Artificial magnetism and optical flux lattices for ultra cold atoms

Gediminas Juzeliūnas
Institute of Theoretical Physics and Astronomy, Vilnius University, Vilnius, Lithuania

Kraków, QTC, 31 August 2011

Vilnius, Lithuania

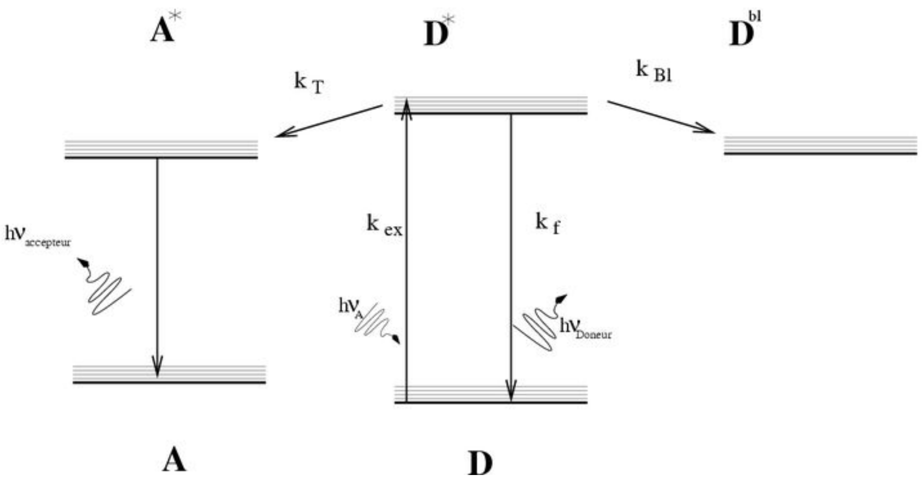
Vilnius University

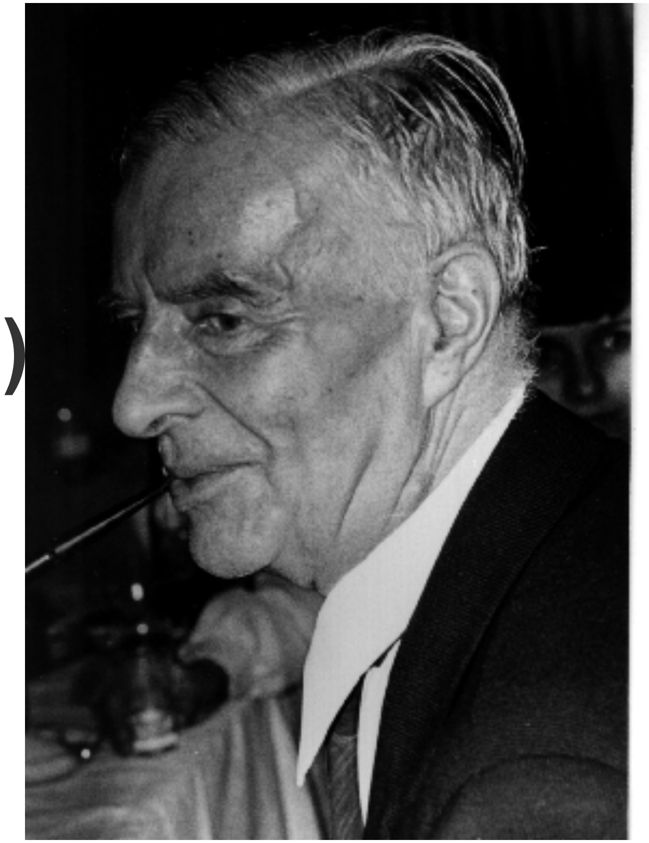
QTC'2010 Toruń:

Vilnius-Toruń

Vilnius University – NCU in Toruń

Aleksander Jablonski (1898-1980)





A Jablonski diagram representing
fluorescence resonance energy transfer

Vilnius University – NCU in Toruń

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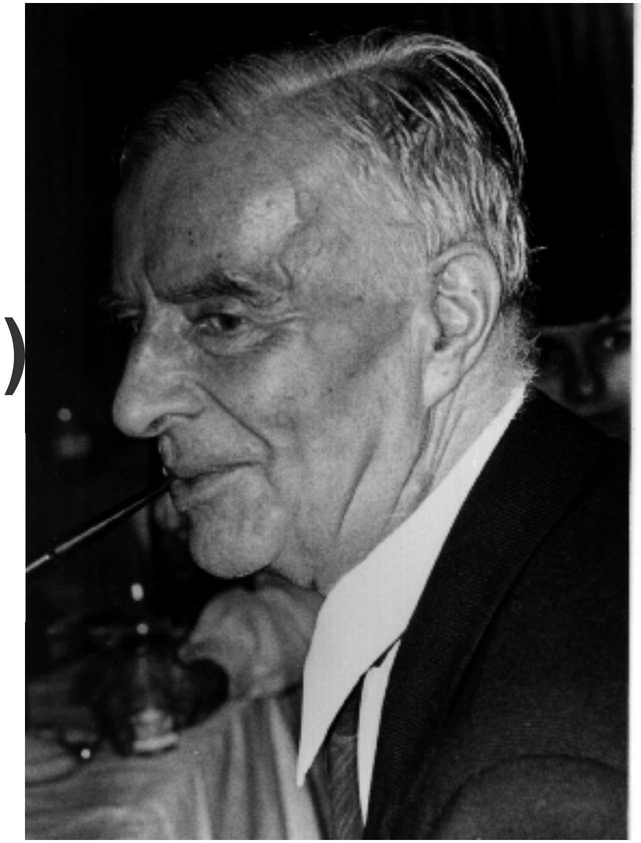

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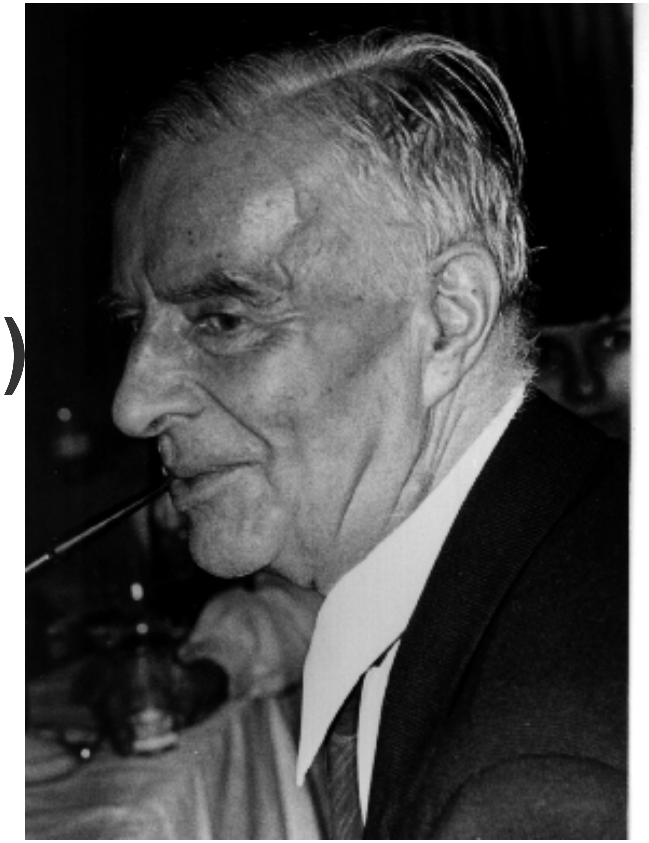
Letters to Editor

nature **142**, 1122-1122 (24 December 1938) | doi:10.1038/1421122a0

Temperature Influence on the Pressure Broadening of Spectral Lines

H. HORODNICZY & A. JABLOÅƒSKI

Vilnius University – NCU in Toruń



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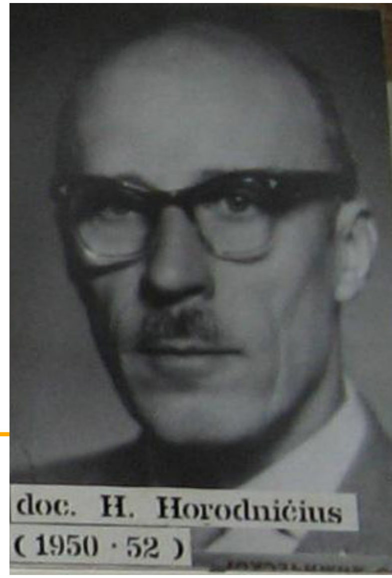
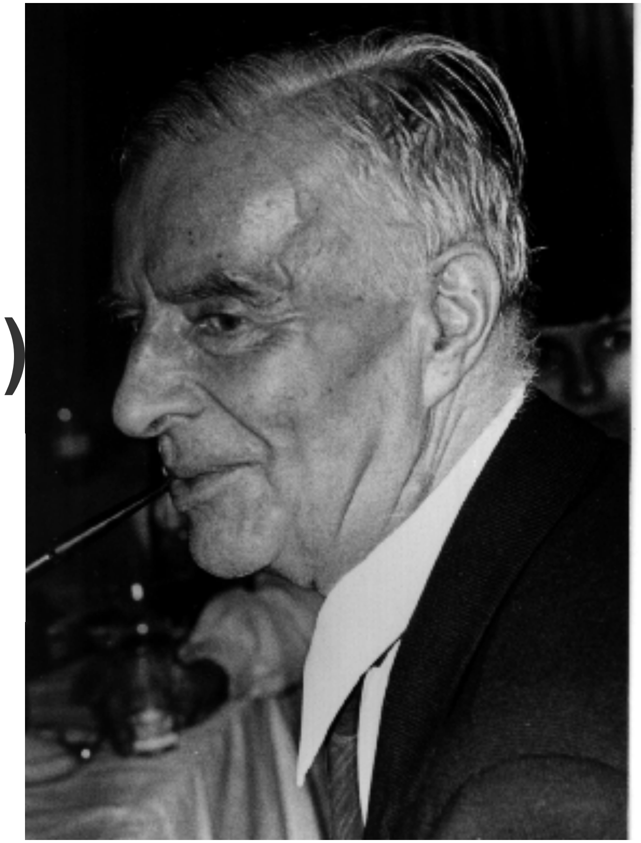


Vilnius University – NCU in Torun



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After the war in November 1945, Jablonski returned to Poland and started to work again at the Department of Physics of the University of Warsaw under Stefan Pienkowski. Soon, however, he moved to Torun, where in the fall of 1945 a new University holding the name of Nicholas Copernicus, who was born in that town, was founded. For many years it was the only University in North Poland. On January 1, 1946 Jablonski was nominated as the full professor of N. Copernicus University and his first historic lecture for students of science at Torun took place on February 17, 1946. This date is considered at Torun as the beginning of physics at N. Copernicus University.

QTC'2011 Kraków:

Vilnius - Kraków

QTC'2011 Kraków:

Vilnius - Kraków

King Władysław II Jagiełło (1386-1434) (resided in Kraków since 1386)

King Władysław II Jagiełło (1386-1434) (In Lithuanian: Jogaila Algirdaitis)

Władysław II Jagiełło

Z Wikipedii, wolnej encyklopedii

Władysław II Jagiełło, lit. Jogaila Algirdaitis, biał. Ягайла Уладзіслаў, ukr. Владислав Ягайло (ur. zapewne ok. 1362[1], zm. 1 czerwca 1434 w Gródku) – wielki książę litewski w latach 1377-1381 i 1382-1392, król Polski i najwyższy książę litewski w latach 1386-1434. Syn Olgierda i jego drugiej żony Julianny, córki księcia twerskiego Aleksandra, wnuk Giedymina. Założyciel dynastii Jagiellonów.

Spis treści

Tytuł królewski

Wladislaus dei gracia Rex Polonie, nec non terrarum Cracovie, Sandomirie, Siradie, Lancicie, Cuyauie, Lituanie princeps supremus, Pomeranie, Russieque dominus et heres, etc.

Tłumaczenie: Władysław z Bożej Łaski król Polski, pan i dziedzic ziemi krakowskiej, sandomierskiej, sieradzkiej, łęczyckiej, Kujaw, Pomorza i Rusi Czerwonej, najwyższy książę Litwy.

