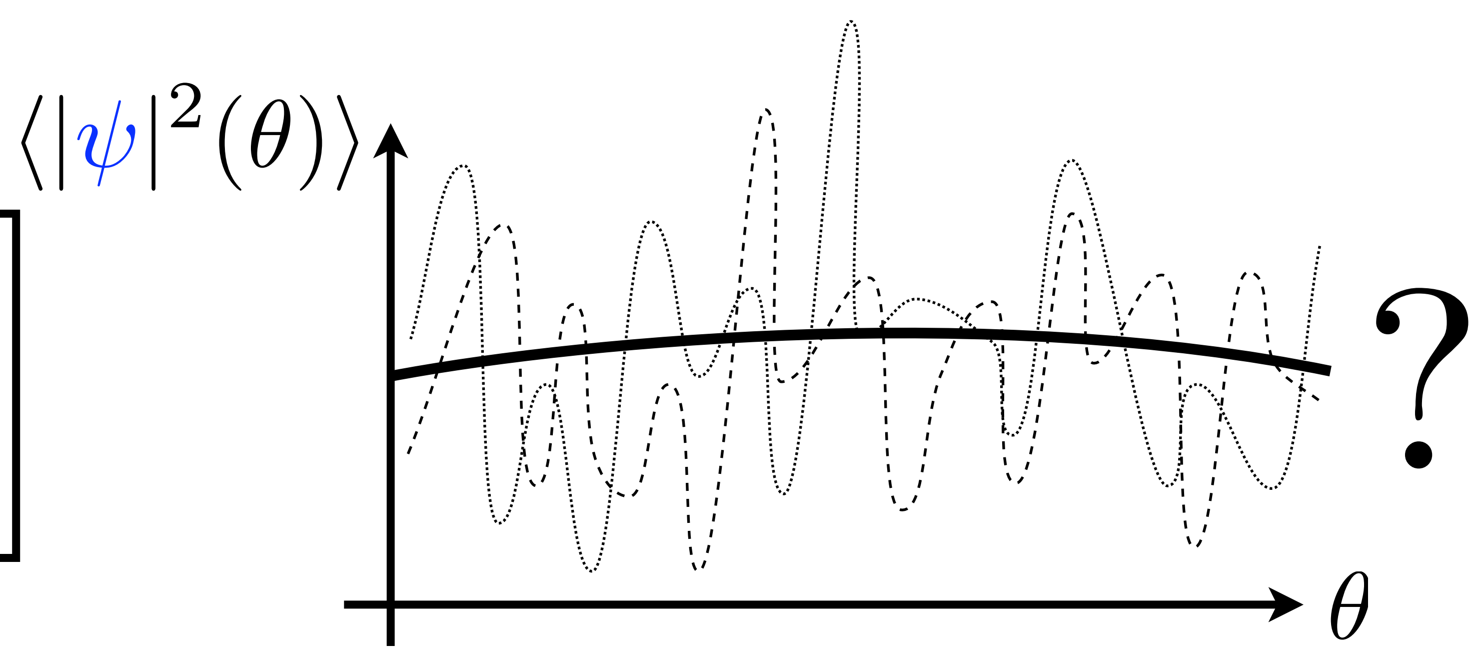


$$\langle|\psi|^2\rangle = \sum_{i,j} \langle \psi_i \psi_j^* \rangle$$

$$= \sum_i \langle|\psi_i|^2\rangle + \sum_{i \neq j} \langle \psi_i \psi_j^* \rangle$$

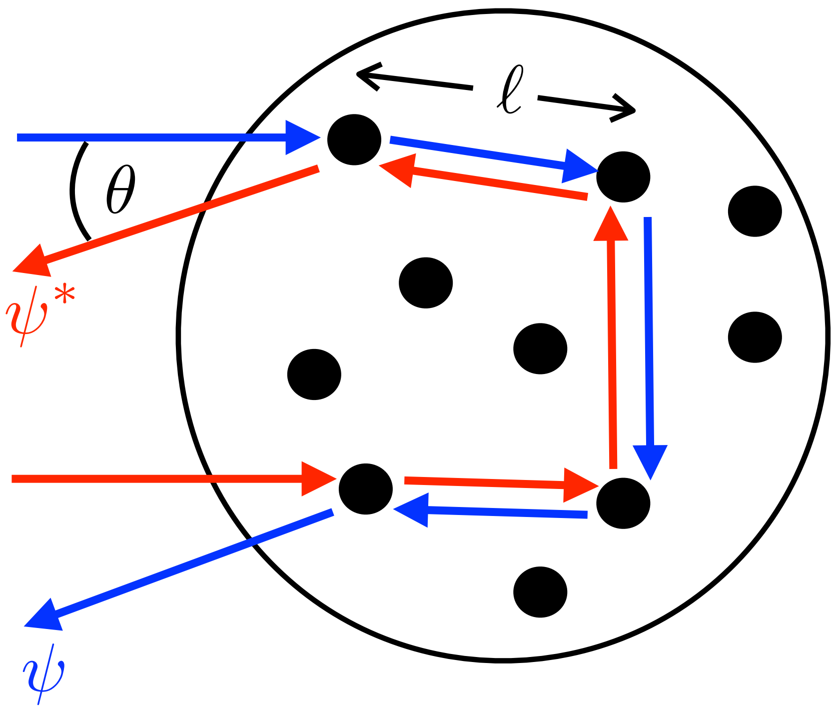
Ensemble average:

Interferences vanish?

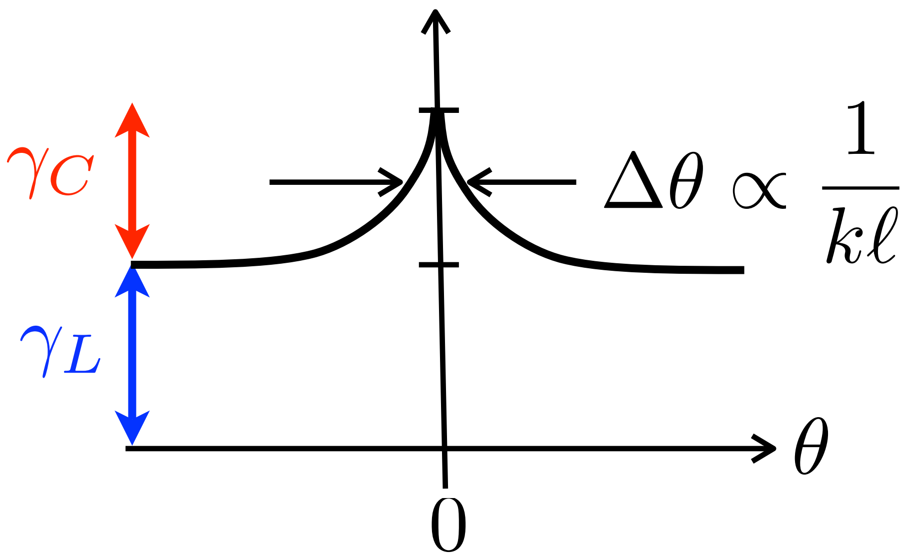


Coherent backscattering (CBS)

Interference between reversed paths
survives disorder average!



$\langle I \rangle = \langle \psi\psi^* \rangle$

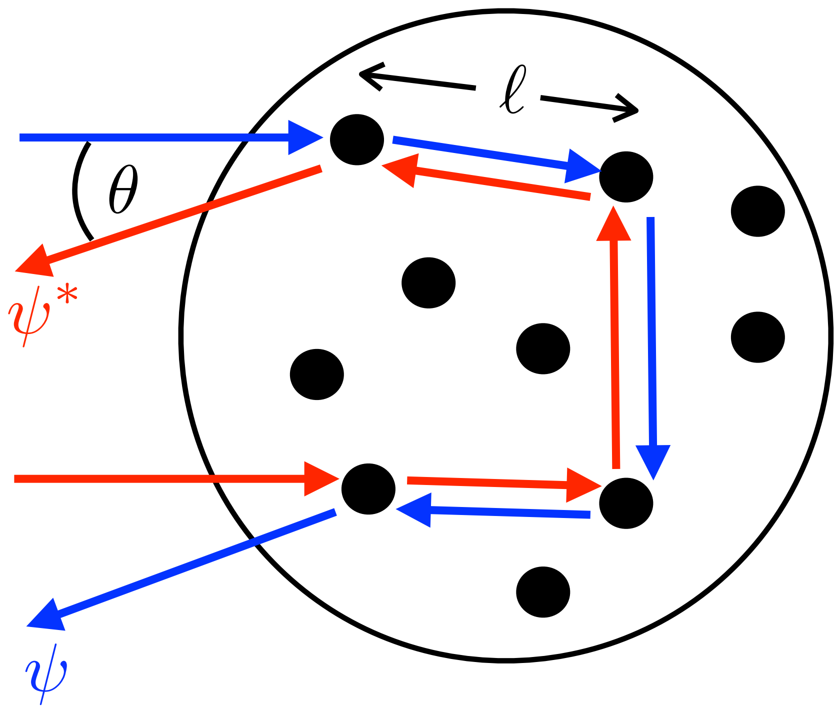


coherent backscattering cone

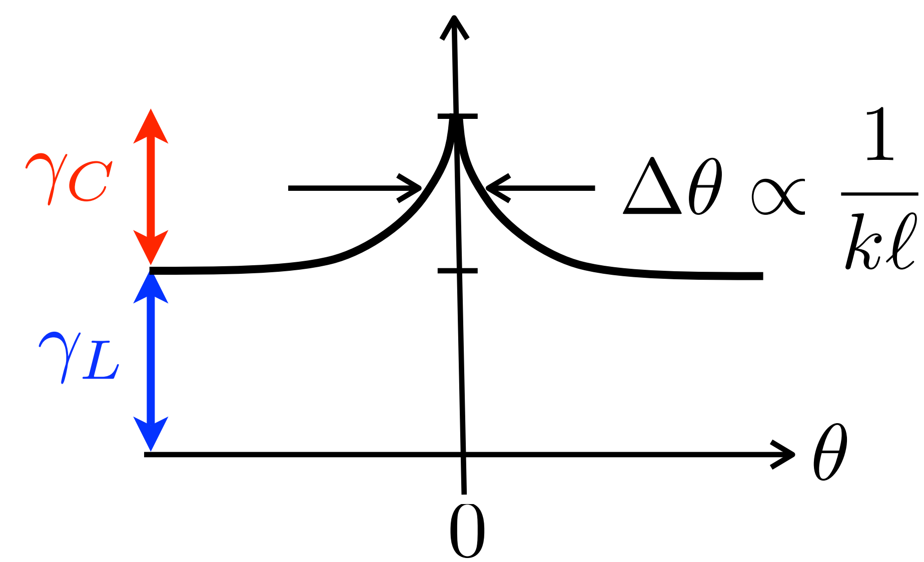
$\gamma_C \leq \gamma_L$

Coherent backscattering (CBS)

Interference between reversed paths
survives disorder average!



$$\langle I \rangle = \langle \psi \psi^* \rangle$$



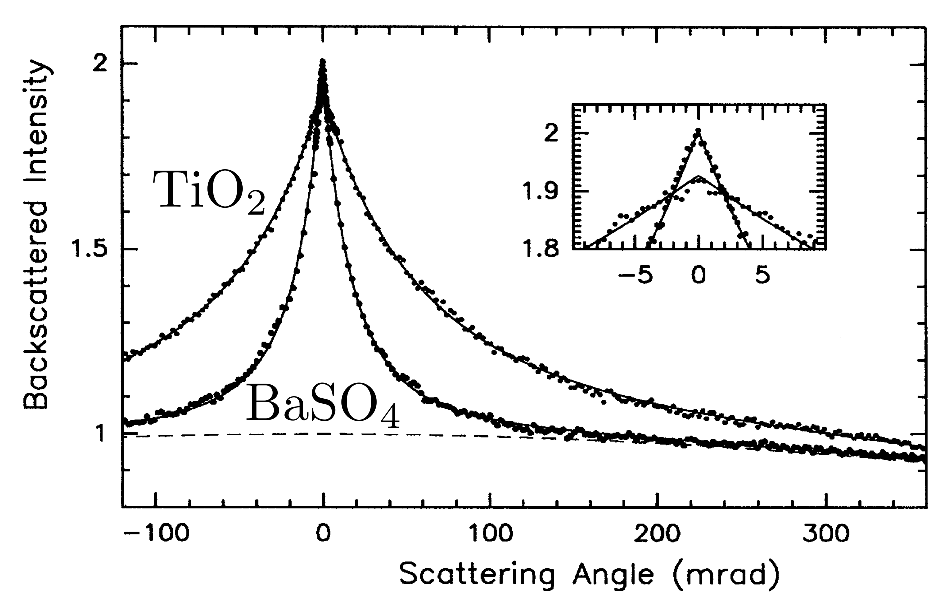
coherent backscattering cone

$$\gamma_C \leq \gamma_L$$

First observation: Saturn‘s rings (1893)

