A Quantum Simulator for Relativistic Field Theories and Topological Insulators

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– Quantum Technologies Conference II –
Krakow - September 2, 2011

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Outline

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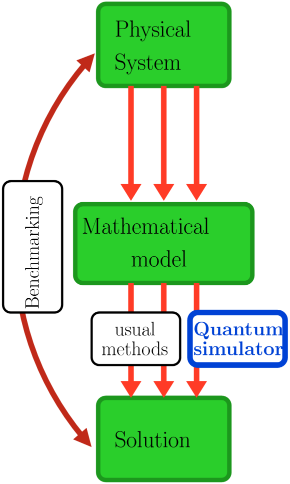
1. Introduction
2. A Superlattice-Based Quantum Simulator
3. Simulation of Relativistic Lattice Theories and Topological Insulators
4. Conclusions and Perspectives

Quantum Simulator



Quantum Simulator:

- attacking numerically-hard problems
- engineering of a model with a real system
- information obtained via measurement

Wanted: a controllable quantum system

- highly detailed microscopic knowledge
- high external tunability
- good possibility of measuring the system

one possibility: **COLD ATOMS!**

- quantum system ($T \sim nK$)
- good microscopic knowledge
- increasing possibility of manipulating the system

Quantum Simulations with Optical Lattices

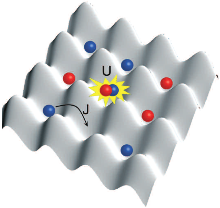
Exploiting a gas trapped in an optical lattice

1. Discrete structure → Lattice theories

Hubbard model: e^- in crystals

$$\hat{H} = -J \sum_{<ij>,\sigma} \hat{c}^\dagger_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\downarrow} \hat{n}_{i,\uparrow} - \sum_{i,\sigma} \mu_i \hat{n}_{i,\sigma}$$

See e.g. work in groups of Bloch and Esslinger



2. Many-species atomic gases → Many-field & many-component theories

Hubbard model: 2 fields

- e^- with spin ↑
- e^- with spin ↓

Cold atoms: take two hyperfine species

- ^{40}K with hyperfine spin $m_F = -9/2$
- ^{40}K with hyperfine spin $m_F = -7/2$

3. No atomic interactions → Non-interacting theories

Study of a model for different interactions and no interactions

See e.g. work in groups of Bloch, Esslinger, Aspect, Inguscio

Feshbach resonances

More Complicated: the Dirac Equation

Simulate the **Dirac Hamiltonian** in 3+1 dimensions

$$\hat{H} = \int \mathbf{dr}\, \hat{\Psi}^\dagger(\mathbf{r}) \left(c\boldsymbol{\alpha}\cdot\mathbf{p} + mc^2\beta\right) \hat{\Psi}(\mathbf{r}) \quad \text{with } \alpha_\nu \text{ and } \beta\text{: 4x4 matrices}$$

Many-Component Theory

$\hat{\Psi}(\mathbf{r})$ is 4 component spinor $\{\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3, \hat{\psi}_4\}$

4 atomic sublevels { ●, ●, ●, ●}

Interactions

Non-interacting theory → Feshbach resonances tune to zero the interactions

Discretization of the continous Hamiltonian

$$\hat{\Psi}^\dagger \alpha_1 \partial_1 \hat{\Psi}(\mathbf{x}, t) \to \sum_{\tau,\tau'} [\alpha_1]_{\tau,\tau'} \hat{c}^\dagger_{\mathbf{x},\tau} \hat{c}_{\mathbf{x}+\Delta\mathbf{x}_1,\tau'}$$

When α_1 is non-diagonal, the hopping process changes the spin

How to do spin-flipping hopping?

Motivation

Our Work

Development of a three-dimensional setup able to implement a generic hopping operator for a multi-species atomic gas trapped in a lattice potential.

Realization of the following Hamiltonian:

$$\hat{H}_{\text{sys}} = \sum_{\mathbf{r}\boldsymbol{\nu}} \sum_{\tau\tau'} t_\nu \hat{c}^\dagger_{\mathbf{r}+\boldsymbol{\nu}\tau'} [U_{\boldsymbol{\nu}}]_{\tau'\tau} \hat{c}_{\mathbf{r}\tau} + \sum_{\mathbf{r}} \Omega \hat{c}^\dagger_{\mathbf{r}\tau'} [\Lambda]_{\tau'\tau} \hat{c}_{\mathbf{r}\tau} + \mathrm{H.c.}$$