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Wielki książę litewski	
Okres panowania	od 1377 do 1381
Poprzednik	Olgierd
Następca	Kiejstut
Wielki książę litewski	
Okres panowania	od 1382 do 1392
Poprzednik	Kiejstut
Następca	Witold Wielki
Król Polski	
Okres panowania	od 4 marca 1386 do 1 czerwca 1434
Poprzednik	Jadwiga Andegaweńska
Następca	Władysław III Warneńczyk

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Collaboration

(in the area of the artificial magnetic field for cold atoms)

- **P. Öhberg & group,** Heriot-Watt University, *Edinburgh*
- ***M. Fleischhauer & group,*** *TU Kaiserslautern,*
- ***L. Santos & group,*** *Universität Hannover*
- ***J. Dalibard, F. Gerbier & group, ENS, Paris***
- ***I. Spielman, D. Campbel, C. Clark and J. Vaishnav, NIST***
- ***M. Lewenstein, ICFO, ICREA***

Quantum Optics Group @ ITPA, Vilnius University

V. Kudriasov, J. Ruseckas, G. J., A. Mekys, T. Andrijauskas and V. Pyragas (not in the picture)

Quantum Optics Group @ ITPA, Vilnius University

Research activities:

- Light-induced gauge potentials for cold atoms (both Abelian and non-Abelian)
- Ultra cold atoms in optical lattices
- Slow light (with OAM, multi-component, …)
- Graphene
- Metamaterials

Quantum Optics Group @ ITPA, Vilnius University

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- Light-induced gauge potentials for cold atoms
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This talk:

A combination of first two topics

Quantum Optics Group @ ITPA, Vilnius University

Research activities:

- Light-induced gauge potentials for cold atoms
- Cold atoms in optical (flux) lattices

This talk:

A combination of first two topics

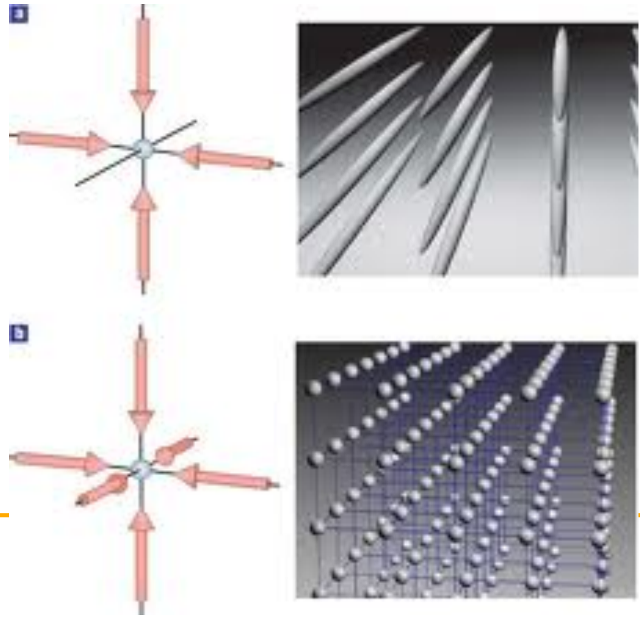
OUTLINE

- Optical lattices (for ultracold atoms)
- Light-induced gauge potentials
- Optical flux lattices (OFL): Non-staggered magnetic flux
- Ways of producing of OFL
- Conclusions

Optical lattices (ordinary): [Last 10 years]

- A set of **counter-propagating** light beams (*off resonance to the atomic transitions*)

I. Bloch, Nature Phys. 1, 23 (2005)



Optical lattices (ordinary)

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- → Atoms are trapped at **intensity minima** (or **intensity maxima**) of the interference pattern

  (depending on the sign of atomic polarisability)

$$V_{dip}(\mathbf{r}) = -\mathbf{d}\cdot\mathbf{E}(\mathbf{r}) = -\alpha(\omega_L)\left|\mathbf{E}(\mathbf{r})\right|^2$$

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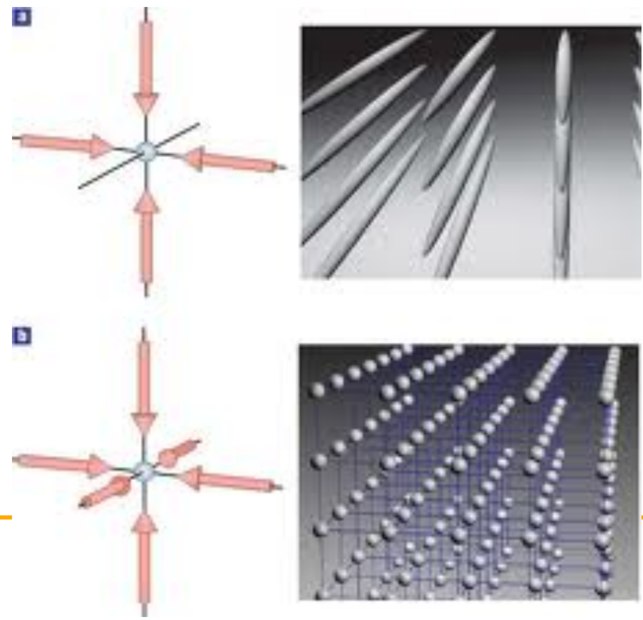

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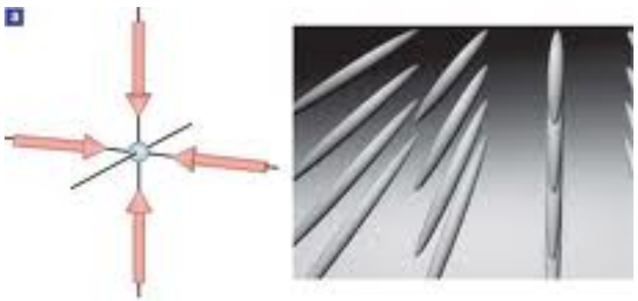
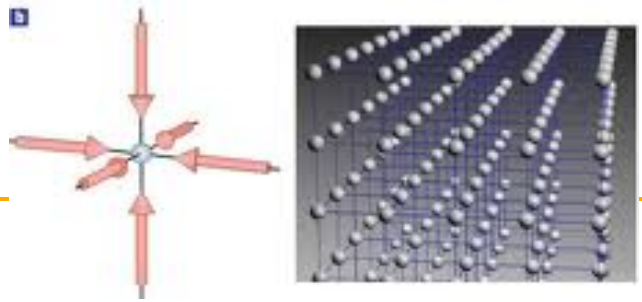
2D square optical lattice:



3D cubic optical lattice:



Optical lattices (more sophisticated)

- Triangular or hexagonal optical lattices using three light beams (propagagating at 120^0)

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Experiment:

New Journal of Physics
The open-access journal for physics

Ultracold quantum gases in triangular optical lattices

New Journal of Physics **12** (2010) 065025 (17pp)

C Becker[1,3], P Soltan-Panahi[1], J Kronjäger[2], S Dörscher[1], K Bongs[2] and K Sengstock[1]

nature physics
ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NPHYS1916

Multi-component quantum gases in spin-dependent hexagonal lattices

P. Soltan-Panahi[1], J. Struck[1], P. Hauke[2], A. Bick[1], W. Plenkers[1], G. Meineke[1], C. Becker[1], P. Windpassinger[1], M. Lewenstein[2,3] and K. Sengstock[1]*

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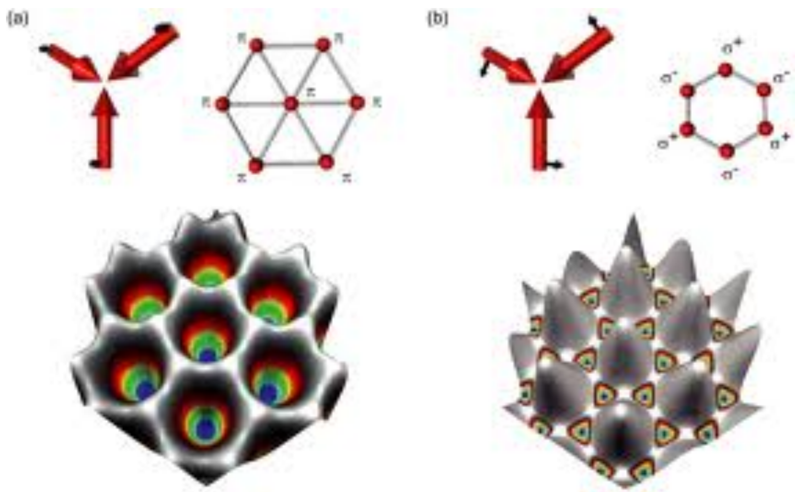
- Triangular or hexagonal optical lattice using three light beams (propagagating at 120^0)



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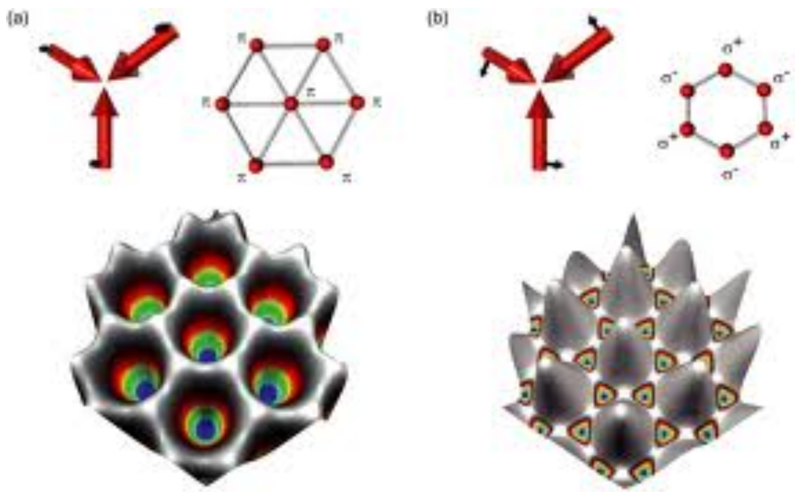

- (a) Polarisations are perpendicular to the plane
  → **<u>Triangular</u>** lattice
- (b) Polarisations are rotating in the plane
  → **<u>Hexagonal</u>** lattice:
  → Analogies with electrons graphene

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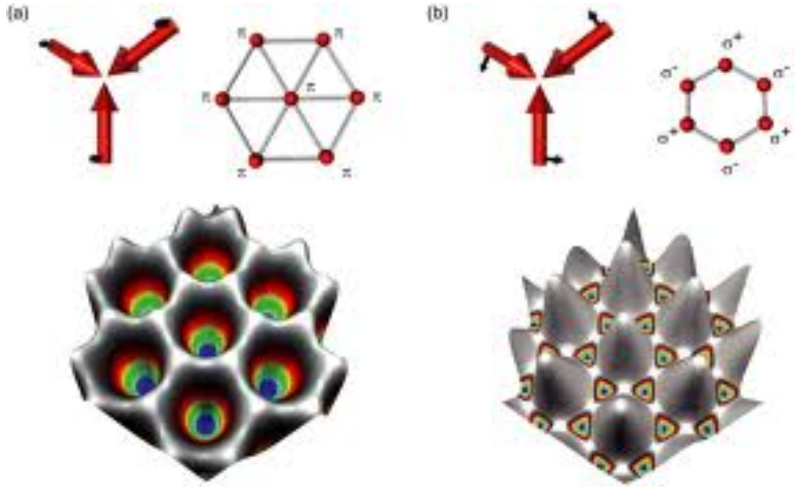

- (a) Polarisations are perpendicular to the plane
  → **Triangular** lattice
- (b) Polarisations are rotating in the plane
  → **Hexagonal** lattice (polarisation-dependent):
  → Analogies with electrons graphene

Optical lattices (more sophisticated)

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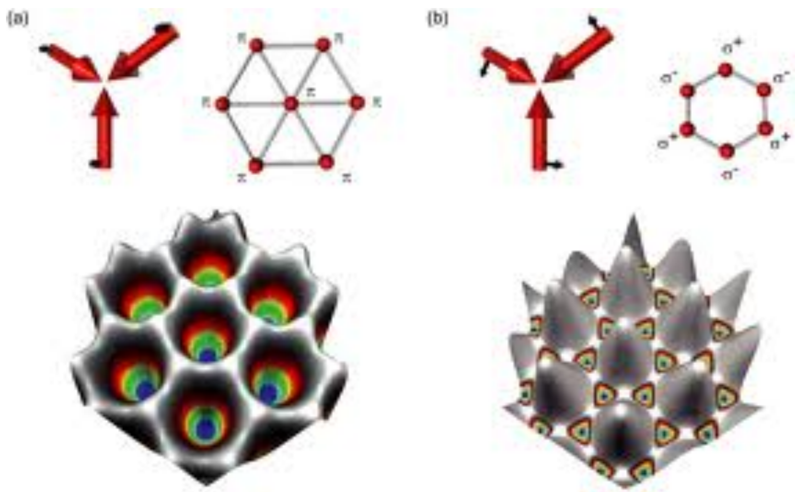
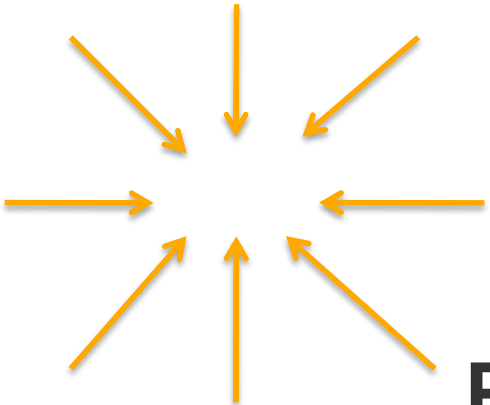
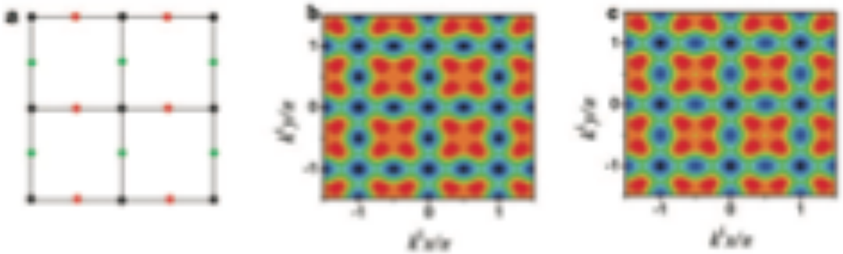