Laboratory experiments: since 1985

D. Wiersma et. al., PRL **74**, 4193 (1995)

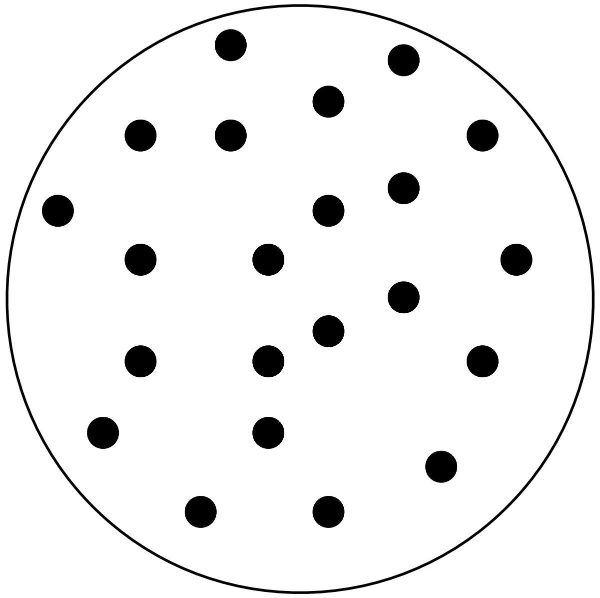


2.) Nonlinear coherent backscattering

T. Wellens (U Freiburg)

B. Grémaud (NUS Singapore/LKB Paris)

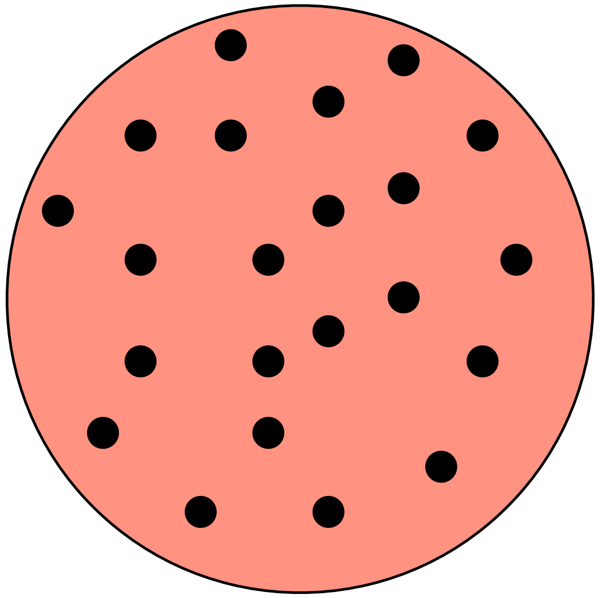
Disorder and Nonlinearity



disordered scatterers

$$\Delta\psi(\mathbf{r}) + k^2\big(1 + V(\mathbf{r})\big)\psi(\mathbf{r}) = 0$$

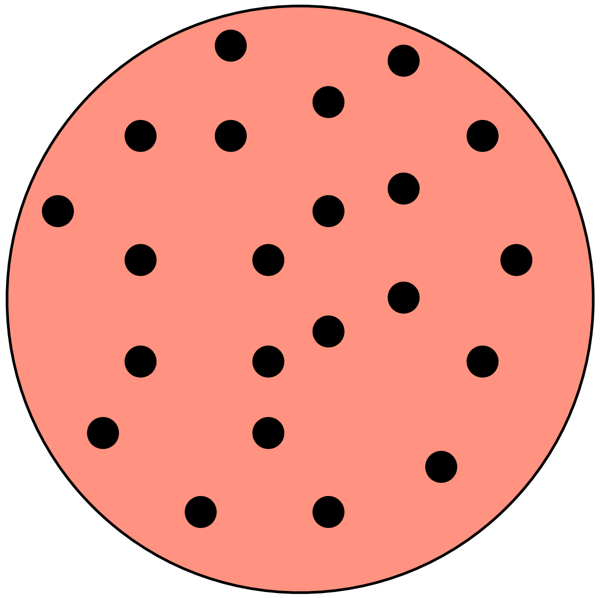
Disorder and Nonlinearity



disordered scatterers
in nonlinear medium

$$\Delta\psi(\mathbf{r}) + k^2\left(1 + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = 0$$

Disorder and Nonlinearity

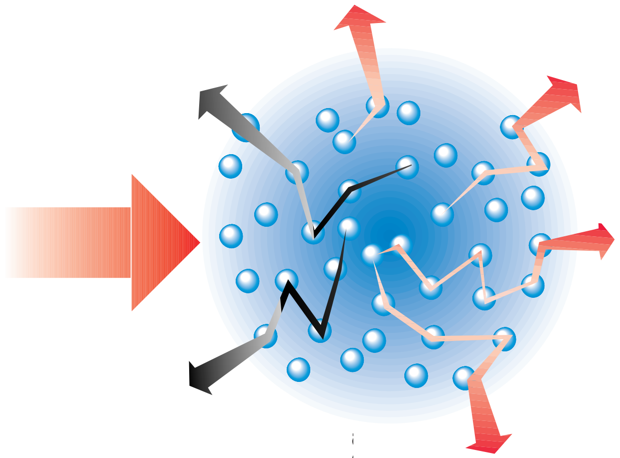


disordered scatterers
in nonlinear medium

$$\Delta\psi(\mathbf{r}) + k^2\Big(1 + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2\Big)\psi(\mathbf{r}) = 0$$

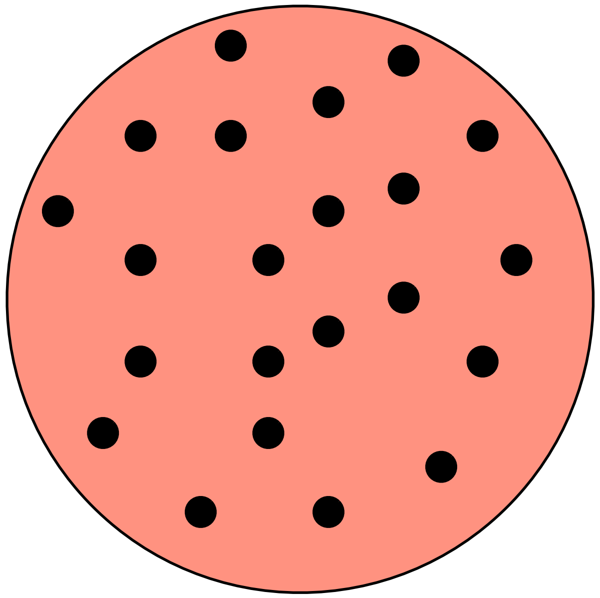
Physical scenarios:

- Multiple scattering of light in nonlinear media
- Matter waves in random potentials
- Random laser



⋮

Disorder and Nonlinearity



disordered scatterers
in nonlinear medium

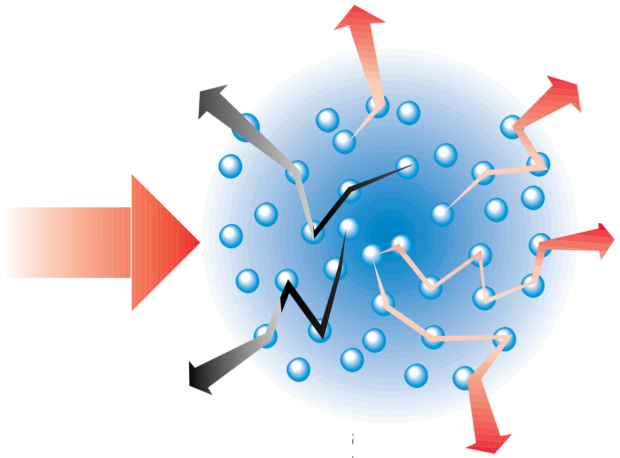
$$\Delta\psi(\mathbf{r}) + k^2\left(1 + V(\mathbf{r}) + g|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = 0$$

Influence of nonlinearity on coherent propagation:

- CBS cone height enhanced or reduced?

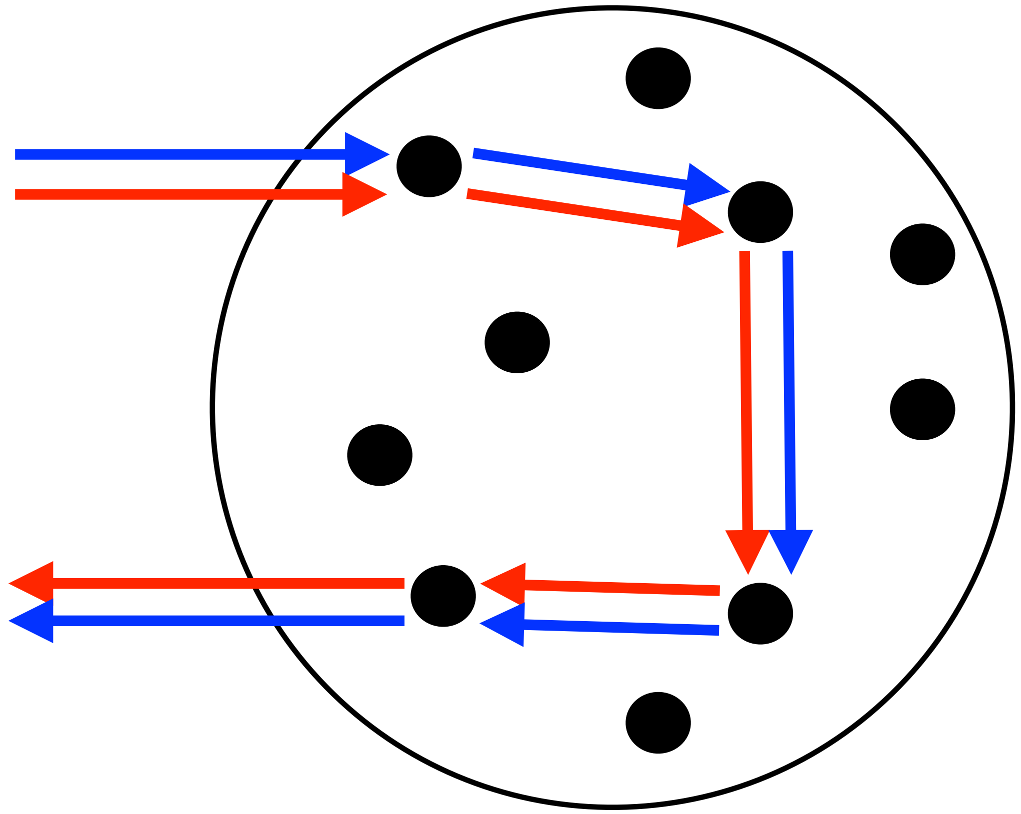
Physical scenarios:

- Multiple scattering of light in nonlinear media
- Matter waves in random potentials
- Random laser



⋮

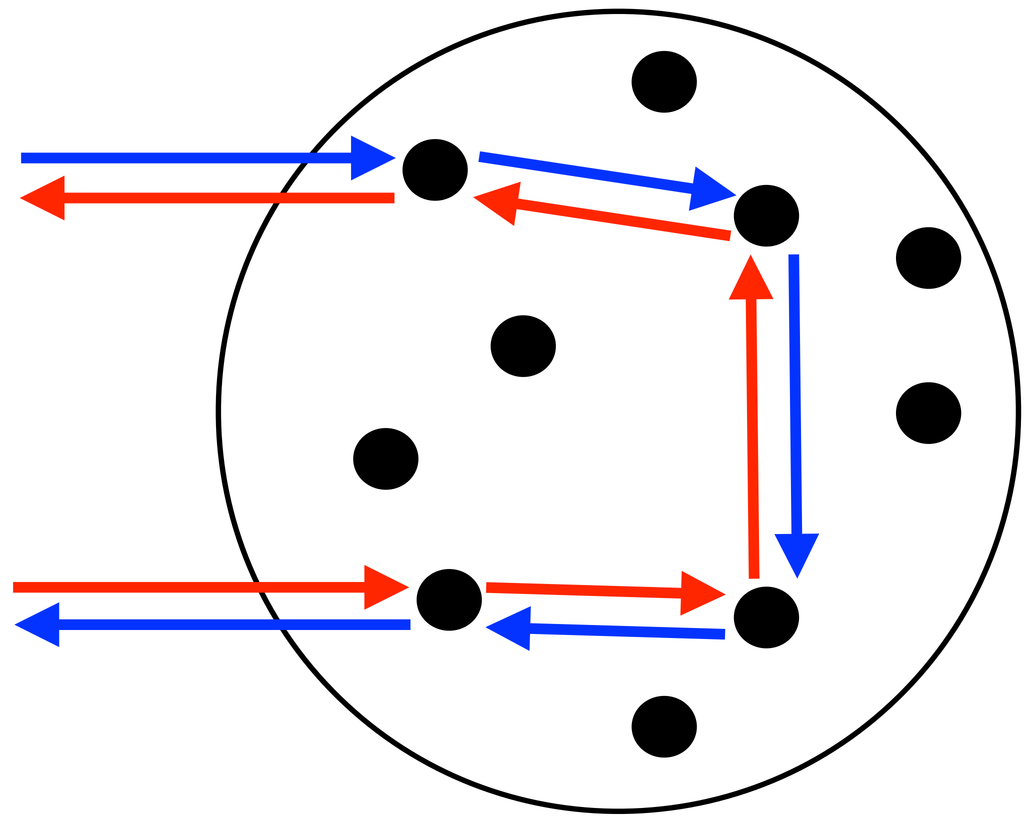
Nonlinear coherent backscattering



Linear CBS:

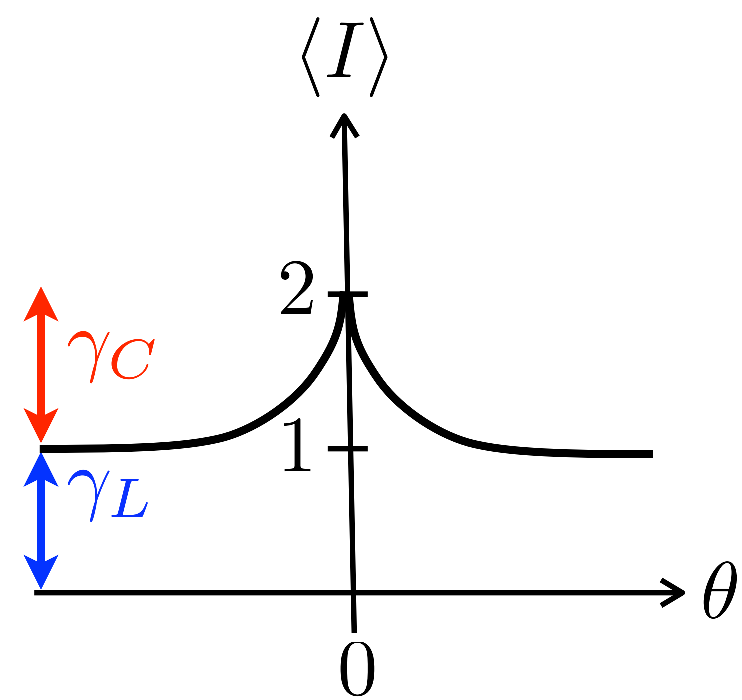
Nonlinear CBS:

Nonlinear coherent backscattering

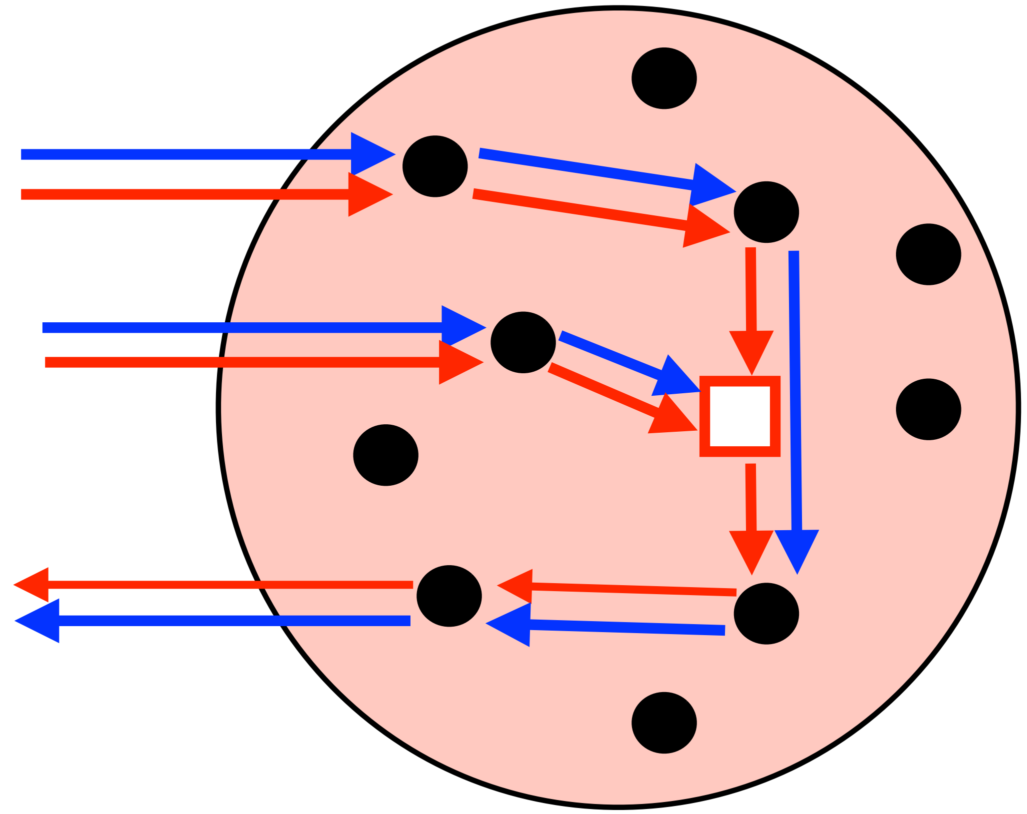


Linear CBS: Two-wave interference

Nonlinear CBS:

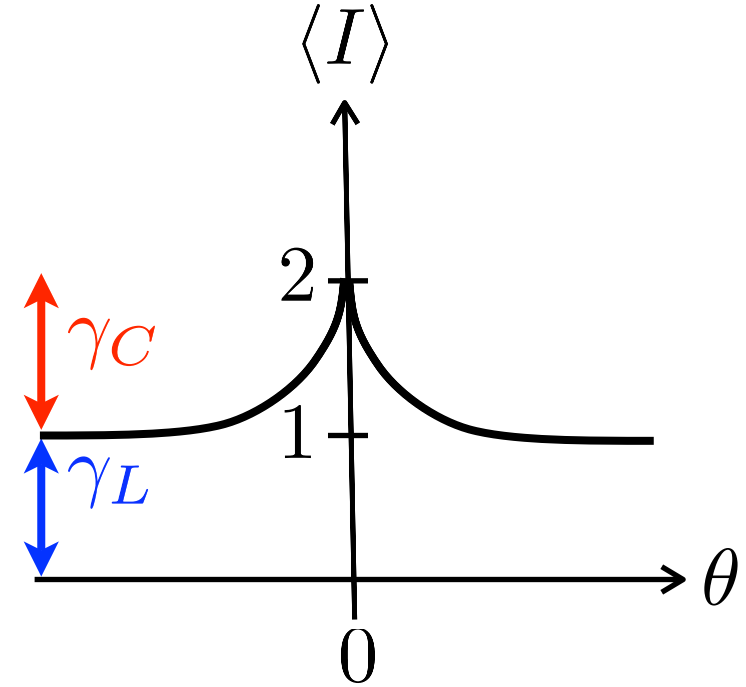


Nonlinear coherent backscattering

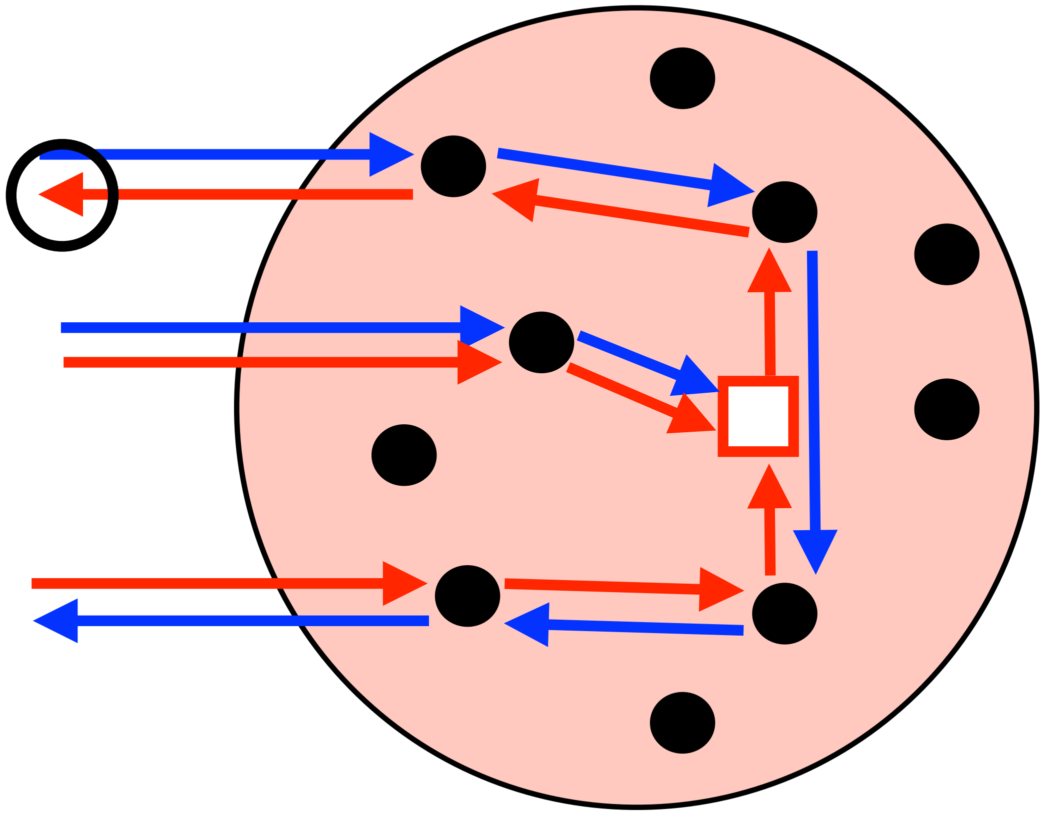


Linear CBS: Two-wave interference

Nonlinear CBS:

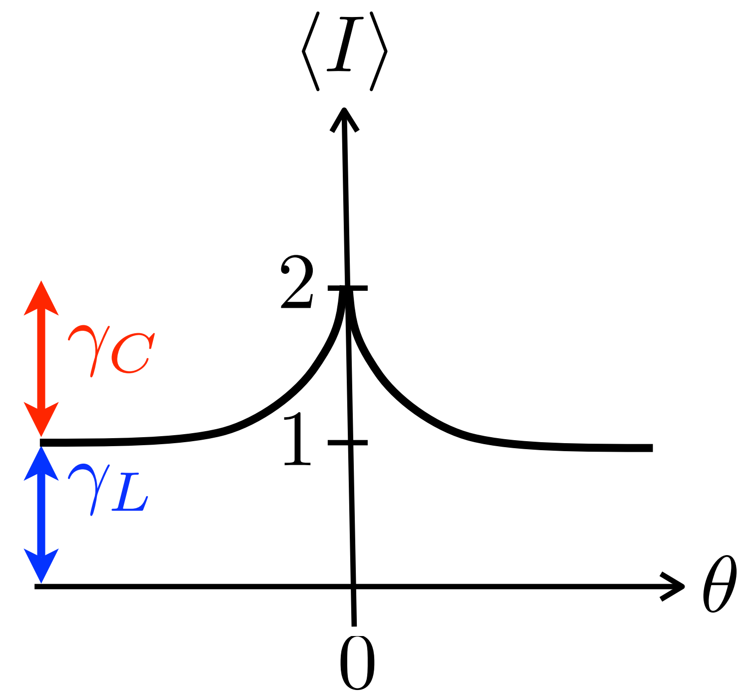


Nonlinear coherent backscattering

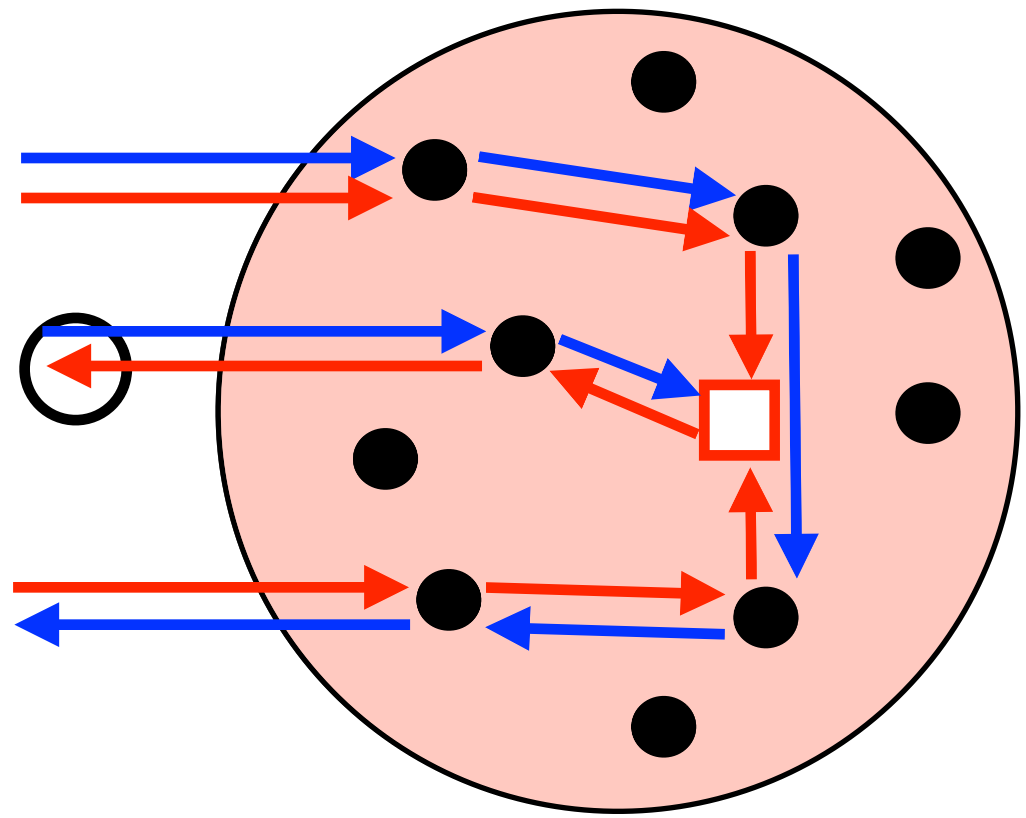


Linear CBS: Two-wave interference

Nonlinear CBS:

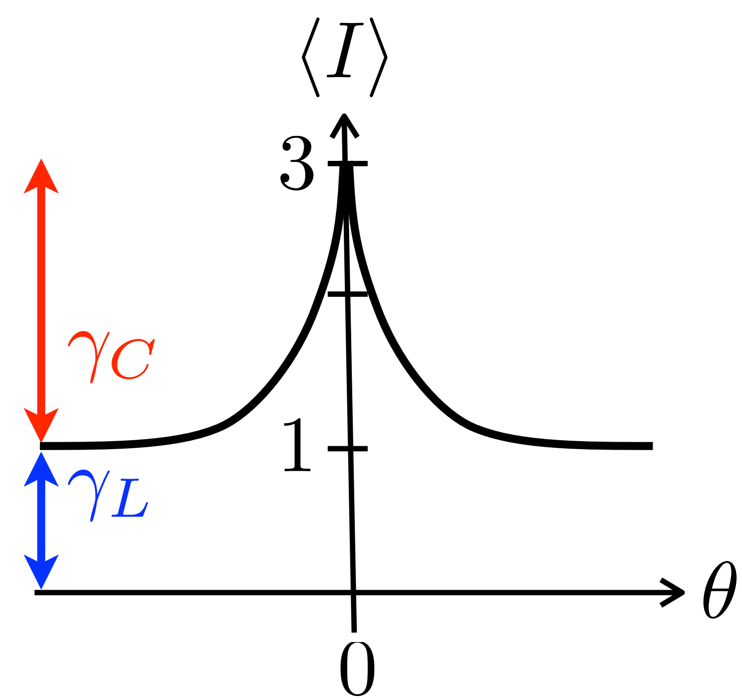


Nonlinear coherent backscattering



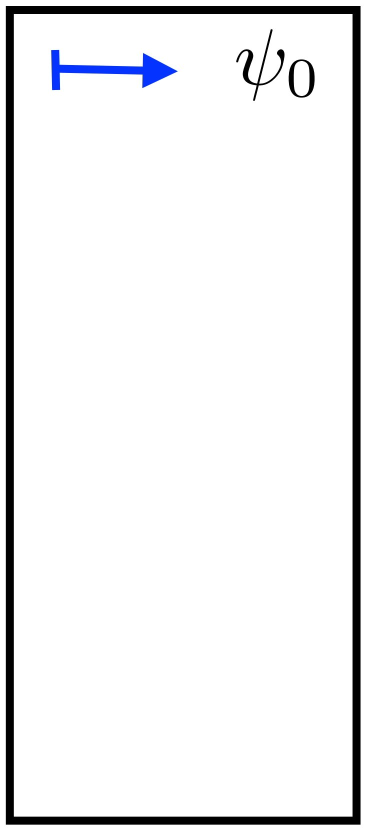
Linear CBS: Two-wave interference

Nonlinear CBS: Many-wave interference!

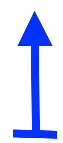


Diagrammatic theory

$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}'\; G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$

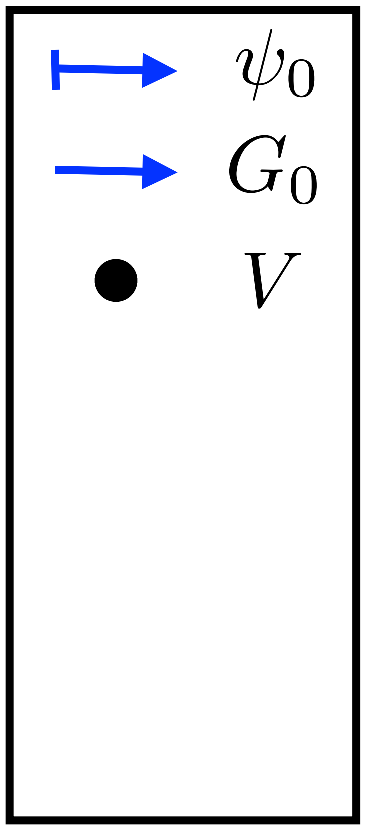


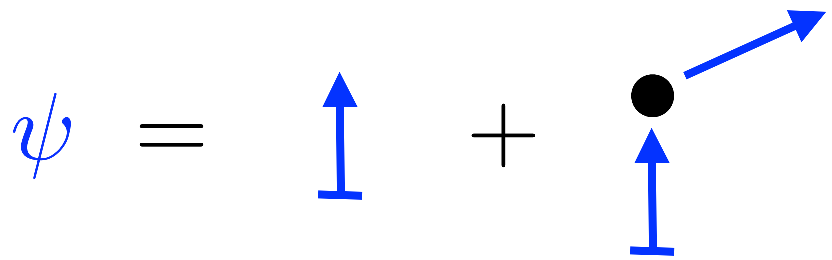
$\psi =$



Diagrammatic theory

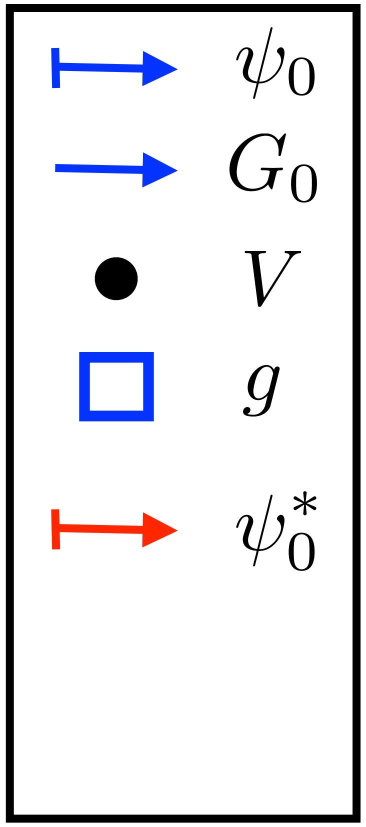
$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}'\ G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$

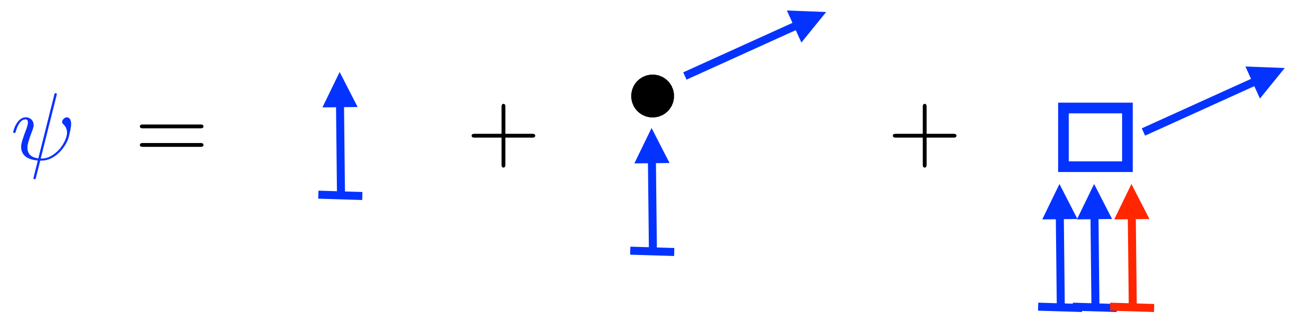




Diagrammatic theory

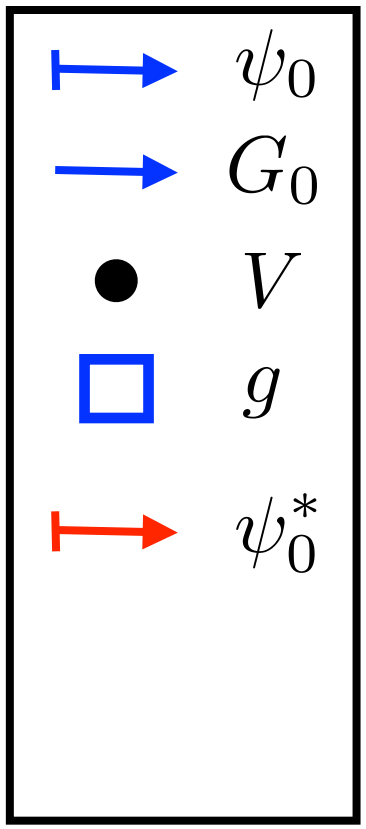
$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}' \; G_0(\mathbf{r},\mathbf{r}') \Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2 \Big) \psi(\mathbf{r}')$$

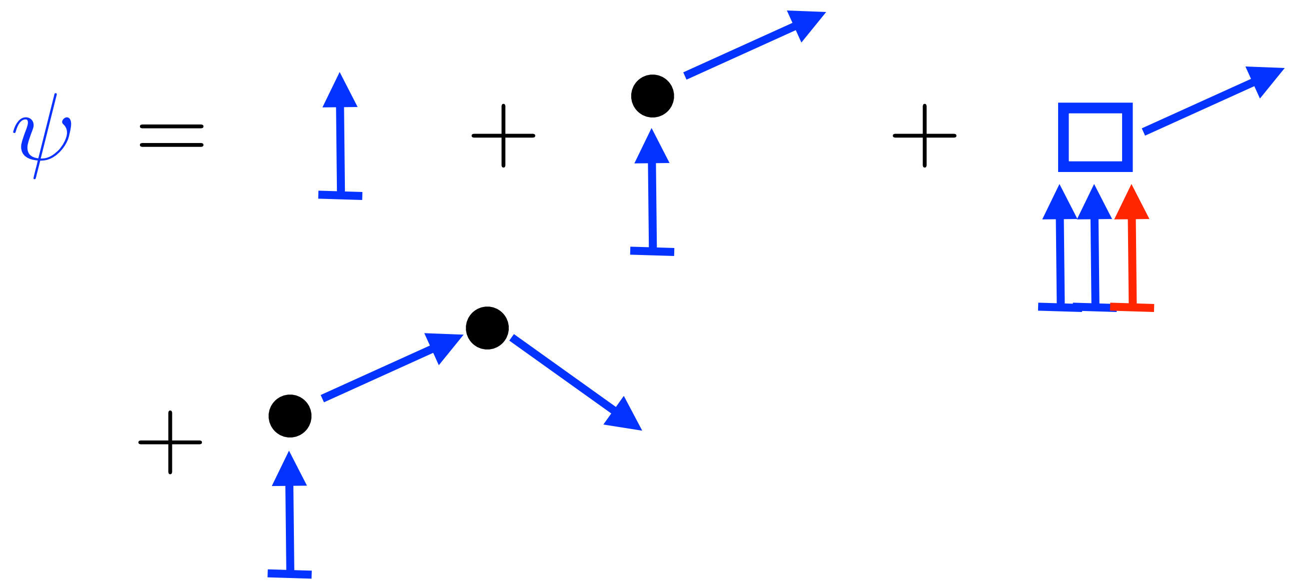




Diagrammatic theory

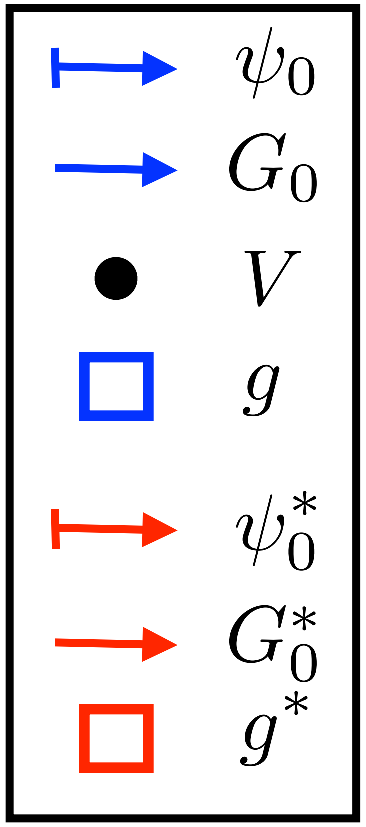
$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}'\; G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$