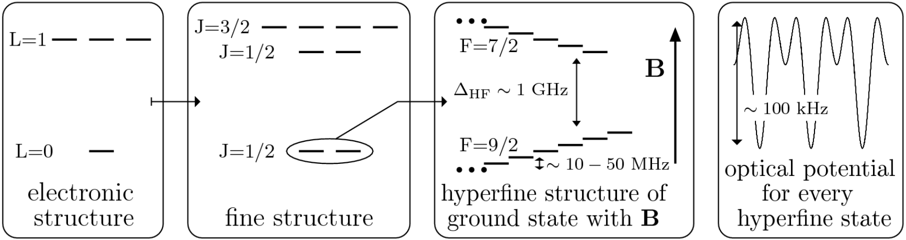
Control on:

1. D, the optical lattice dimension
2. t_ν, the tunneling strength
3. U_ν, the spin-dependent hopping operator
4. Ω, Λ, on site transitions

- Non-interacting is easy
  no need of quantum simulation
- First step towards the **interacting** case
  highly non-trivial

Outline

Alkali Atoms - ^{40}K

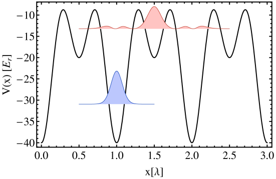


- $\hat{\mathbf{L}}$: orbital angular momentum of the electron
- $\hat{\mathbf{S}}$: spin of the electron
- $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$: total angular momentum of the electron
- $\hat{\mathbf{I}}$: angular momentum of the nucleus
- $\hat{\mathbf{F}} = \hat{\mathbf{J}} + \hat{\mathbf{I}}$: hyperfine spin of the atom

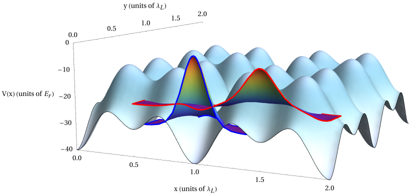
Optical Superlattice

The trapping potential: $V(\mathbf{x}) = \sum_i -V_0 \left[\cos^2(kx_i) + \cos^2(2kx_i)\right]$

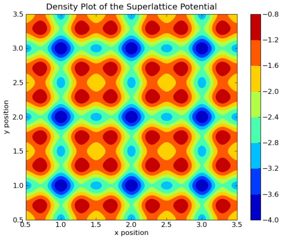
One dimension



Two dimensions



Two dimensions

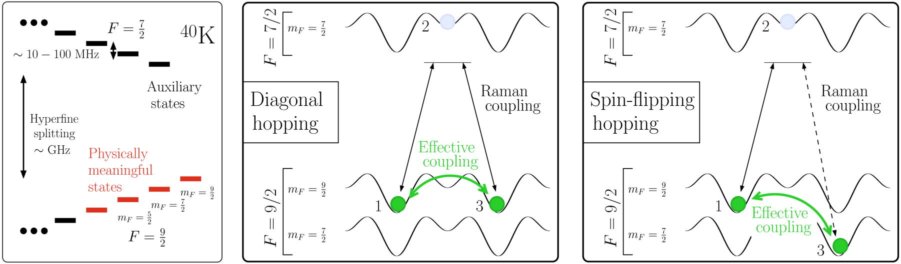


- Square lattice of main minima (blue)
- Presence of secondary minima in the middle of each link (light blue)
- Similar in 3D

- Wannier functions localized in main and secondary minima
- All the hyperfine levels are trapped with this potential

Laser-Assisted Hopping: Sketch

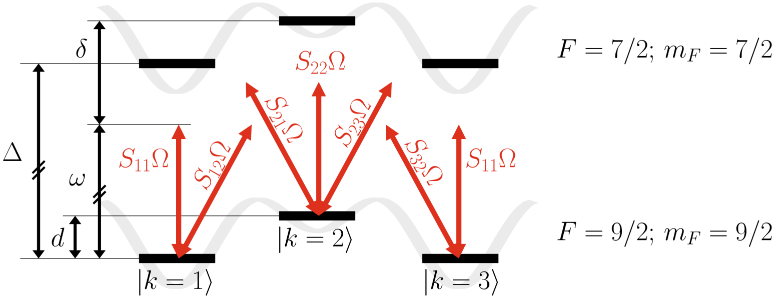
- ^{40}K: Fermionic System for Relativistic Theories
- Realization of diagonal and non-diagonal hopping



- Diagonal hopping breaking spin-symmetry
- Couplings Ω realised via optical Raman transitions
- Momentum transfer parallel to the hopping needed

Thanks to U.Schneider (LMU & MPQ, München)
Other ideas on laser-assisted hopping:
Jaksch and Zoller, NJP **5** 56 (2003);
Gerbier and Dalibard, NJP **12** 033007 (2010)