- (a) Polarisations are perpendicular to the plane → **Triangular** lattice
- (b) Polarisations are rotating in the plane → **Hexagonal** (spin-dependent) lattice

Optical lattices (more sophisticated)

- Line-centered (Lieb) lattice – several counter-propagating light beams making 45^0 and 90^0





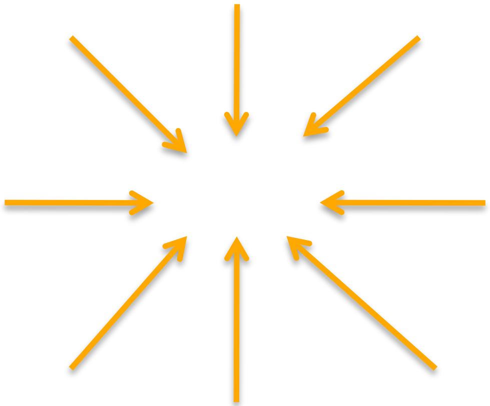
Proposal to produce such lattice:

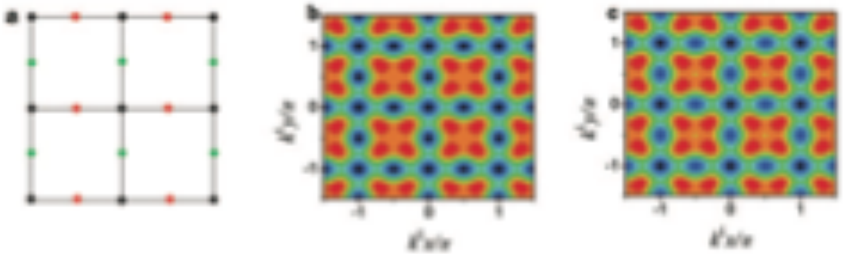
PHYSICAL REVIEW B **81**, 041410(R) (2010)

R. Shen,* L. B. Shao, Baigeng Wang, and D. Y. Xing

Optical lattices (more sophisticated)

- Line-centered (Lieb) lattice – several counter-propagating light beams making 45^0 and 90^0





- Formation of a flat band and Dirac cones in the dispersion
- Interesting many-body effects:
  - More – **talk by Tomas Andrijauskas (<u>next</u>)**

Optical lattices

- Analogies with the solid state physics
  - Fermionic atoms ↔ Electrons in solids
  - Atoms in optical lattices – Hubbard model
  - Simulation of various many-body effects
- **Advantage :**
  - Freedom in changing experimental parameters that are often inaccessible in standard solid state experiments

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  - Simulation of various many-body effects
- **Advantage :**
  - Freedom in changing experimental parameters that are often inaccessible in standard solid state experiments
  - e.g. number of atoms, atom-atom interaction, lattice potential

Trapped atoms - **electrically neutral** species

- No direct analogy with **magnetic** properties due to electrons in solids
- A possible method to create an **effective** magnetic field (an **artificial** Lorentz force):

  Rotation

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- A possible method to create an **effective** magnetic field (an **artificial** Lorentz force):

  <u>Rotation</u>, e.g.

PRL **104,** 050404 (2010) PHYSICAL REVIEW LETTERS wee 5 FEB

Observation of Vortex Nucleation in a <u>Rotating Two-Dimensional Lattice</u> of Bose-Einstein Condensates

R. A. Williams, S. Al-Assam, and C. J. Foot

Trapped atoms - **electrically neutral** particles

- No direct analogy with **magnetic** properties due to electrons in solids
- A possible method to create an **effective** magnetic field (an **artificial** Lorentz force):

  <u>Rotation</u> → Coriolis force →

Trapped atoms - **electrically neutral** particles

- No direct analogy with **magnetic** properties due to electrons in solids
- A possible method to create an **effective** magnetic field:

  <u>Rotation</u> → Coriolis force

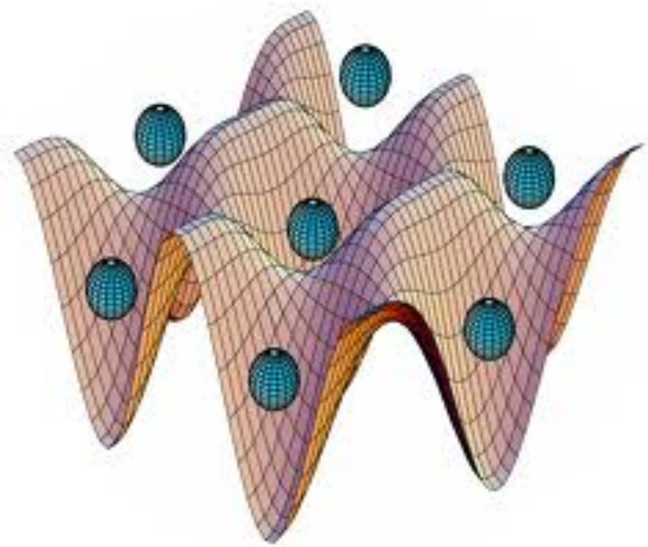
  (Mathematically equivalent to Lorentz force)

ROTATION

- Can be applied to utracold atoms both in **usual traps** and also in **optical lattices**

(a) Ultracold atomic cloud (trapped):

(b) Optical lattice:



Trap rotation

Hamiltonian in the rotating frame

[see e.g. A. Fetter, RMP 81, 647 (2009)]

Trapping potential

$$H_0' = p^2/(2M) + \tfrac{1}{2} M \omega_\perp^2 r^2 - \boldsymbol{\Omega} \cdot \boldsymbol{r} \times \boldsymbol{p}$$

Trap rotation

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Trapping potential

$$H_0' = p^2/(2M) + \tfrac{1}{2}M\omega_\perp^2 r^2 - \boldsymbol{\Omega}\cdot\boldsymbol{r}\times\boldsymbol{p}$$

or

rotation vector

$$H_0' = \frac{(\boldsymbol{p} - M\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2M} + \frac{1}{2}M(\omega_\perp^2 - \Omega^2)r^2$$

Effective vector potential

(constant B_{eff}~Ω)

Trap rotation

Hamiltonian in the rotating frame

[see e.g. A. Fetter, RMP 81, 647 (2009)]

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Effective vector potential (constant $B_{eff} \sim \Omega$)

Centrifugal potential (anti-trapping)

Trap rotation

Hamiltonian in the rotating frame

[see e.g. A. Fetter, RMP 81, 647 (2009)]

Trapping potential

$$H_0' = p^2/(2M) + \tfrac{1}{2}M\omega_\perp^2 r^2 - \boldsymbol{\Omega}\cdot\boldsymbol{r}\times\boldsymbol{p}$$

or **rotation vector**

$$H_0' = \frac{(\boldsymbol{p} - M\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2M} + \frac{1}{2}M(\omega_\perp^2 - \Omega^2)r^2$$

Effective vector potential **Centrifugal potential**

(constant B_{eff}~Ω) **(anti-trapping)**

Coriolis force (equivalent to **Lorentz** force)

Trap rotation:
Summary of the **main features**

- Constant B_{eff}: $B_{eff} \sim \Omega$
- Trapping frequency: $\omega_{eff} = \sqrt{\omega_\perp^2 - \Omega^2}$
- $\Omega \to \omega_\perp$ ⇨ Landau problem

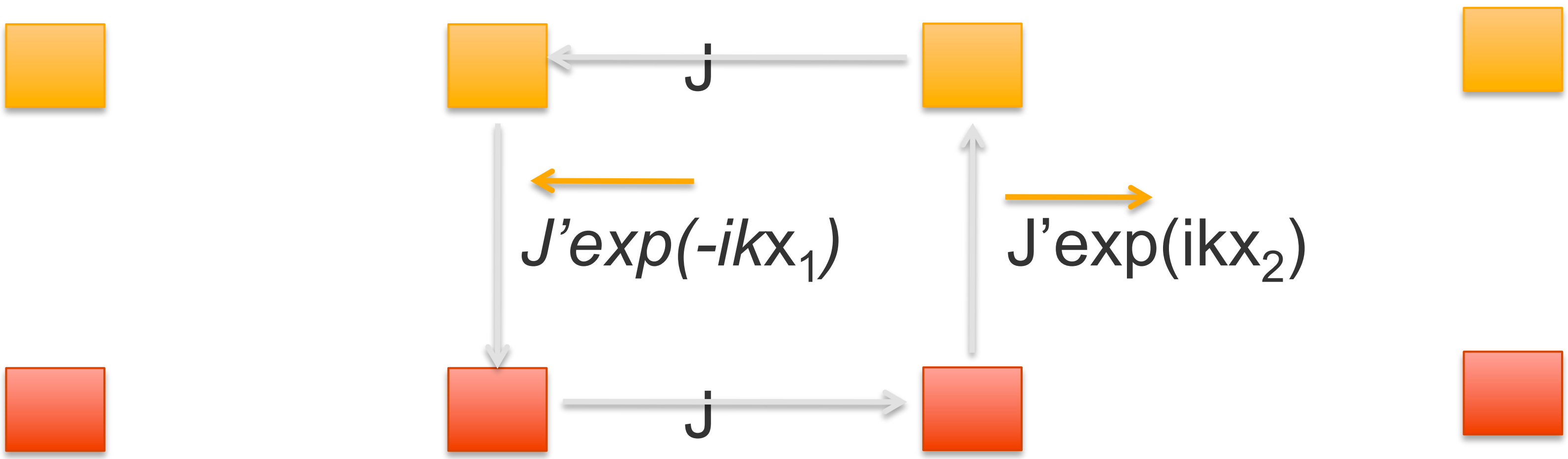
Effective magnetic fields without rotation

- **<u>Using (unconventional) optical lattices</u>**

  Initial proposals:
  - J. Ruostekoski, G. V. Dunne, and J. Javanainen, Phys. Rev. Lett. 88, 180401 (2002)
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - E. Mueller, Phys. Rev. A **70**, 041603 (R) (2004)
- B_{eff} *is produced by inducing an* assymmetry in atomic transitions between the lattice sites.
- Non-vanishing phase for atoms moving along a closed path on the lattice
- $\rightarrow$ Simulates non-zero magnetic flux $\rightarrow B_{eff} \neq 0$

Effective magnetic fields without rotation

- **Optical lattices**
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - J. Dalibard and F. Gerbier, NJP **12**, 033007 (2010).
  - -Ordinary tunneling along *x* direction (J).
  - -Laser-assisted tunneling between atoms in different internal states along *y* axis (**with recoil** along x).