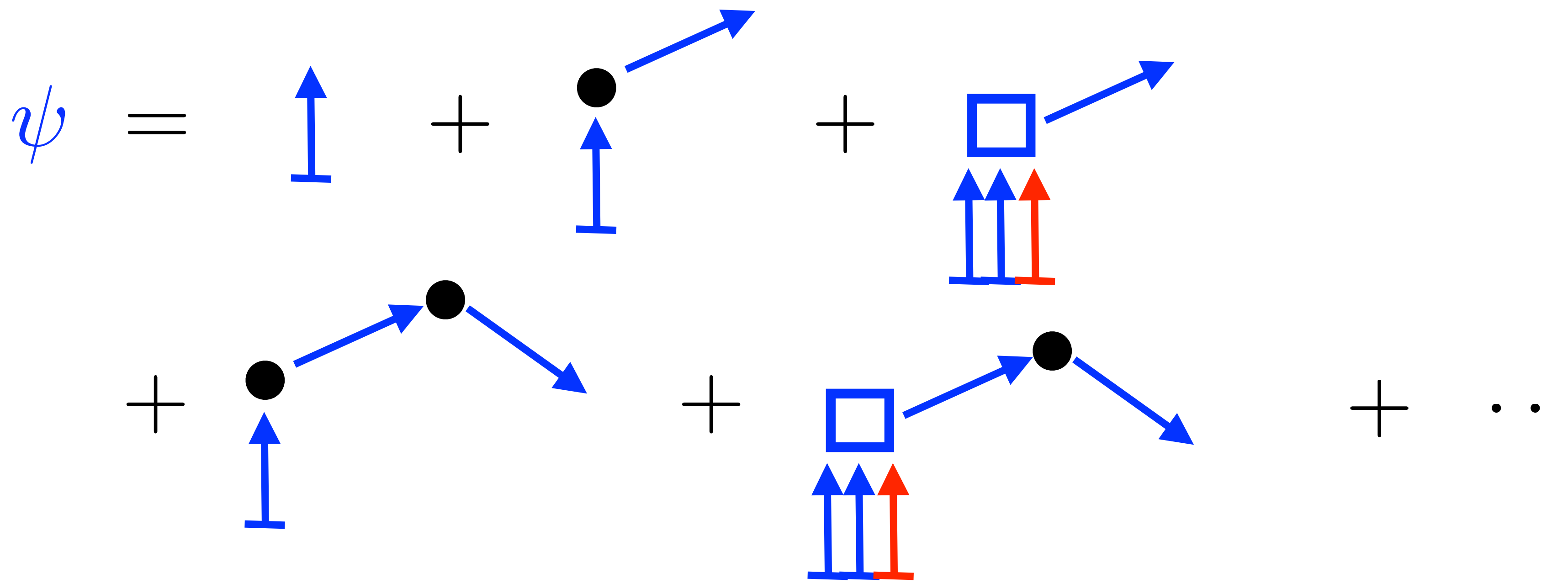




Diagrammatic theory

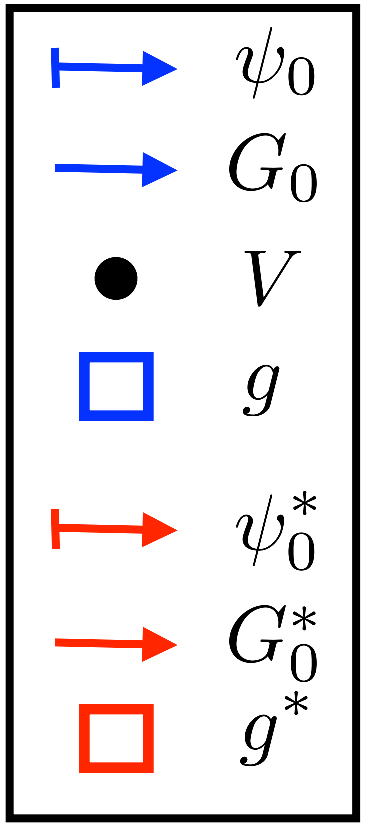
$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}'\ G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$



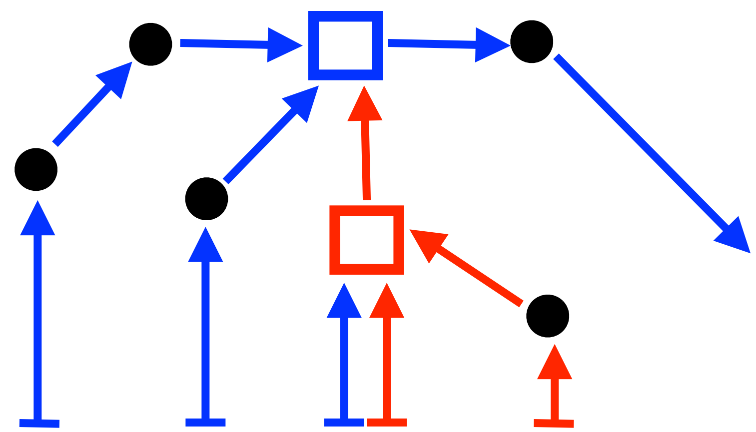


Diagrammatic theory

$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}'\; G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$

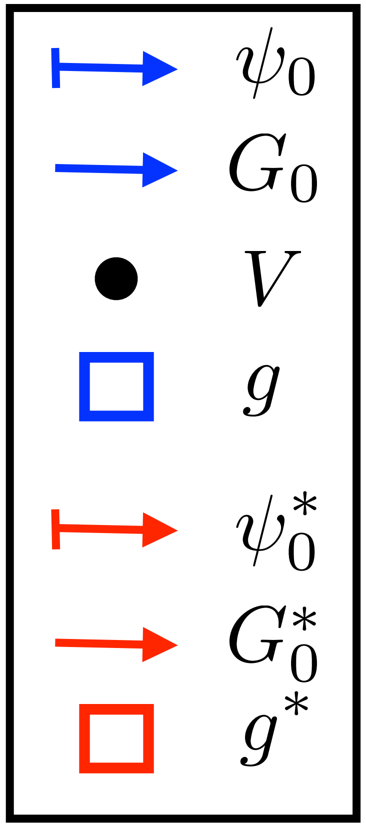


Nonlinear Born series:

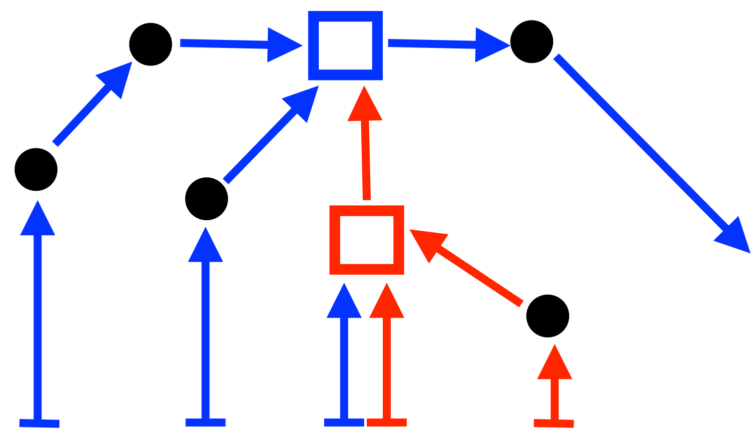
$\psi = \cdots +$  $+ \cdots$

Diagrammatic theory

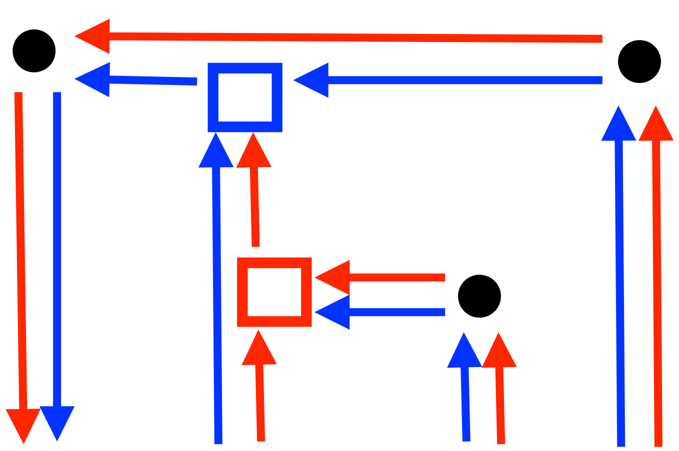
$$\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{k_L}\cdot\mathbf{r}} + k^2 \int d\mathbf{r}' \; G_0(\mathbf{r},\mathbf{r}')\Big(V(\mathbf{r}') + g|\psi(\mathbf{r}')|^2\Big)\psi(\mathbf{r}')$$

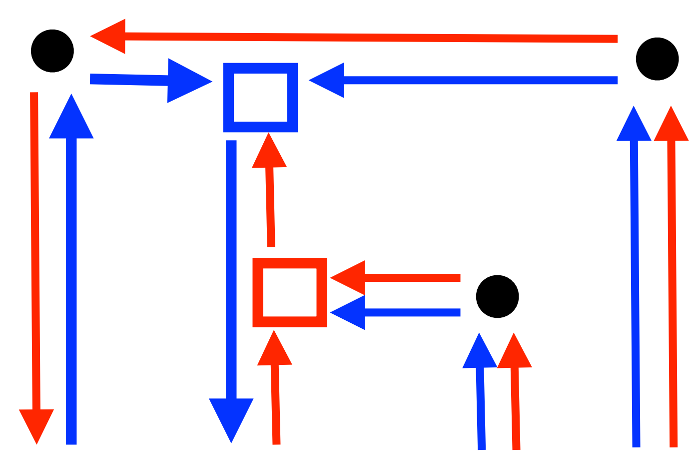


Nonlinear Born series:

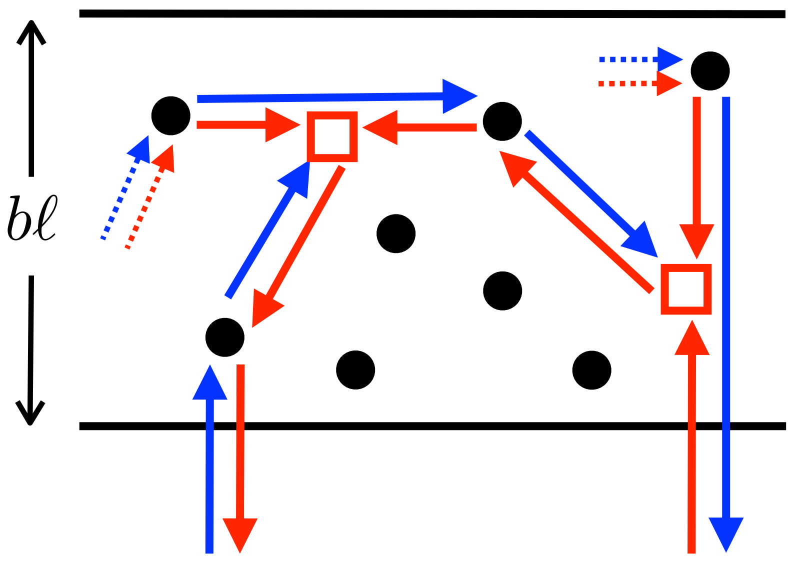
$\psi \;=\; \cdots +$  $+ \;\cdots$

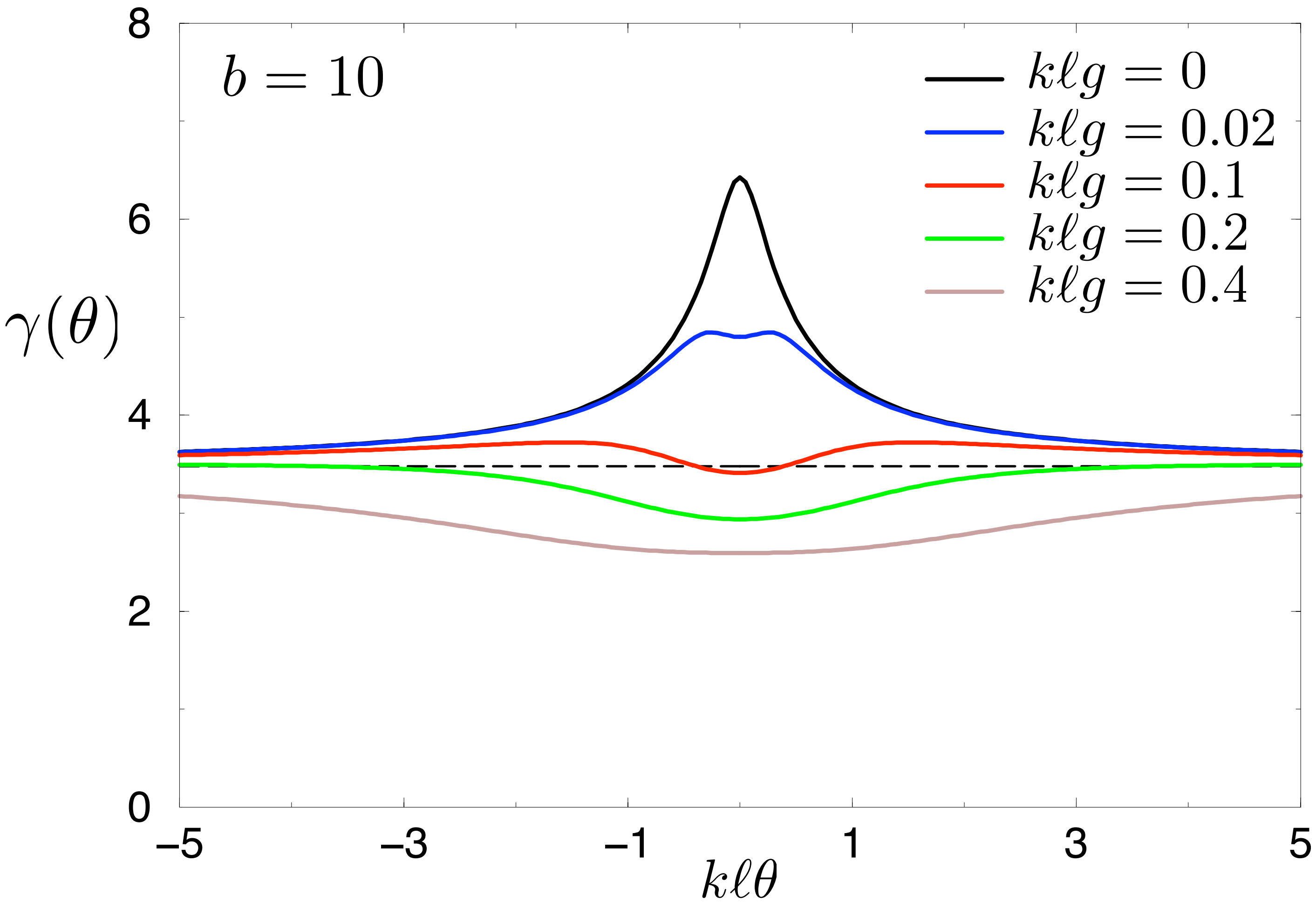
$\langle \psi\psi^* \rangle$: only **Ladder** and **Crossed** diagrams survive disorder average!