Laser Assisted Hopping: Link Model

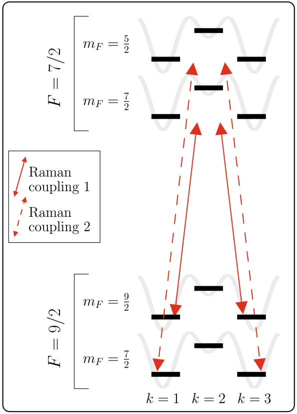


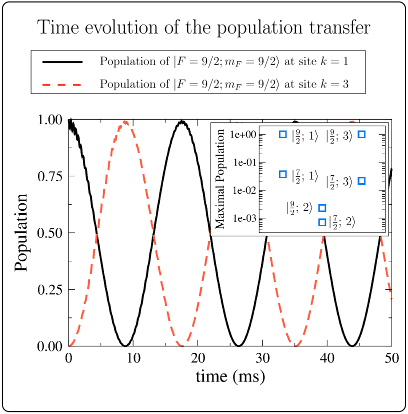
Develope an **effective link model**

- Quantify the fidelity of the lasser-assisted hopping $|m_F = 9/2, k = 1\rangle \rightarrow |m_F = 9/2, k = 3\rangle$
- Many couplings induced by the Raman transfer Ω
- $S_{i,j}$ is the overlap factor between Wannier functions

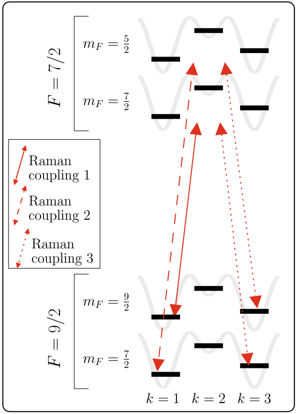
Laser-Assisted Hopping: Time-Evolution of the Link Model

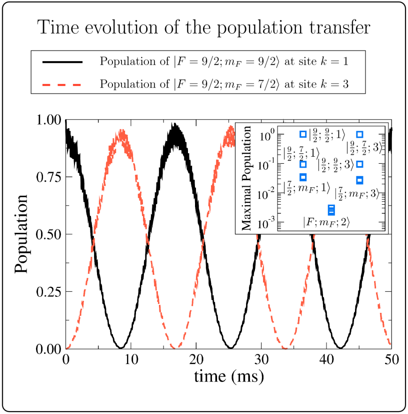
- Exact real-time evolution of the link model
- Initial population in $|F = 9/2, m_F = 9/2\rangle$
- Study of two species system: **diagonal** and off-diagonal hopping





- Exact real-time evolution of the link model
- Initial population in $|F = 9/2, m_F = 9/2\rangle$
- Study of two species system: diagonal and **off-diagonal** hopping
- Staggering of the lattice required





Summarizing

Existence of a hyerarchy of energies in atoms and optical lattices exploitable for laser-assisted tunneling

- Hyperfine Splitting: $1 - 10$ GHz
- Zeeman Splitting: $10 - 100$ MHz
- Lattice-bands gaps: $50 - 100$ kHz
- Staggering: 20 kHz

Effective Coupling:
$\sim \mathbf{100}$ Hz

Approximations

- use of Wannier functions as eigenstates of the system
- continuum states not considered

OK only if

- band width $\ll$ detuning
- detuning $\sim$ depth of the optical potential

otherwise:

- spurious next-nearest-neighbours couplings
- problems with the overlap between neighbouring wavefunctions

The Final Model

$$\hat{H}_{\text{sys}} = \sum_{\mathbf{r}\boldsymbol{\nu}} \sum_{\tau\tau'} t_\nu \hat{c}^\dagger_{\mathbf{r}+\boldsymbol{\nu}\tau'} [U_{\boldsymbol{\nu}}]_{\tau'\tau} \hat{c}_{\mathbf{r}\tau} + \sum_{\mathbf{r}} \Omega \hat{c}^\dagger_{\mathbf{r}\tau'} [\Lambda]_{\tau'\tau} \hat{c}_{\mathbf{r}\tau} + \text{H.c.}$$

- Hopping part: described laser-assisted tunneling
- On-site part: on-site raman couplings

GOAL: Simulation of quadratic theories

1. Relativistic Theories
2. Topological Insulators

"Elegant" Approach

For every model I want to quantum-simulate I find a clever mapping to an optical lattice

Drawbacks: models for which I cannot find the needed clever mapping

"Systematic" Approach - THIS WORK

If I can engineer whatever hopping and on-site quadratic term I can simulate almost all the quadratic theories

Drawbacks: space-dependencies and not-cubic lattices

"Elegant approach": see e.g. works by: Pachos, Garcia-Ripoll, Lepori, Trombettoni, Ruostekoski, Schützold, Solano, etc etc...