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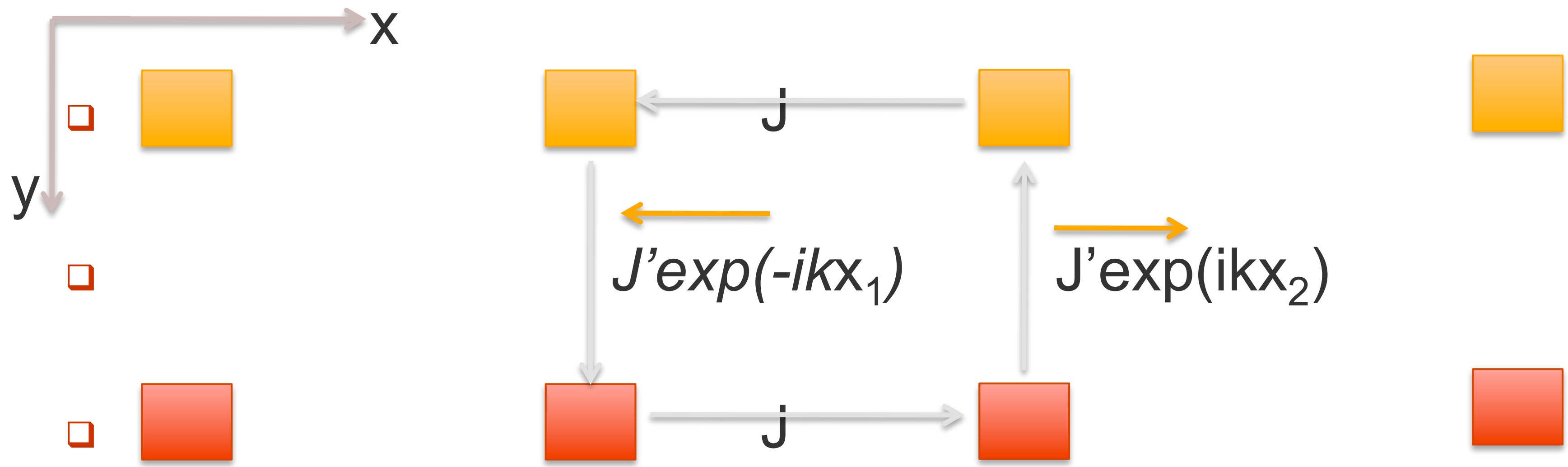

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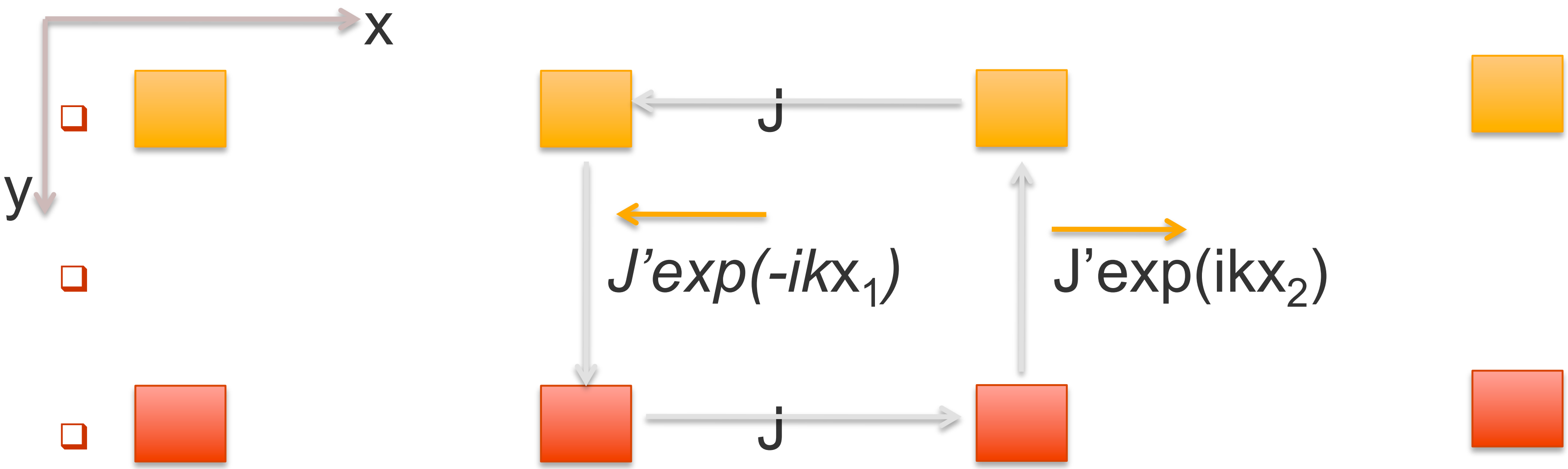

  - Non-vanishing phase for the atoms moving over a plaquette: $S=k(x_2-x_1)=ka$

Effective magnetic fields without rotation

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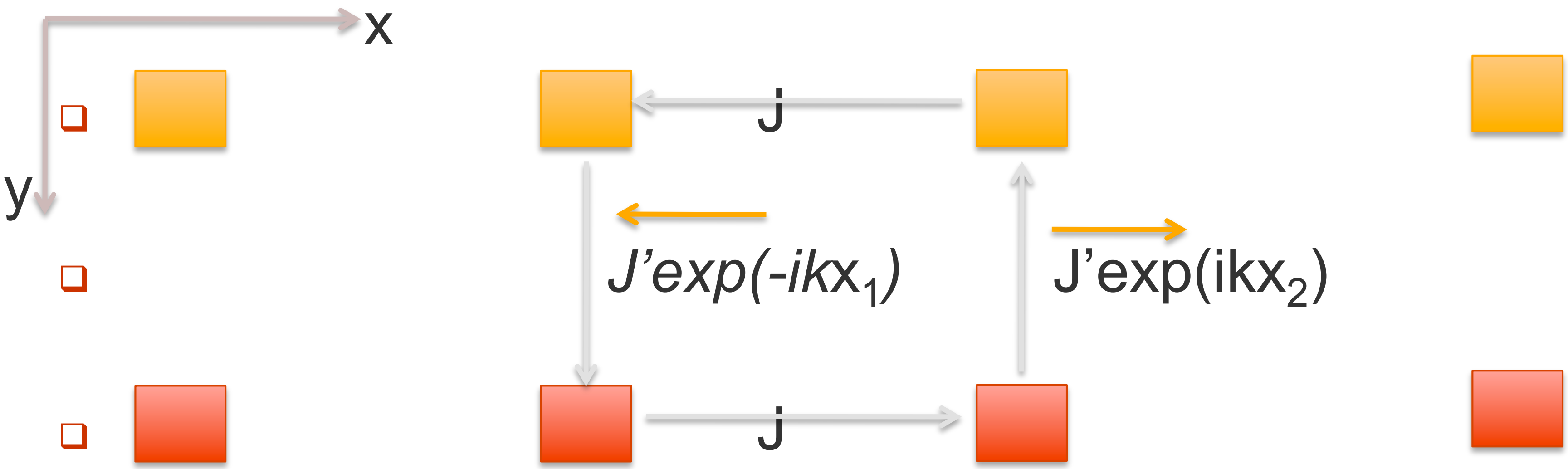

  - <u>Non-vanishing phase</u> for the atoms moving over a plaquette: $S=k(x_2-x_1)=ka$
  - → Simulates <u>non-zero magnetic flux</u>

Effective magnetic fields without rotation

- **Optical lattices**:

The method can be extended to create Non-Abelian gauge potentials

(Laser assisted, state-sensitive tunneling)

- K. Osterloh, M. Baig, L. Santos, P. Zoller and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005)

Here: Creation of B_{eff} using geometric potentials

- Distinctive features:
  - No rotation is necessary
  - No lattice is needed
  - Yet lattices can be an important ingredient in creating B_{eff} using geometric potentials → **Optical flux lattices**

Geometric potentials

- Emerge in various areas of physics (molecular, condensed matter physics etc.)
- First considered by Mead, Berry, Wilczek and Zee and others in the 80's (initially in the context of molecular physics).
- More recently – in the context of motion of cold atoms affected by laser fields

  (**Currently:** a lot of activities)
- Advantage of such atomic systems: possibilities to control and shape gauge potentials by choosing proper laser fields.

Creation of B_{eff} using geometric potentials

Atomic dynamics taking into account both internal degrees of freedom and also center of mass motion.

- The full atomic Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t).$$

Creation of B_{eff} using geometric potentials

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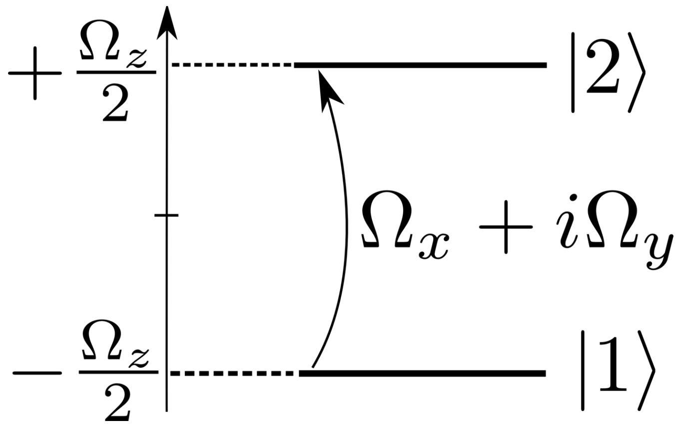
For instance: Two atomic internal states



- Position-dependent detuning $\Delta(\mathbf{r}) \equiv \Omega_z$
- Position-dependence of the Rabi frequencies of atom-light coupling $\Omega_\pm(\mathbf{r}) \equiv \Omega_x \pm i\Omega_y$
- **Atom-light Hamiltonian:**

$$\hat{H}_0(\mathbf{r}) = -\hbar \begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

(2×2 matrix)

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  - $\hat{V}(\mathbf{r})$ — the external trapping potential (for c.m. motion)
- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r}, t)$.

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↖ (**r**-dependent "dressed" eigenstates)

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- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r}, t)$.
- Full atomic wave function ↖ (**r**-dependent "dressed" eigenstates)

$$|\Phi\rangle = \sum_n \Psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle.$$

- Adiabatic atomic energies $\varepsilon_n(\mathbf{r})$

- Full state vector:

$$|\Phi\rangle = \sum_{n=1}^{N} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$

$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal “dressed” state $|\chi_n(\mathbf{r})\rangle$

Non-degenerate state with n=1

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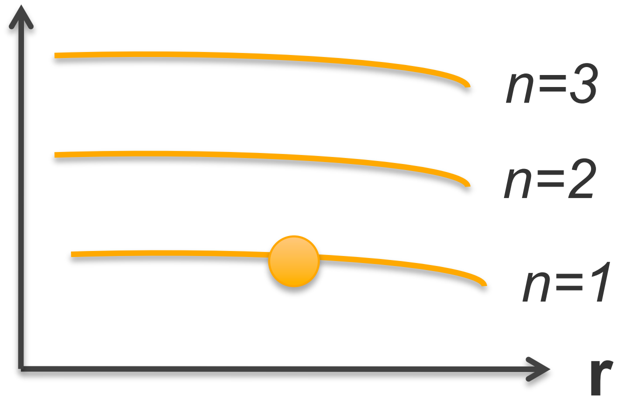

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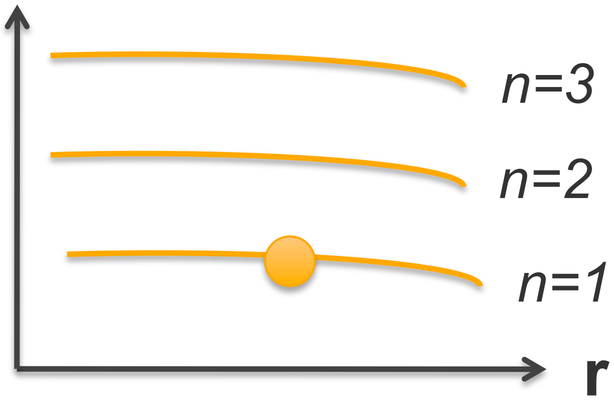

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$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$

(only the atomic internal state with *n=1* is included)

Non-degenerate state with n=1

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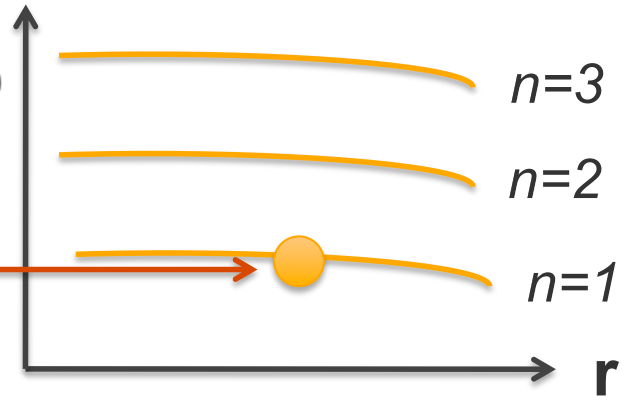

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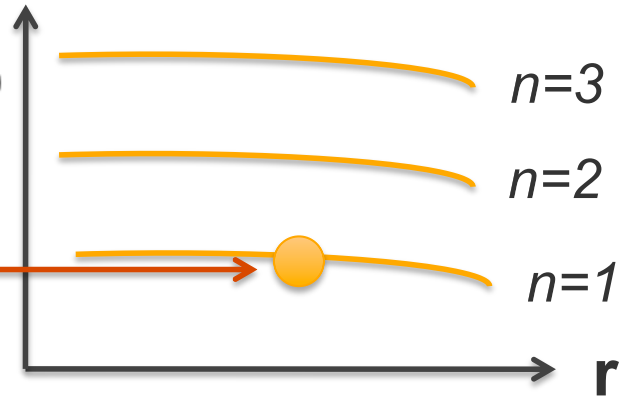

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- What is the equation of motion for $\Psi_1(\mathbf{r},t)$?