$\gamma_L =$  $+ \cdots$

$\gamma_C =$  $+ \cdots$

Results I: Conservative medium ($g \in \mathbb{R}$)

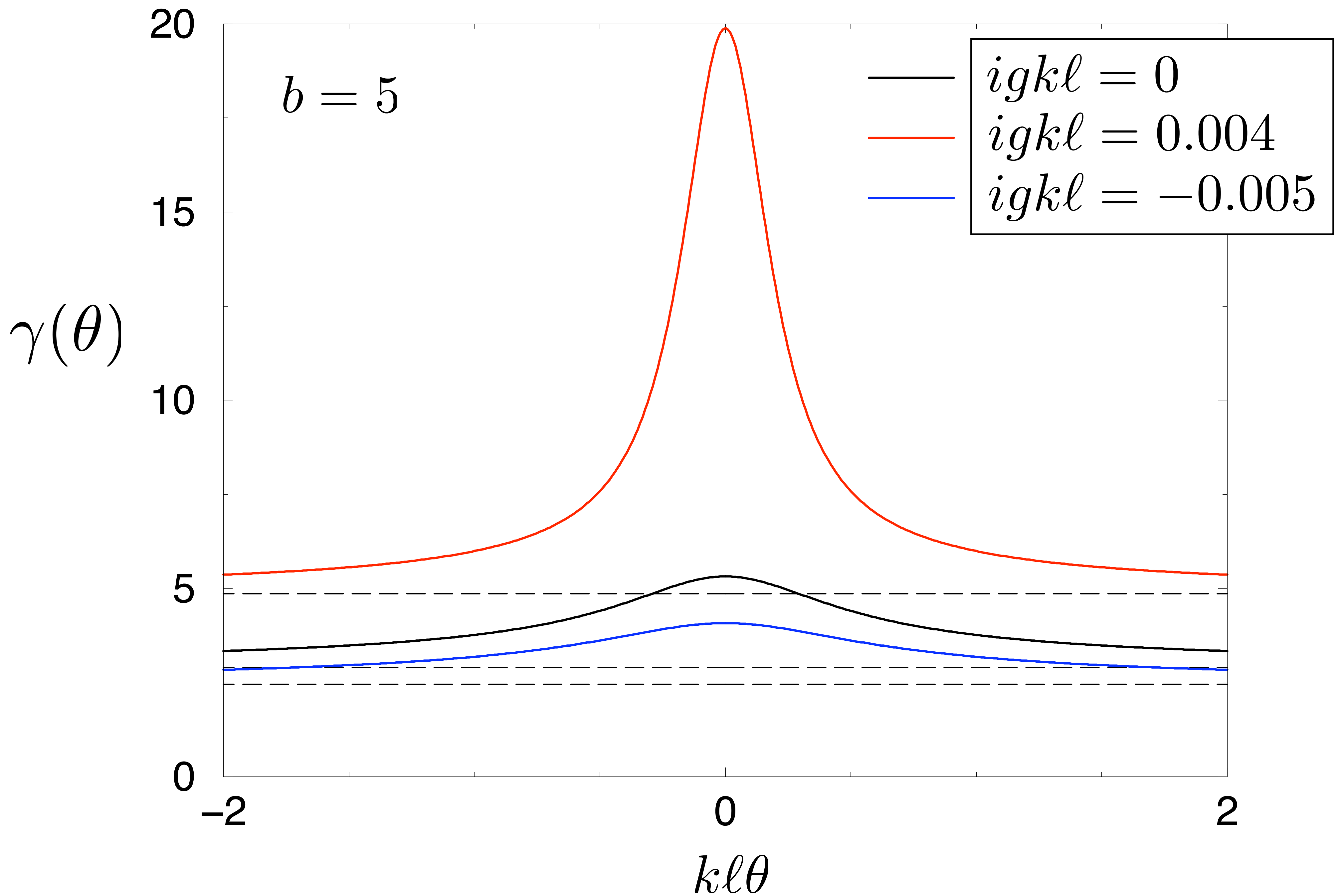




Nonlinearity turns constructive into destructive interference!

T. Wellens and B. Grémaud, PRL **100**, 033902 (2008); T. Wellens, Appl. Phys. B **95,** 189 (2009)

Results II: Amplifying/absorbing medium $(ig \in \mathbb{R})$



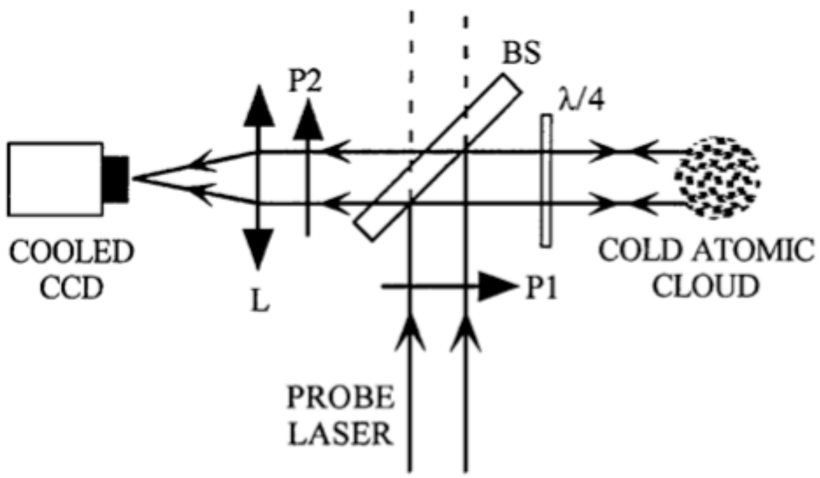
Amplifying nonlinearity enhances coherent backscattering!

T. Wellens and B. Grémaud, PRL **100**, 033902 (2008); T. Wellens, Appl. Phys. B **95,** 189 (2009)

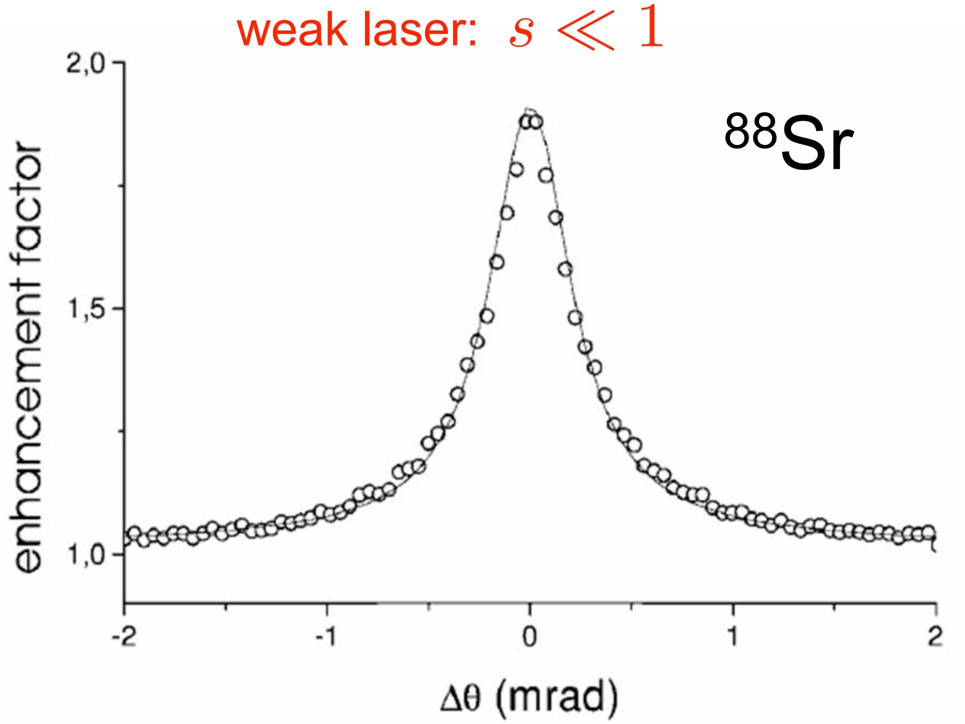
3.) Multiple scattering of intense laser light by cold atoms

T. Geiger, T. Wellens, V. Shatokhin, A. Buchleitner (U Freiburg)

CBS saturation experiment

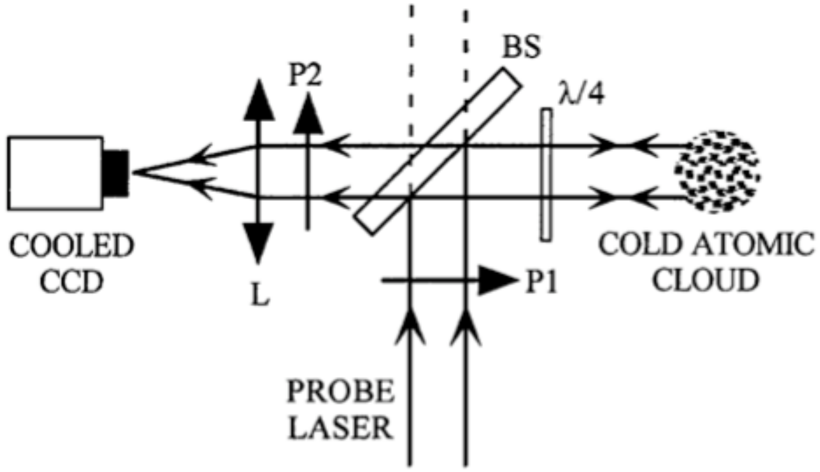


weak laser: $s \ll 1$



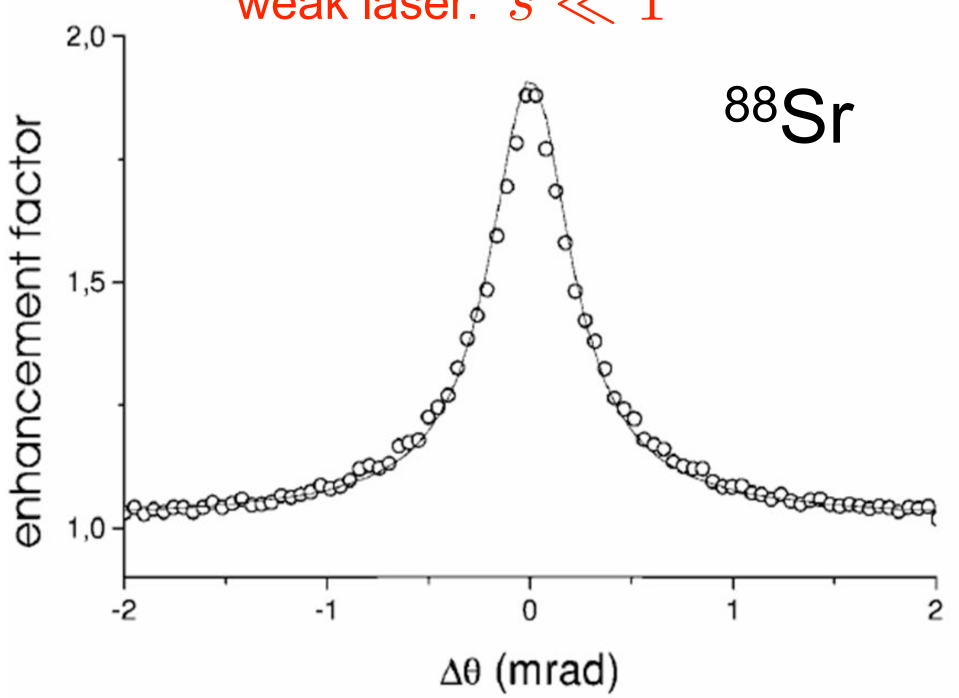
Y. Bidel et al. PRL **88**, 20902 (2002)

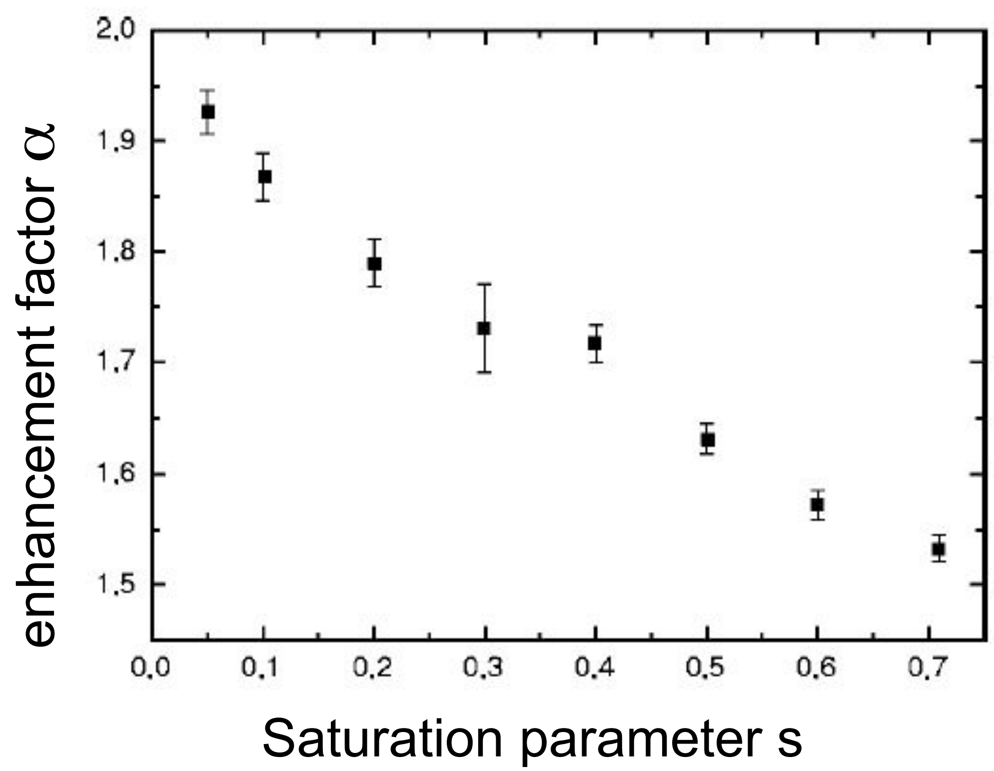
CBS saturation experiment



CBS interference goes down with increasing laser intensity!