Outline

3D Massless Dirac Fermions

The Hamiltonian to be simulated:

- D-dimensional optical lattice
- translationally invariant hopping operators $U_{\mathbf{a}_j} \in SU(N_D)$
- π-flux regime: $\phi_\nu = \pi/2$

D	$U_{\mathbf{a}_1}$	$U_{\mathbf{a}_2}$	$U_{\mathbf{a}_3}$
1	$\mathrm{e}^{\mathrm{i}\phi_1\sigma_x}$		
2	$\mathrm{e}^{\mathrm{i}\phi_1\sigma_x}$	$\mathrm{e}^{\mathrm{i}\phi_2\sigma_y}$	
3	$\mathrm{e}^{\mathrm{i}\phi_1\sigma_z\otimes\sigma_x}$	$\mathrm{e}^{\mathrm{i}\phi_2\sigma_z\otimes\sigma_y}$	$\mathrm{e}^{\mathrm{i}\phi_3\sigma_z\otimes\sigma_z}$

Results:

- **8 points** in the 3D Brillouin Zone: the energy bands touch K_d
- **Half-filled lattice**: Fermi surface are 8 isolated points
- **Low energy excitations**: described by a Hamiltonian of massless Dirac fermions

$$H_{\mathrm{eff}} = \sum_{\mathbf{d}} \sum_{\mathbf{p_d}} \Psi^\dagger(\mathbf{p_d})\, H_{\mathrm{DI}}^{\mathbf{d}}\, \Psi(\mathbf{p_d}), \qquad H_{\mathrm{DI}}^{\mathbf{d}}(\mathbf{p_d}) = c\alpha^{\mathbf{d}} \cdot \mathbf{p_d},$$

3D Massive Dirac Fermions and beyond

Fermion doubling problem Nielsen-Ninomiya theorem

- The number of Dirac points is always even in a lattice theory
- Symmetry breaking required to get an odd number:
  - **chiral symmetry** for D odd $\{H^{\mathbf{d}}_{\mathrm{DI}}, \Gamma_1\} = 0$
  - **time-reversal** for D even $\Gamma_2^\dagger \left[H^{\mathbf{d}}_{\mathrm{DI}}(-\mathbf{p_d})\right]^* \Gamma_2 = H^{\mathbf{d}}_{\mathrm{DI}}(\mathbf{p_d})$

This correspond to modification of the parameters of the theory

Can we do more?

- from massless to massive case
- individual tailoring of the masses for each of the 8 Dirac modes
- engineering of Wilson fermions
- relativistic topological insulators

Bermudez, LM, Rizzi, Goldman, Lewenstein, Martín-Delgado PRL **105** 190404 (2010)

LM, Bermudez, Goldman, Rizzi, Martín-Delgado, Lewenstein arXiv:1105.0932

Topological Insulators

1. **Insulating bulk bands:** Gapped bulk spectrum
2. **Topological order:** Topological invariant characterizing some observable
3. **Edge states:** Gapless edge excitations

Non interacting models: $\hat{H} = \sum_{\alpha\beta} \hat{\Psi}^\dagger_\alpha \mathbb{H}_{\alpha\beta} \hat{\Psi}_\beta + \mathrm{H.c.}$ with $\mathbb{H}_{\alpha\beta}$ a matrix

Example: 1D Creutz ladder

$$H_{\mathrm{AIII}} = \sum_n K e^{-i\theta} a^\dagger_{n+1} a_n + K e^{i\theta} b^\dagger_{n+1} b_n + K b^\dagger_{n+1} a_n + K a^\dagger_{n+1} b_n + M a^\dagger_n b_n + \mathrm{H.c.}$$

Class	Name	T	C	S
A	Unitary	0	0	0
AIII	Chiral unitary	0	0	1
AI	Orthogonal	$+1$	0	0
AII	Symplectic	-1	0	0
CII	Chiral Symplectic	-1	-1	1

Class	d = 1	QS	d = 2	QS	d = 3	QS
A	0		$\mathbb{Z}$	yes	0	
AIII	$\mathbb{Z}$	yes	0		$\mathbb{Z}$	yes
AI	0		0		0	
AII	0		$\mathbb{Z}_2$	yes	$\mathbb{Z}_2$	yes
CII	$2\mathbb{Z}$	yes	0		$\mathbb{Z}_2$	yes

Interactions? Study their effect with a quantum simulation!

LM, Bermudez, Goldman, Rizzi, Martín-Delgado, Lewenstein arXiv:1105.0932

Conclusions and Perspectives

Superlattice based quantum simulator

- Existence of a hyerarchy of energies in atoms and optical lattices exploitable for laser-assisted tunneling.
- Possibility of engineering diagonal and non-diagonal hopping
- Possibility of studying complicated non-interacting theories

Perspectives:

- Controlled introduction of interaction via Feshbach resonances
  - Interacting Relativistic Theories
  - Interacting Topological Insulators
- Look for the simplest interesting model to be simulated

Bermudez, LM, Rizzi, Goldman, Lewenstein, Martín-Delgado PRL **105** 190404 (2010)
LM, Bermudez, Goldman, Rizzi, Martín-Delgado, Lewenstein `arXiv:1105.0932`

Thank you