Non-degenerate state with n=1

Adiabatic approximation:

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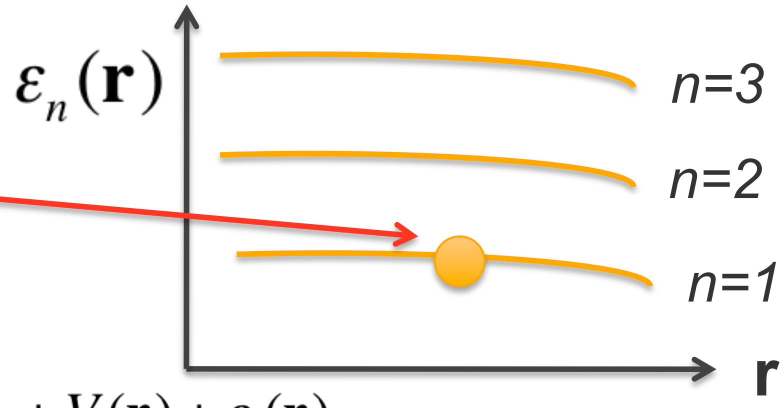

Equation of motion:

- $i\hbar\partial_t|\Phi\rangle = H|\Phi\rangle$ with $\hat{H} \approx \frac{\hat{p}^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

$$\hat{p} = -i\hbar\nabla$$

Non-degenerate state with n=1

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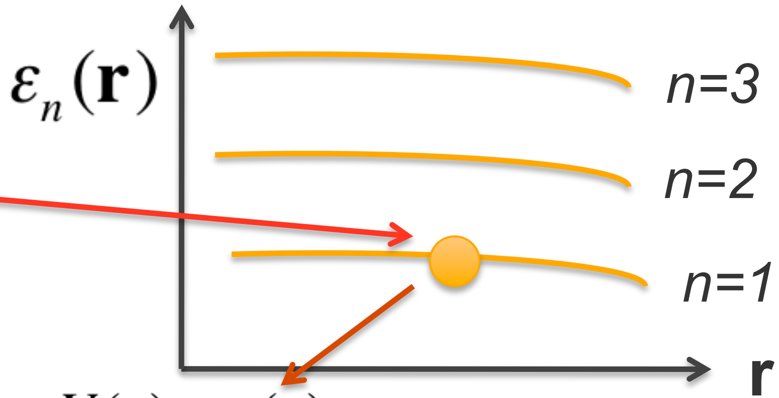

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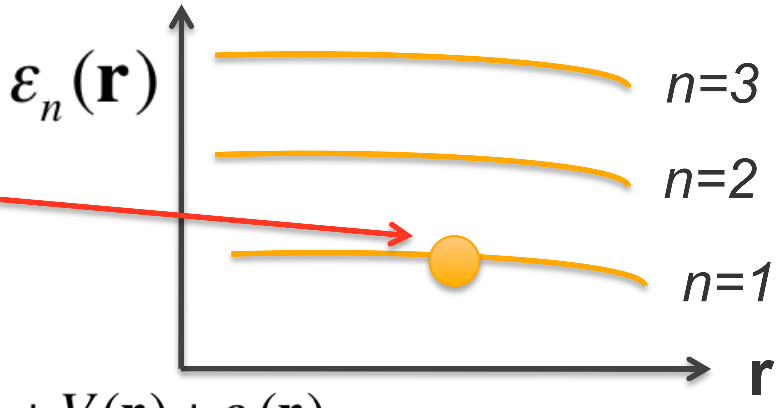

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Action of the momentum operator:

$$\hat{p} = -i\hbar\nabla$$

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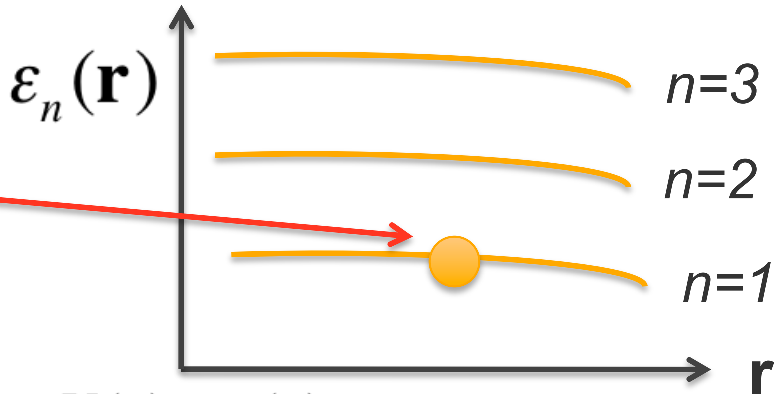

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Projecting onto the state $|\chi_1(\mathbf{r})\rangle$

Non-degenerate state with n=1

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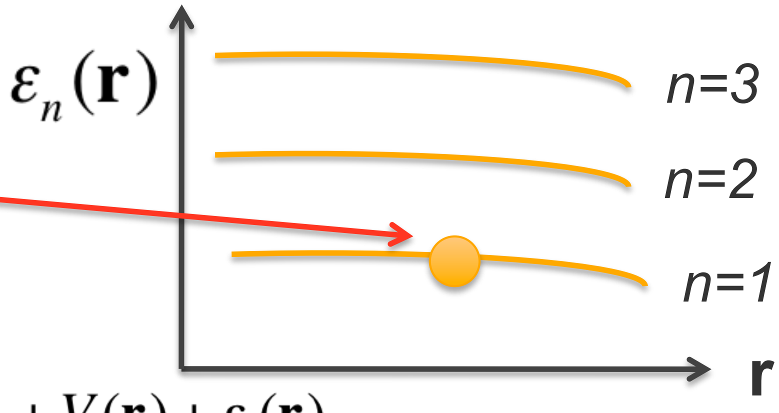

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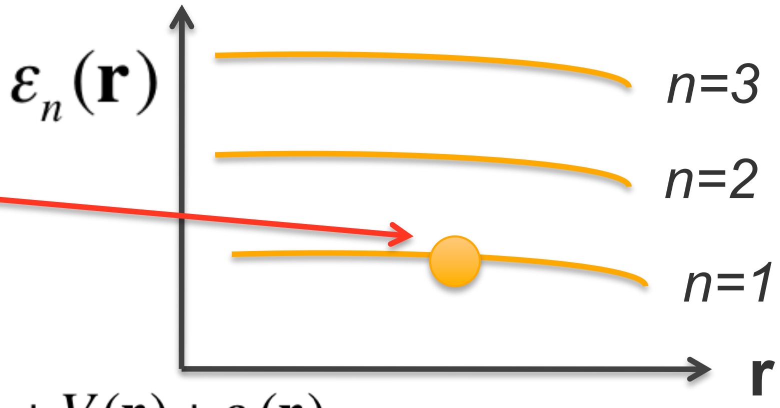
Projecting onto the state $|\chi_1(\mathbf{r})\rangle$

we have $\hat{p}|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle\left(-i\hbar\nabla - i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle\right)\Psi_1(\mathbf{r},t)$

Non-degenerate state with n=1

Adiabatic approximation:



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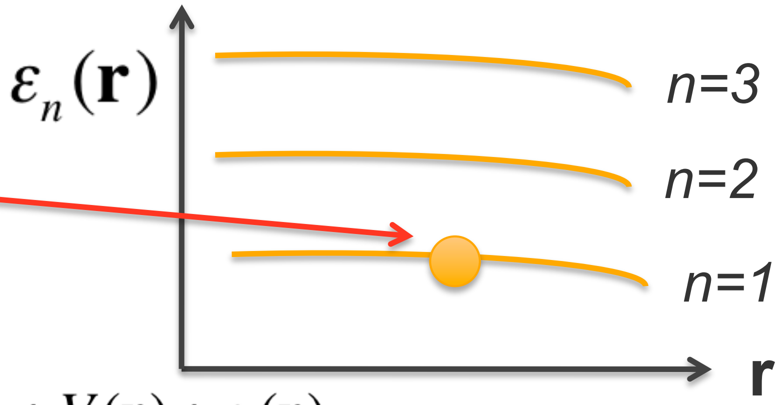
Brings a vector potential

we have $\hat{p}|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle\Big(-i\hbar\nabla \underbrace{- i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle}_{=\mathbf{A}_{11}}\Big)\Psi_1(\mathbf{r},t)$

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



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Brings a vector potential

Projecting

$$\hat{p}|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle\left(-i\hbar\nabla - \mathbf{A}_{11}\right)\Psi_1(\mathbf{r},t)$$

$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

Non-degenerate state with n=1

Adiabatic approximation:

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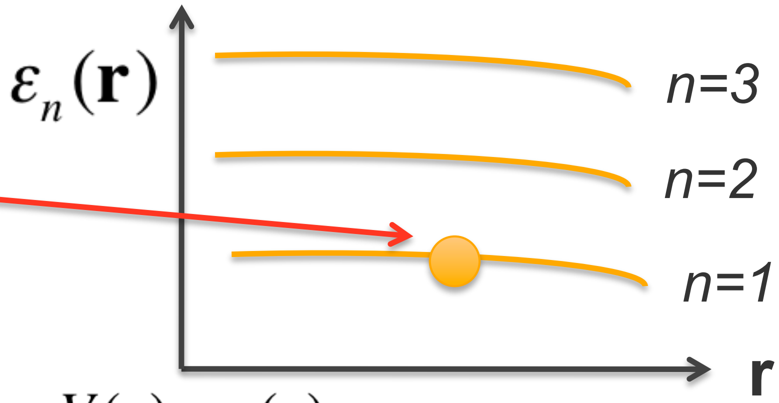

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$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

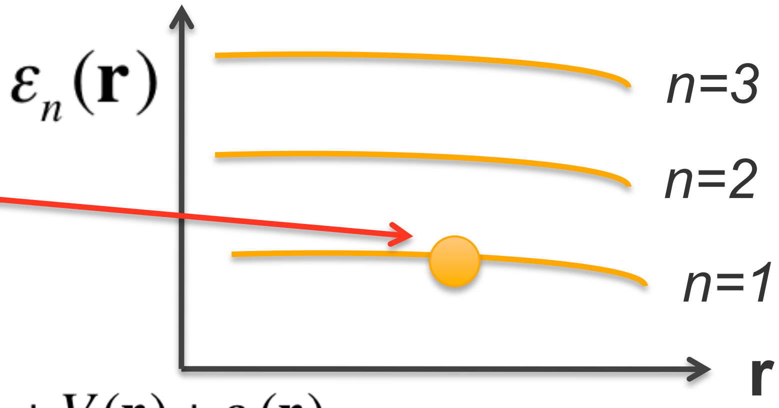
$\rightarrow$ $i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$ $\qquad \hat{H} = \dfrac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



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$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

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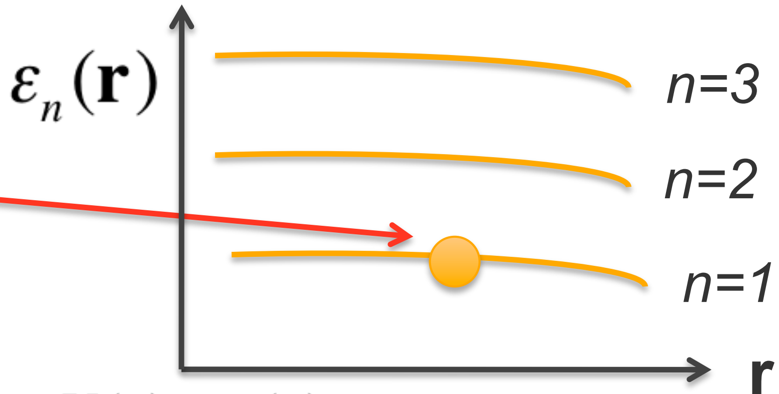
Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears. **What is mising?**

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

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$$\hat{p} = -i\hbar\nabla$$

Action of the momentum operator:

$$\hat{p}|\Phi\rangle = -i\hbar|\chi_1(\mathbf{r})\rangle\nabla\Psi_1(\mathbf{r},t) - i\hbar|\nabla\chi_1(\mathbf{r})\rangle\Psi_1(\mathbf{r},t)$$

Using completeness !!!