weak laser: $s \ll 1$



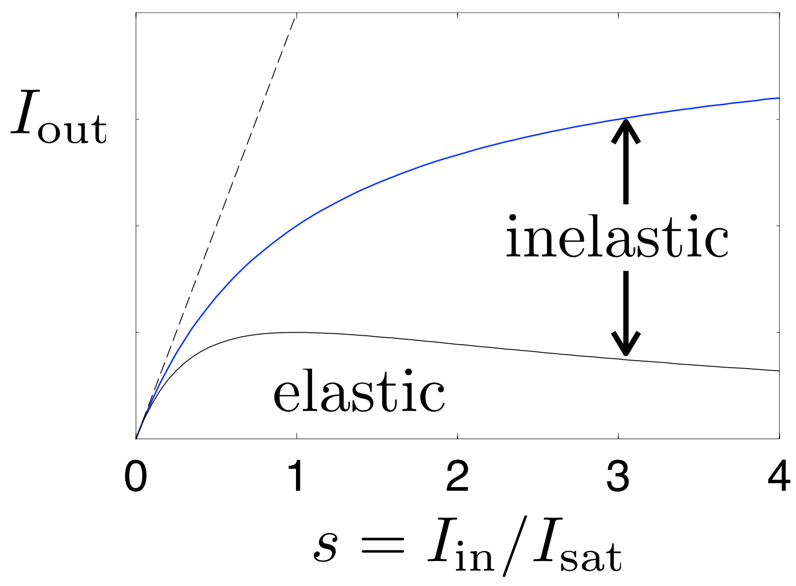


Y. Bidel et al. PRL **88**, 20902 (2002)

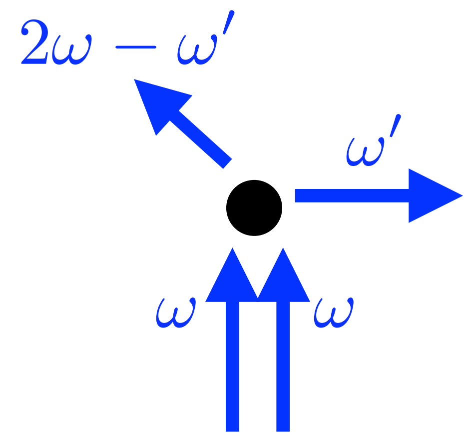
T. Chanelière et al. PRE **70,** 036602 (2004)

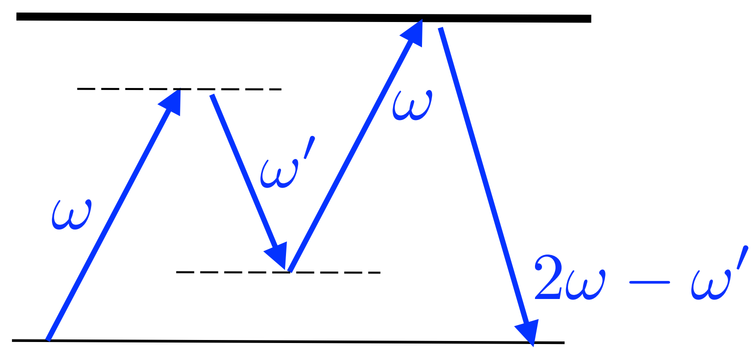
Single atom: resonance fluorescence

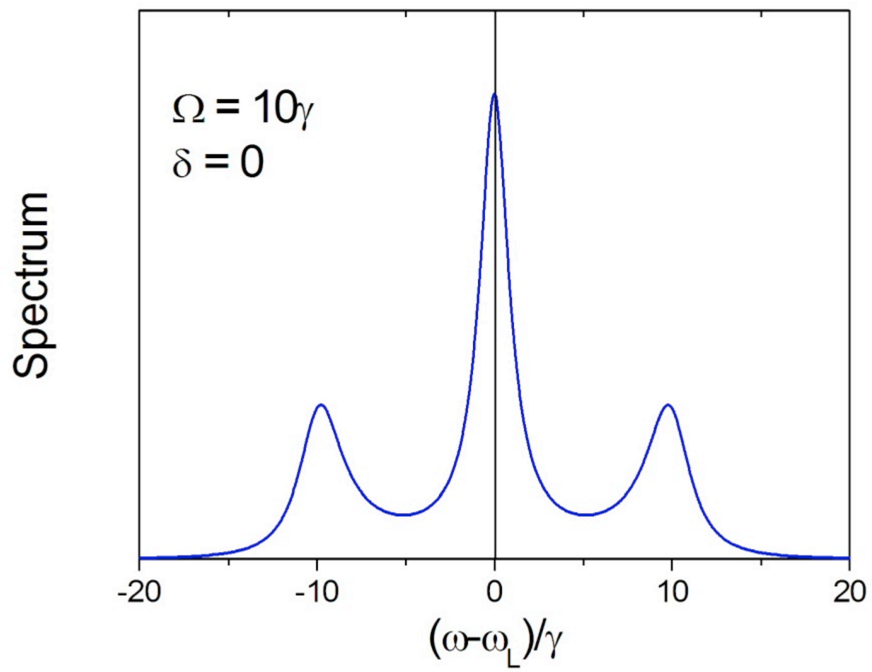
(i) nonlinearity



(ii) inelastic scattering







B.R. Mollow, PR **188**, 1969 (1969)

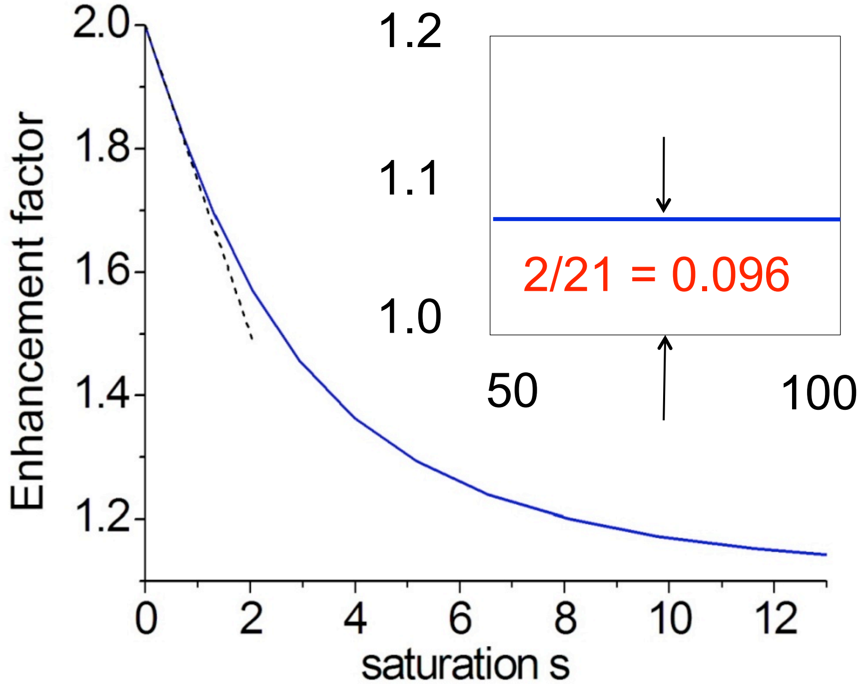
Master equation

$$\dot{\rho}_A = \underbrace{\sum_{i=1}^{N} \hat{L}_i \rho_A}_{\text{independent atoms}} + \underbrace{\sum_{i \neq j}^{N} \hat{L}_{ij} \rho_A}_{\text{interaction via photon exchange}} \qquad \dim \rho_A = 4^N$$

independent atoms

interaction via photon exchange: $\frac{1}{kr_{ij}} \ll 1$

Two atoms:



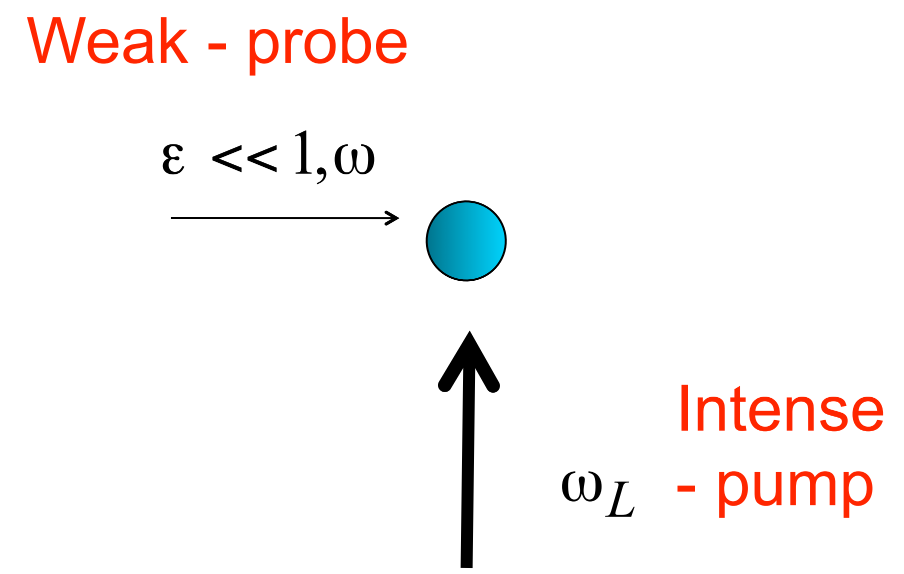
$\alpha_\infty > 1$

residual self-interference of inelastically scattered photons

V. Shatokhin, C. A. Müller, A. Buchleitner, PRL **94**, 043603 (2005)

Pump-probe

exchange of a single photon



modeled by **classical field**:

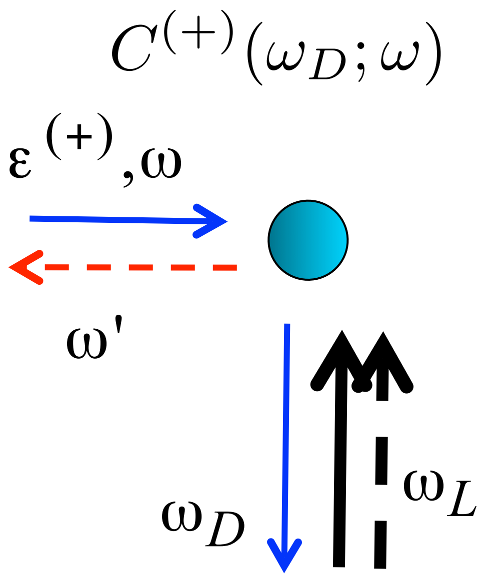
$$E = (\alpha e^{-i\omega_L t} + \alpha e^{i\omega_L t}) + (\varepsilon\, e^{-i\omega t} + \varepsilon\, e^{i\omega t})$$

$\varepsilon^{(+)}$ $\varepsilon^{(-)}$

Solve master equation for **single atom** under bichromatic driving!