Exact:

$$\hat{p}|\Phi\rangle = \left[|\chi_1(\mathbf{r})\rangle(-i\hbar\nabla - \mathbf{A}_{11}) - \sum_{n=2}^{N}|\chi_n(\mathbf{r})\rangle\mathbf{A}_{n1}\right]\Psi_1(\mathbf{r},t)$$

$$\mathbf{A}_{n1} = i\hbar\langle\chi_n(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

!!!

$$\rightarrow \quad i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\boxed{\hat{H} = \frac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})}$$

!!!

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ & scalar potential $\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N}\mathbf{A}_{1n}\mathbf{A}_{n1}$

Non-degenerate state with n=1

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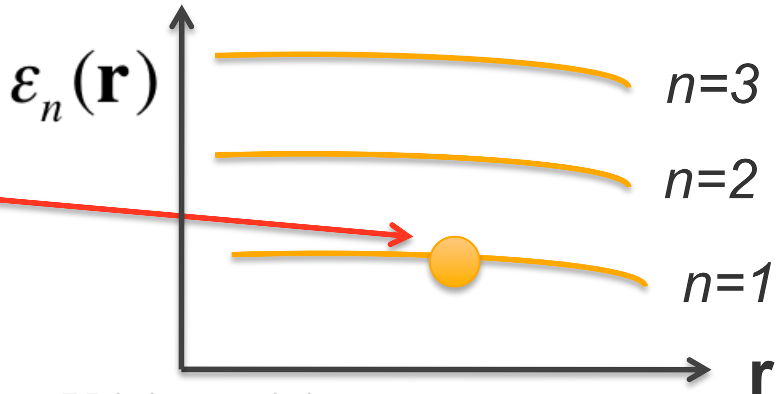

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$$\mathbf{A}_{n1} = i\hbar\langle\chi_n(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

Corrected:

$$\rightarrow \quad i\hbar\partial_t\Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t) \qquad \boxed{\hat{H} = \frac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})}$$

!!!

!!!

Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ & scalar potential $\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N} \mathbf{A}_{1n}\mathbf{A}_{n1}$

Summary: Non-degenerate state with n=1

Adiabatic approximation:

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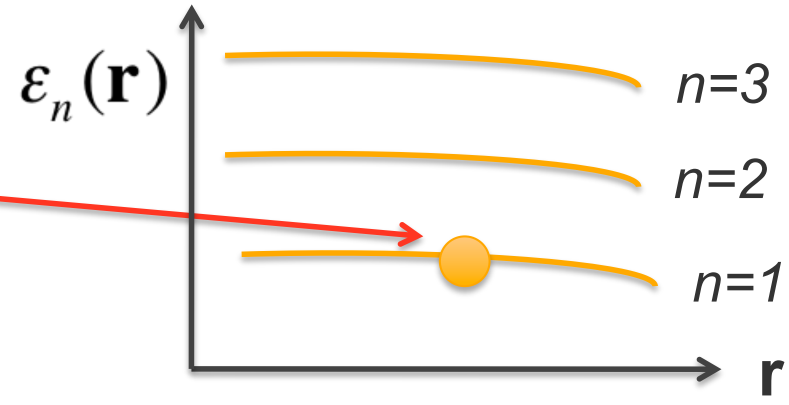

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$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$ - effective vector potential (Berry connection)

$\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N}\mathbf{A}_{1n}\mathbf{A}_{n1}$ - effective scalar potential

$\mathbf{B} = \nabla\times\mathbf{A}$ - effective magnetic field (curvature)

Both $\mathbf{A}$ & Φ appear due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

Summary: Non-degenerate state with n=1

Adiabatic approximation:

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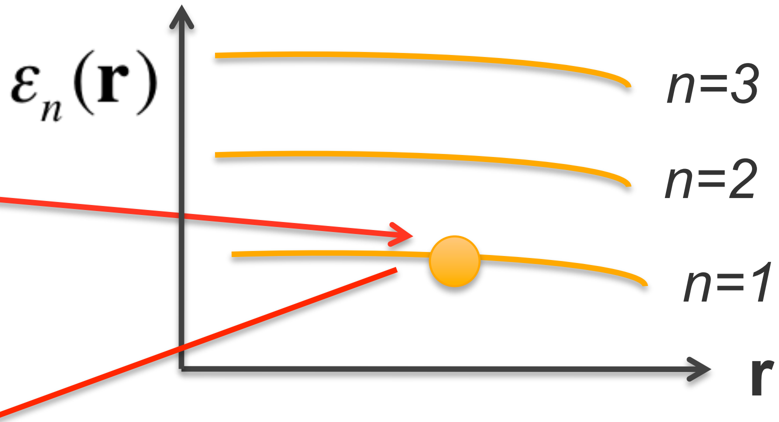

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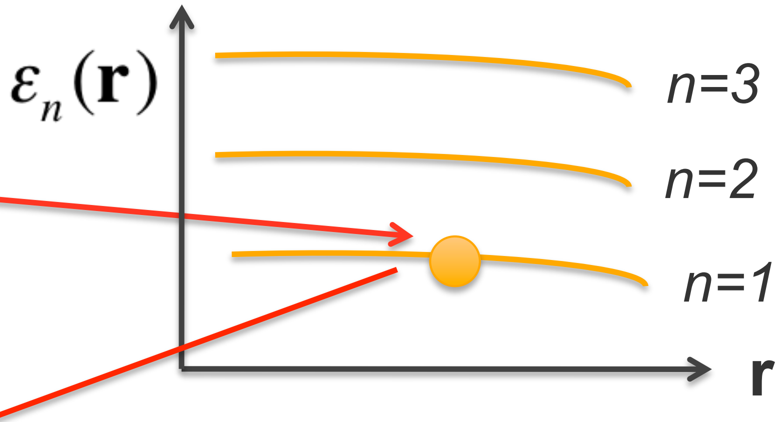
Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of motion:

$$i\hbar\partial_t \Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) + \Phi(\mathbf{r})$$

$$\hat{p} = -i\hbar\nabla$$

Momentum associated with $|\chi_1(\mathbf{r})\rangle$

$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$ - effective vector potential (Berry connection)

$\Phi \equiv \Phi(\mathbf{r}) = \dfrac{1}{2M}\sum_{n=2}^{N} \mathbf{A}_{1n}\mathbf{A}_{n1}$ - effective scalar potential

$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (curvature)

Both $\mathbf{A}$ & Φ appear due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

Summary: Non-degenerate state with n=1

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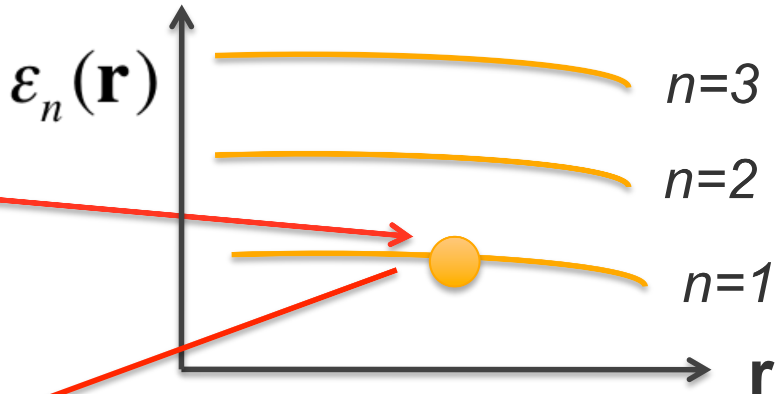

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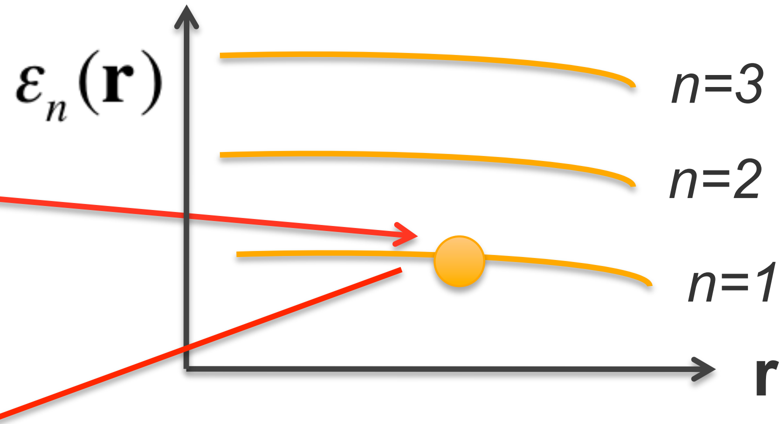

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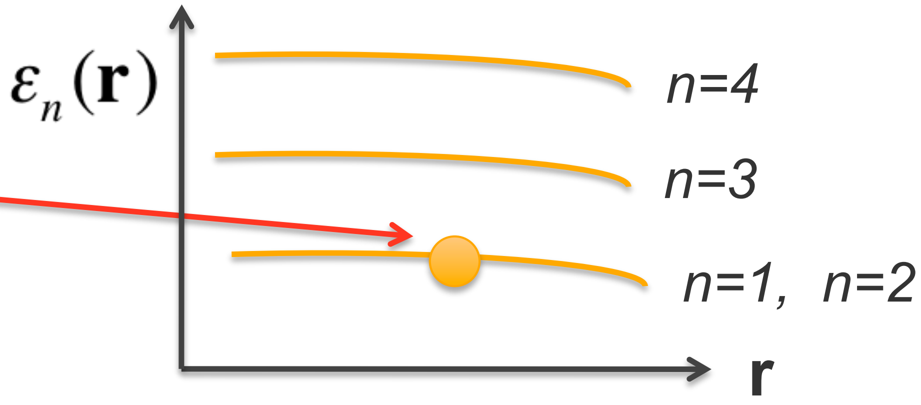
Degenerate states with n=1 and n=2

Adiabatic approximation:

$$|\Phi\rangle \approx \sum_{n=1}^{2} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$



$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal “dressed” state (*n=1,2*)

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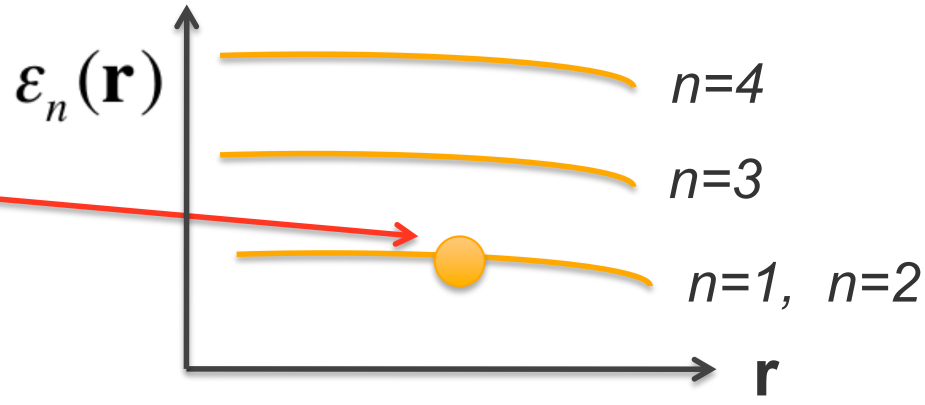

$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal “dressed” state (*n=1,2*)

$$\Psi(\mathbf{r},t) = \begin{pmatrix} \Psi_1(\mathbf{r},t) \\ \Psi_2(\mathbf{r},t) \end{pmatrix}$$

-two-component atomic wave-function (spinor wave-function)

→ **Quasi-spin 1/2**

Repeating the same procedure …

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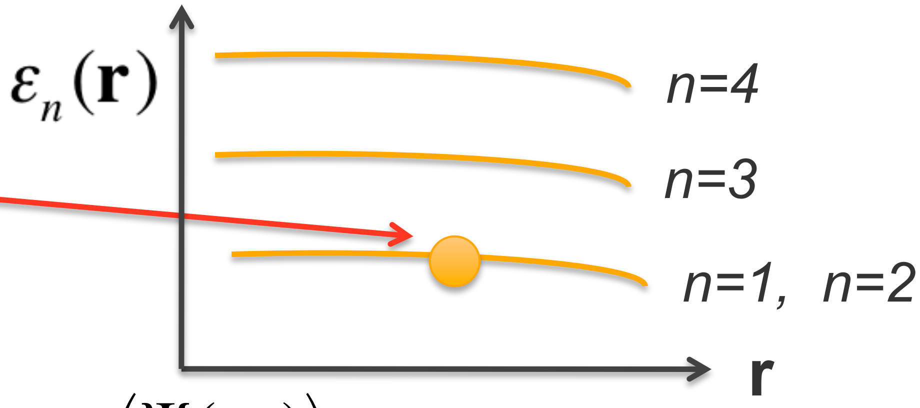

Equation of motion:

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- two-comp. atomic wave-function

2x2 matrix

$$\mathbf{A}_{lj} = i\hbar\langle\chi_l(\mathbf{r})|\nabla\chi_j(\mathbf{r})\rangle, \qquad (l,j=1,2)$$

- effective vector potential

2x2 matrix

$$\mathbf{B} = \nabla\times\mathbf{A} + \frac{1}{i\hbar}\mathbf{A}\times\mathbf{A}$$

- effective magnetic field (non-trivial situation if $\mathbf{B}\neq 0$)

$\mathbf{A}$ appears due to position-dependence of $|\chi_n(\mathbf{r})\rangle$

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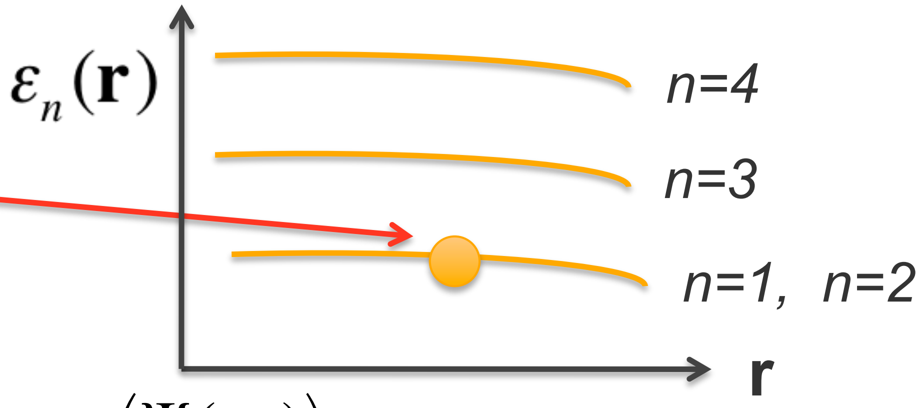

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If A_x, A_y, A_z do not commute, **B**≠0 even if **A** is constant !!!

Degenerate states with n=1 and n=2

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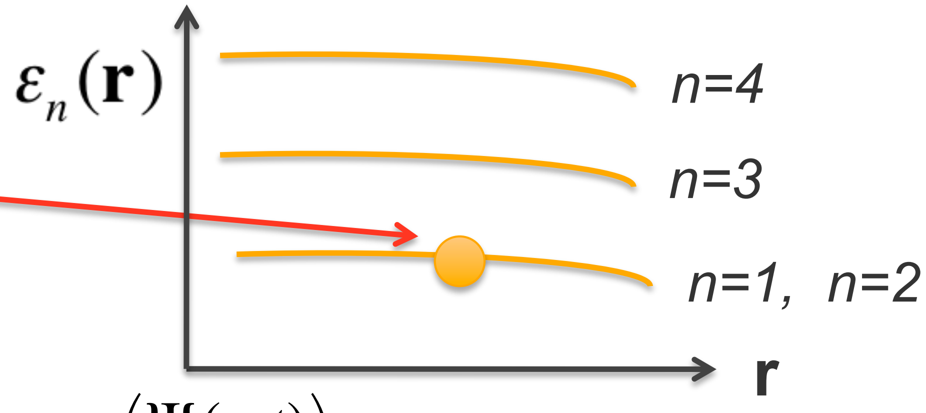

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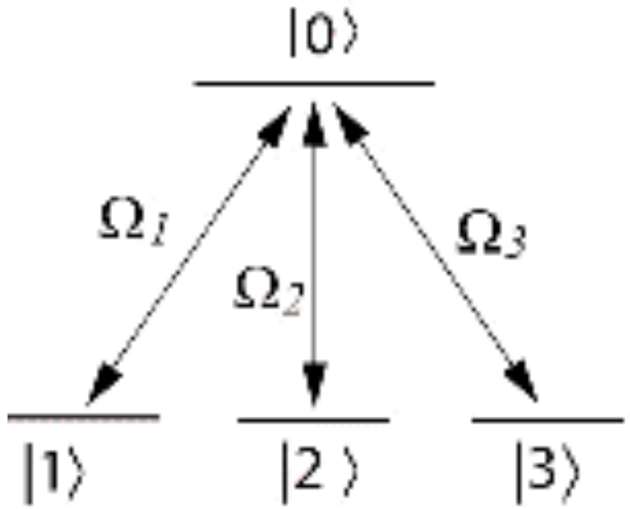
→ Non-Abelian gauge potentials are formed

Tripod setup:
Non-Abelian light-induced gauge potentials



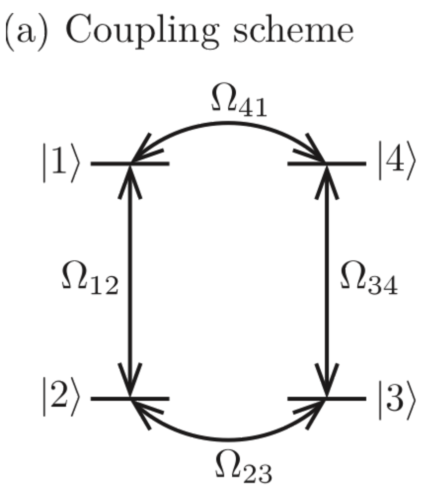
for centre of mass motion of dark-state atoms:

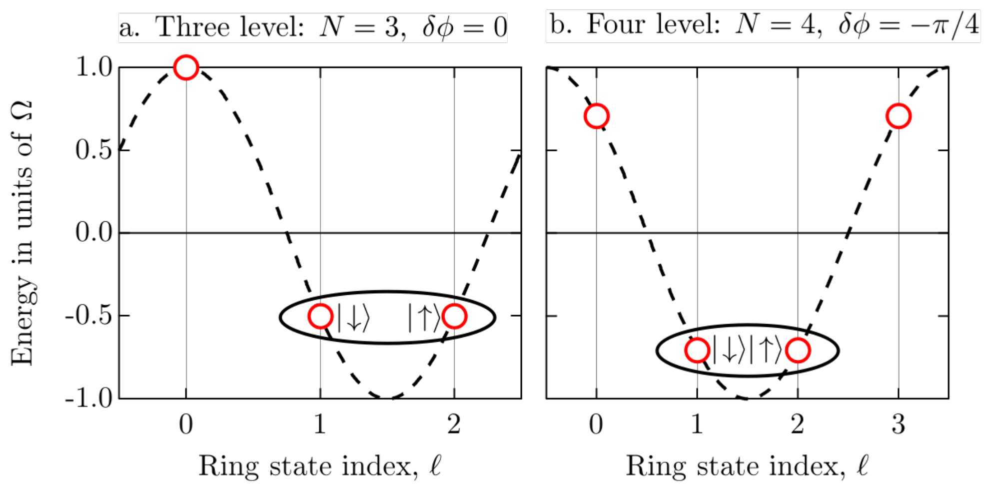
(Due to the spatial dependence of the dark states)

↖ (two degenerate dark states)

J. Ruseckas, G. Juzeliūnas and P.Öhberg, and M. Fleischhauer, Phys. Rev. Letters 95, 010404 (2005).

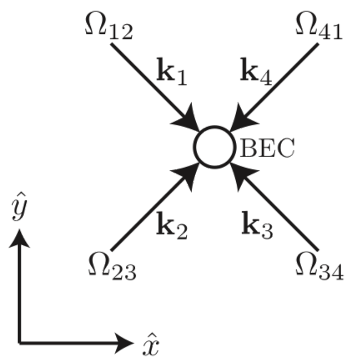
- Drawback of the tripod scheme: degenerate dark states are not the ground atomic dressed states → collision-induced loses
- Closed loop setup overcomes this drawback:





D. L. Campbell, G. Juzeliūnas and I. B. Spielman, arXiv1102.3945 (2011); (to appear in PRA)

- Arrangement of laser fields:



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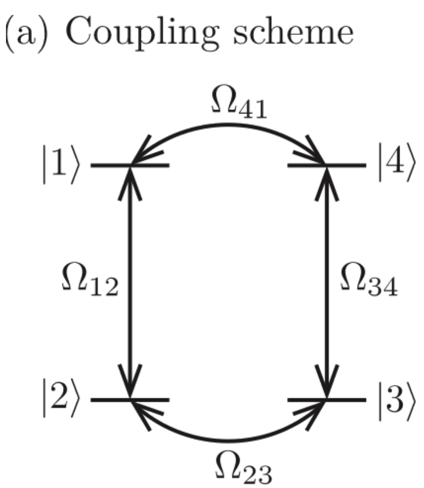
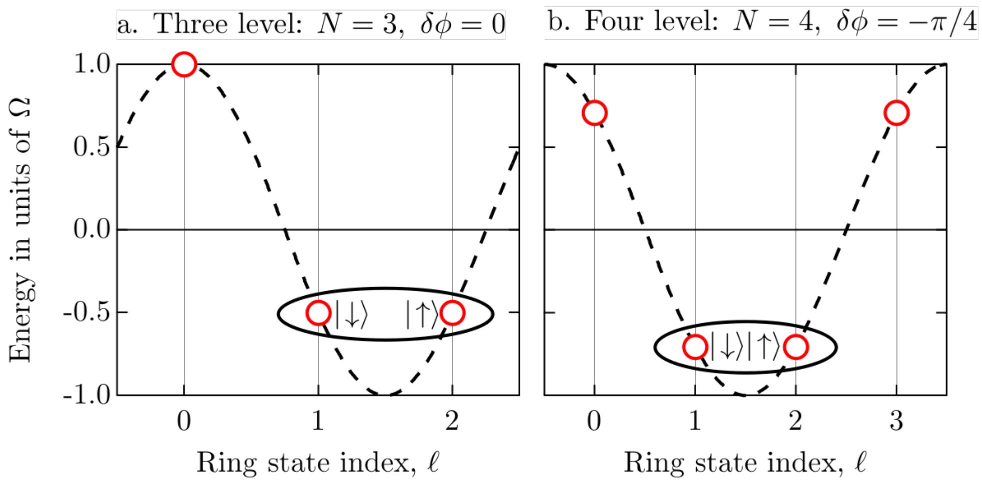




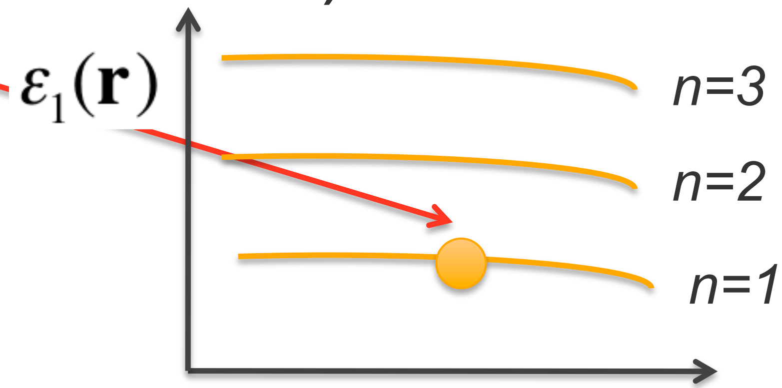
Two degenerate internal ground states

→ **non-Abelian gauge fields** for ground-state manifold

- Here – Abelian gauge potentials

(a single atomic dressed state $|\chi_1(\mathbf{r})\rangle$)

$$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$



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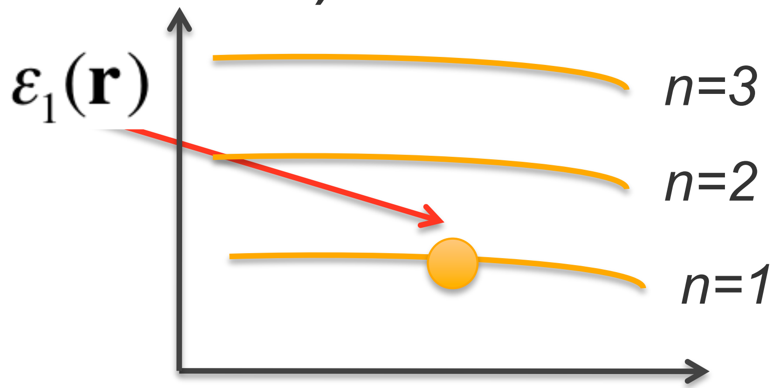