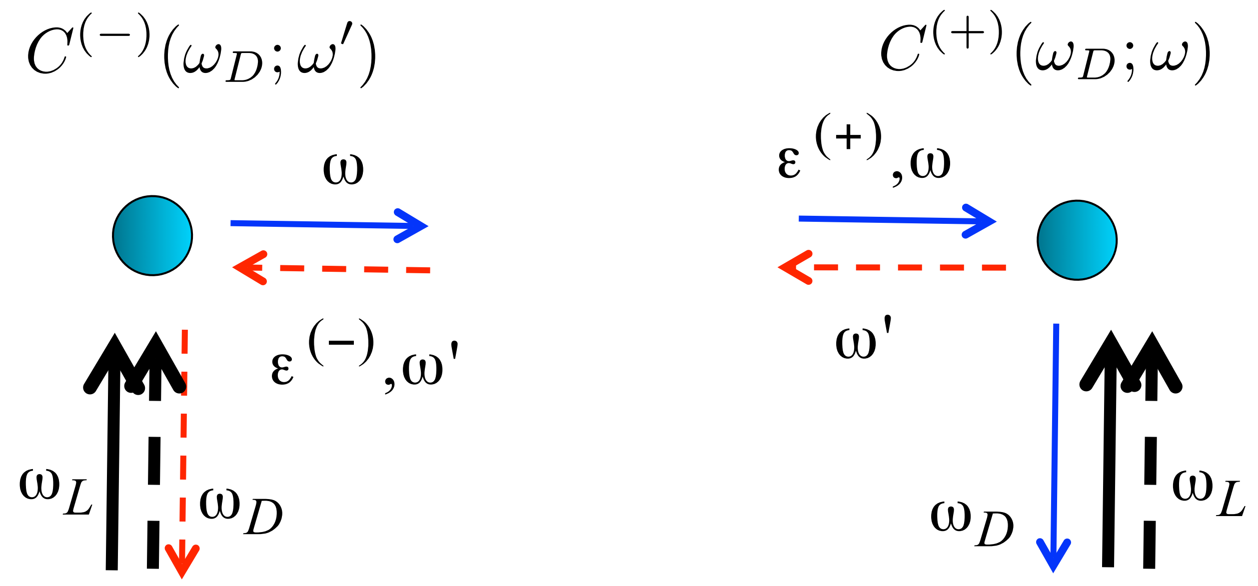
T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

Pump-probe



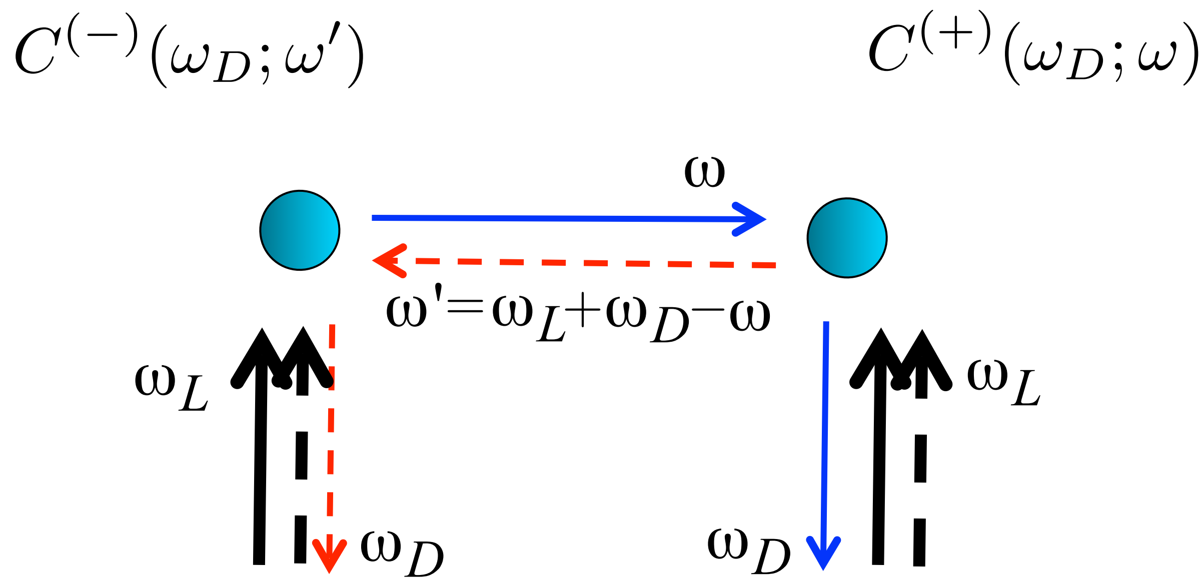
T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

Pump-probe



T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

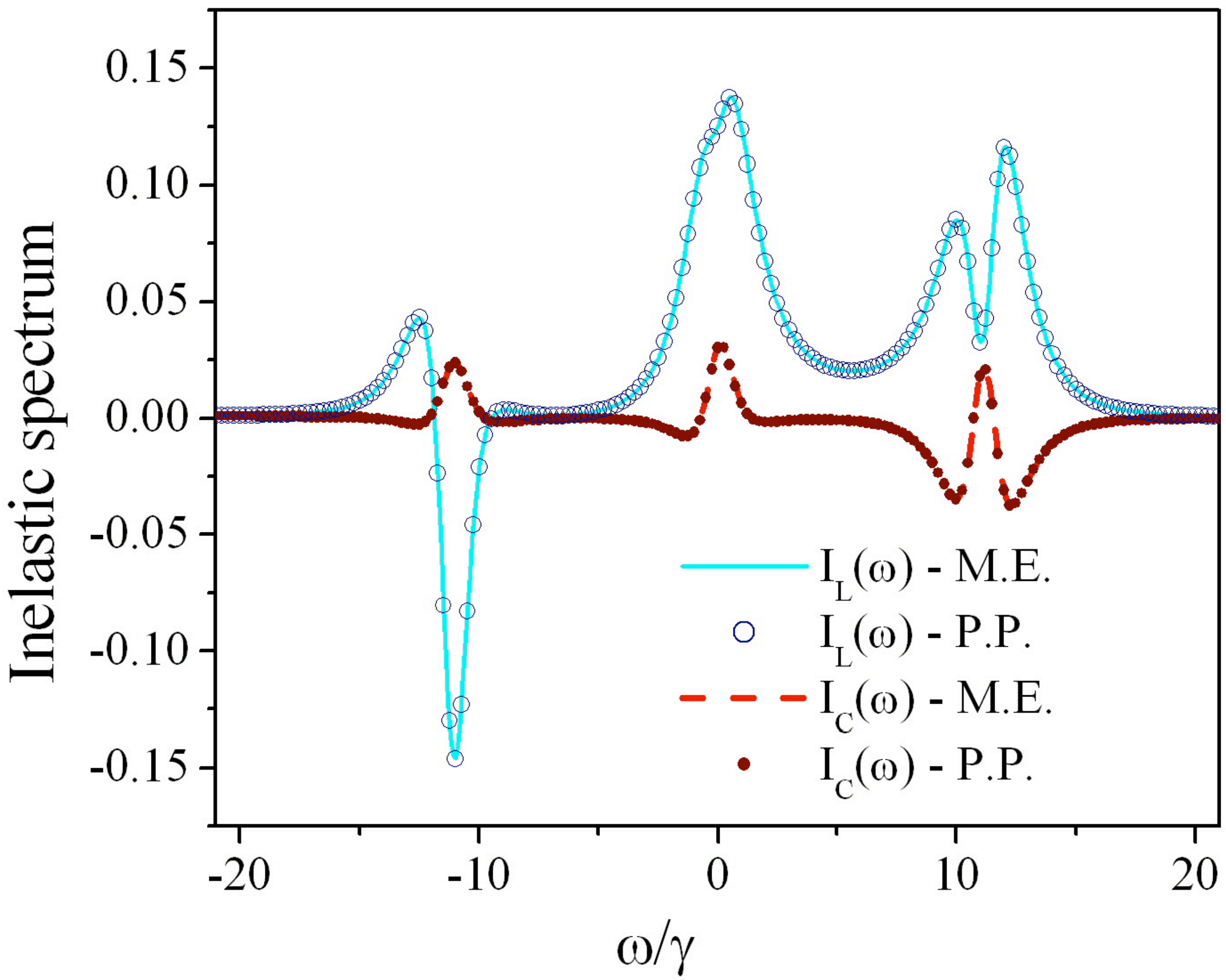
Pump-probe



$$\text{Interference}(\omega_D) \;=\; \int_{-\infty}^{\infty} \mathrm{d}\omega\; C^{(-)}(\omega_D;\omega_L+\omega_D-\omega)C^{(+)}(\omega_D;\omega)$$

T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

Comparison: master equation vs. pump-probe

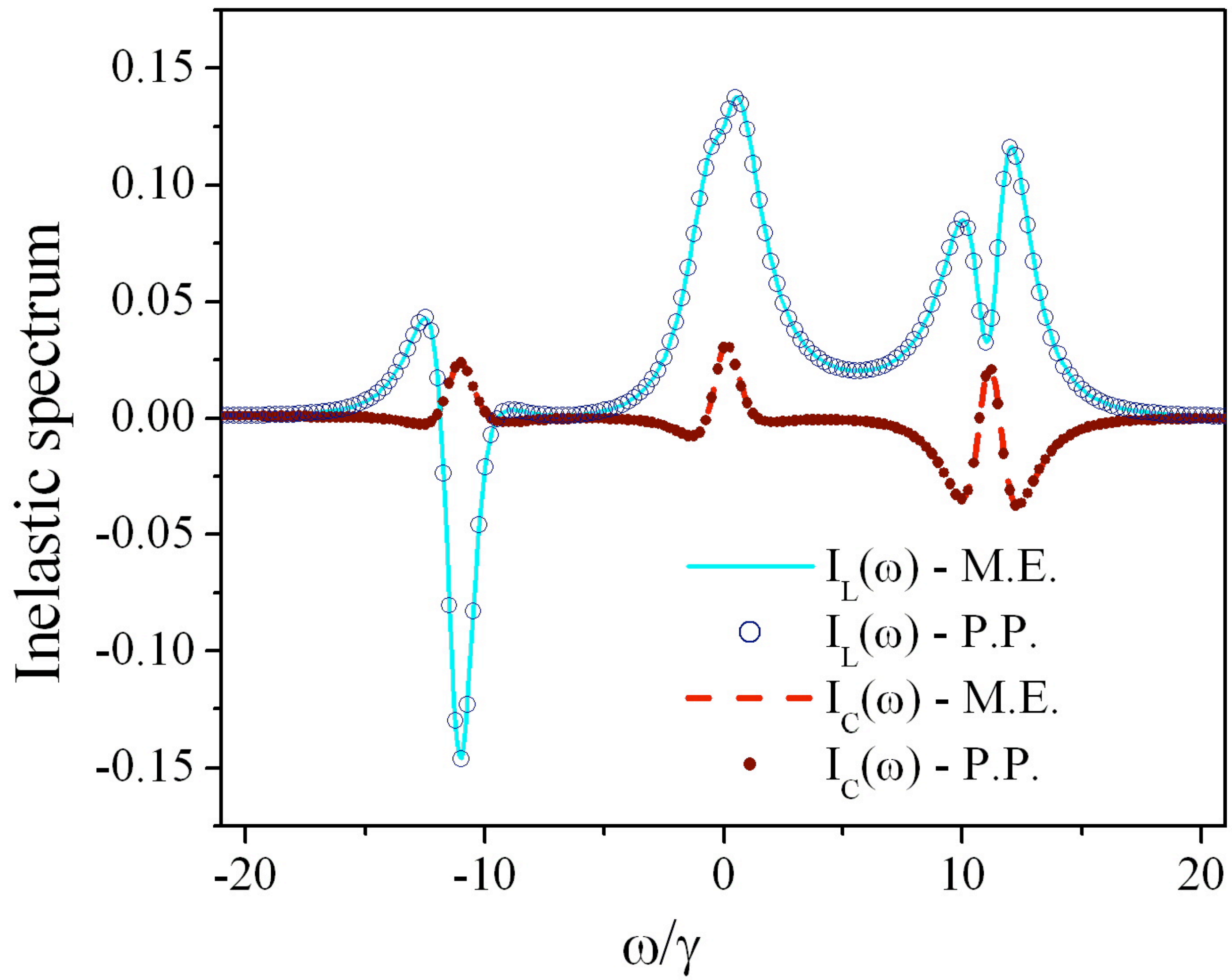


Perfect agreement!

T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

V. Shatokhin, T. Geiger, T. Wellens, A. Buchleitner, Chem. Phys. **375,** 150 (2010)

Comparison: master equation vs. pump-probe



Perfect agreement!

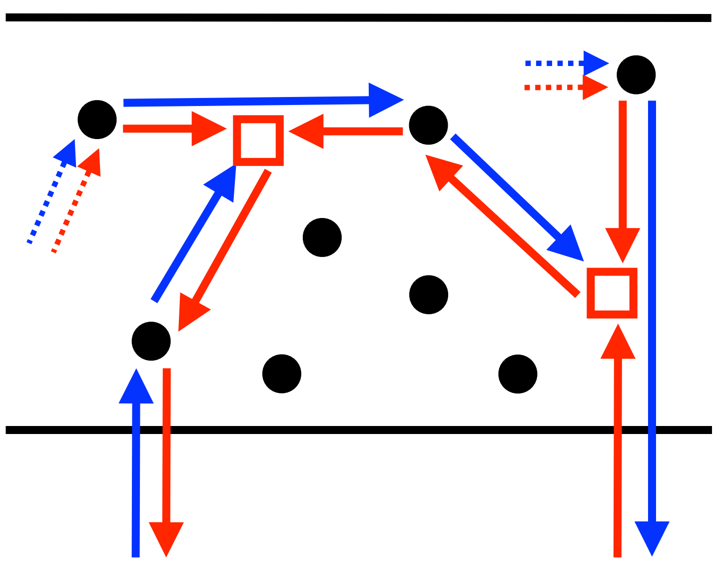
Single-atom equations, coupled by classical fields $\Rightarrow$ generalization to multiple scattering possible!

T. Wellens, T. Geiger, V. Shatokhin, A. Buchleitner, PRA **82,** 013832 (2010)

V. Shatokhin, T. Geiger, T. Wellens, A. Buchleitner, Chem. Phys. **375,** 150 (2010)

Conclusion

- Diagrammatic theory for disorder average of nonlinear wave equations



- Nonlinear coherent backscattering
  - conservative nonlinearity: CBS cone reduced
  - amplifying nonlinearity: CBS cone enhanced

- Multiple scattering of intense laser light by cold atoms
  - decoherence due to inelastic scattering
  - non-perturbative treatment in reach! (for dilute medium)