A appears due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

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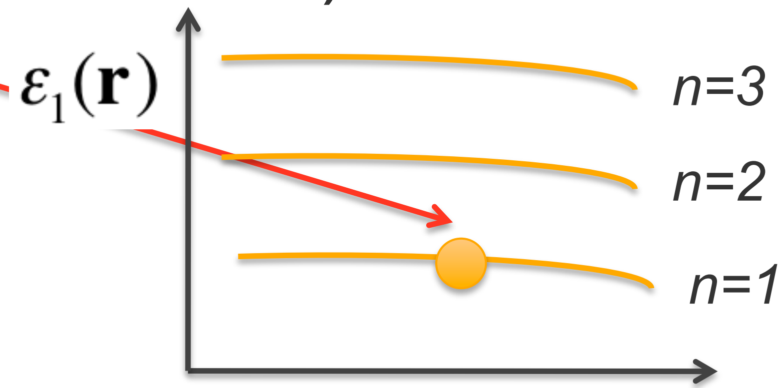


A appears due to position-dependence of $|\chi_1(\mathbf{r})\rangle$

Large possibilities to **control** and **shape** the potential **A** by **changing** the **light beams**

Light induced effective magnetic field can be due to

1. ***Spatial dependence*** of laser amplitudes
2. ***Spatial dependence*** of atom-light detuning
3. ***Spatial dependence*** of **both** the laser amplitudes and also atom-light detuning

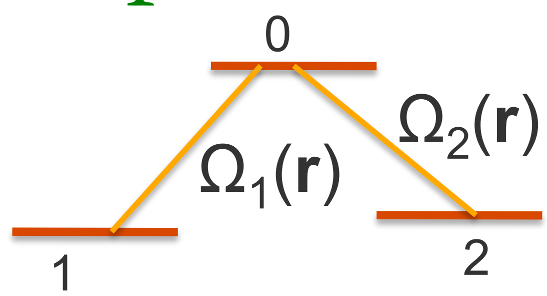
   (e.g. optical flux lattices)

Light induced effective magnetic field can be due to

1. ***Spatial dependence*** of laser amplitudes

Effective magnetic field due to spatial dependence of laser amplitudes.

Three level Λ-type atoms:



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  - *R. Dum and M. Olshanii, Phys. Rev. Lett.* ***76****, 1788 (1996).*

Effective magnetic field due to spatial dependence of laser amplitudes.

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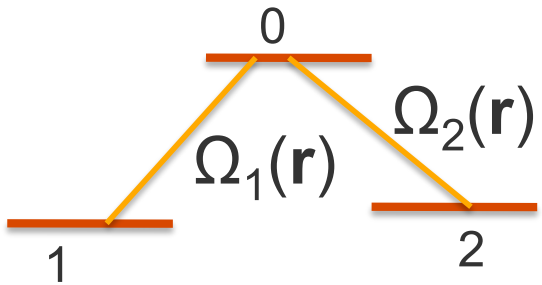

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  - G. Juzeliūnas and P. Öhberg, PRL **93**, 033602 (2004);
  - G. Juzeliūnas, J. Ruseckas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A 73, 025602 (2006).
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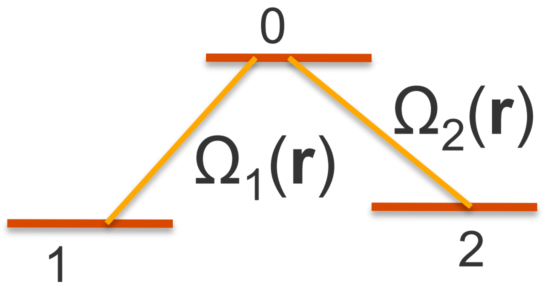

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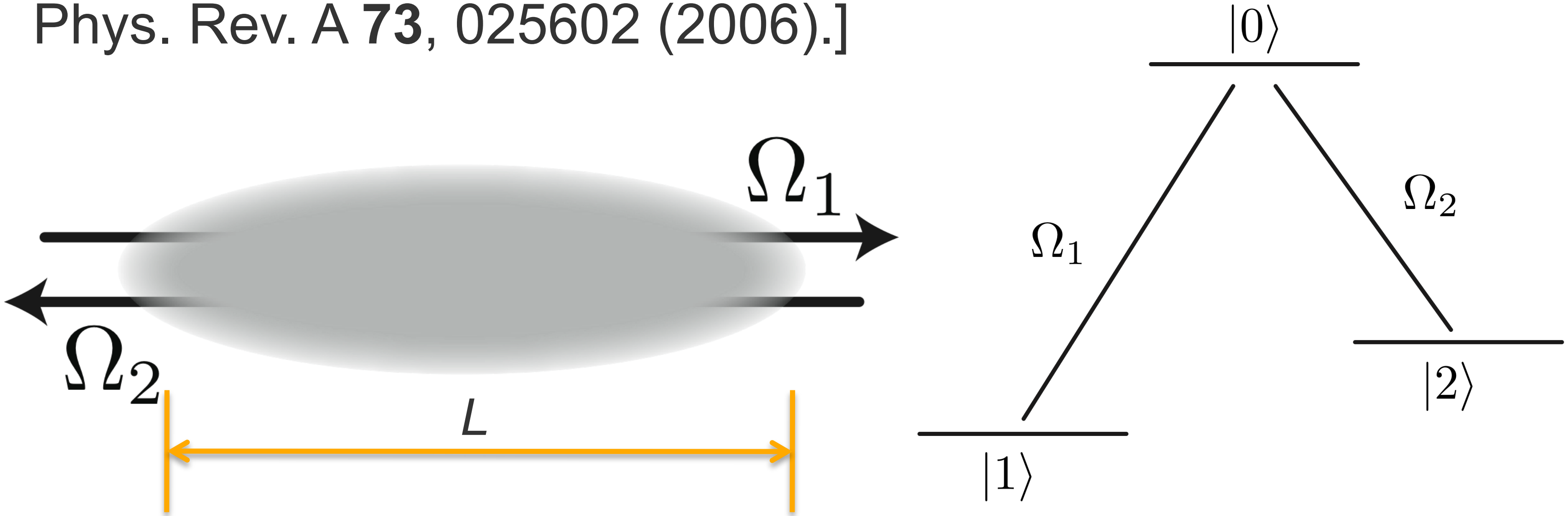
Counter-propagating beams with spatially shifted profiles

[G. Juzeliūnas, J. Ruseckas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A **73**, 025602 (2006).]



Total magnetic flux is proportional to the **sample length** *L*: $\Phi \approx kL$

(one can not increase the total flux in the transverse direction)

No translational symmetry for shifted beams (in the transverse direction):

→ No lattice

Light induced effective magnetic field due to

- ~~***Spatial dependence*** of laser amplitudes~~
- ***Spatial dependence*** of atom-light detuning

Selected for a Viewpoint in *Physics*
PRL **102,** 130401 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*
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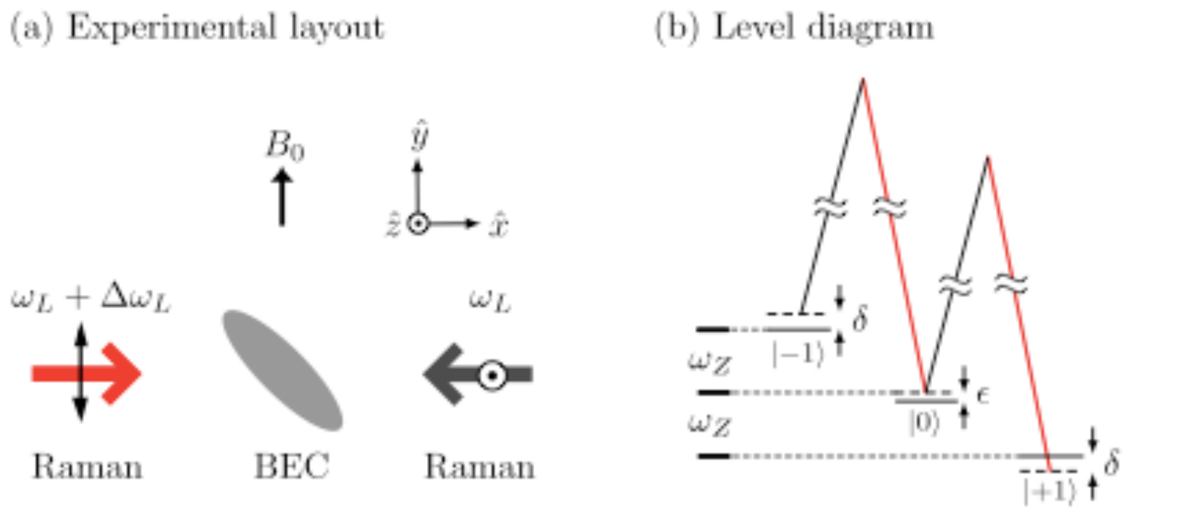

nature Vol 462 | 3 December 2009 | **doi:10.1038/nature08609**

LETTERS

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Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

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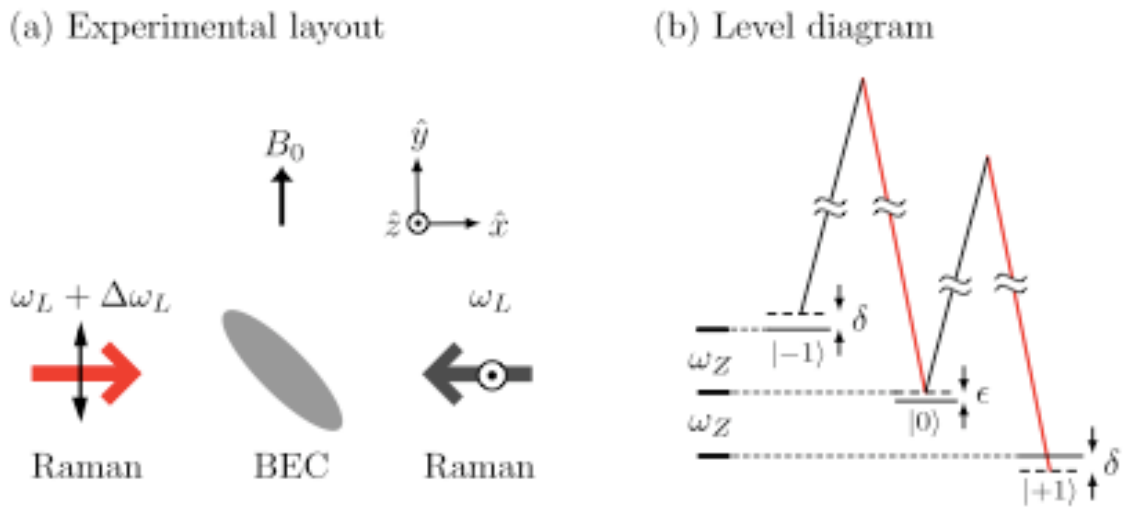

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LETTERS

Position-dependent detuning δ

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Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

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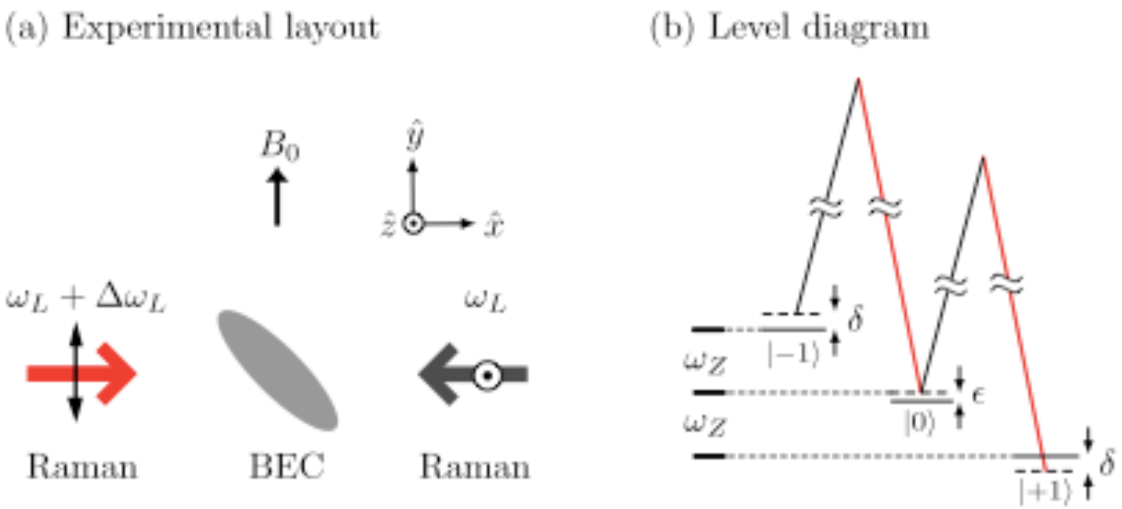

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LETTERS

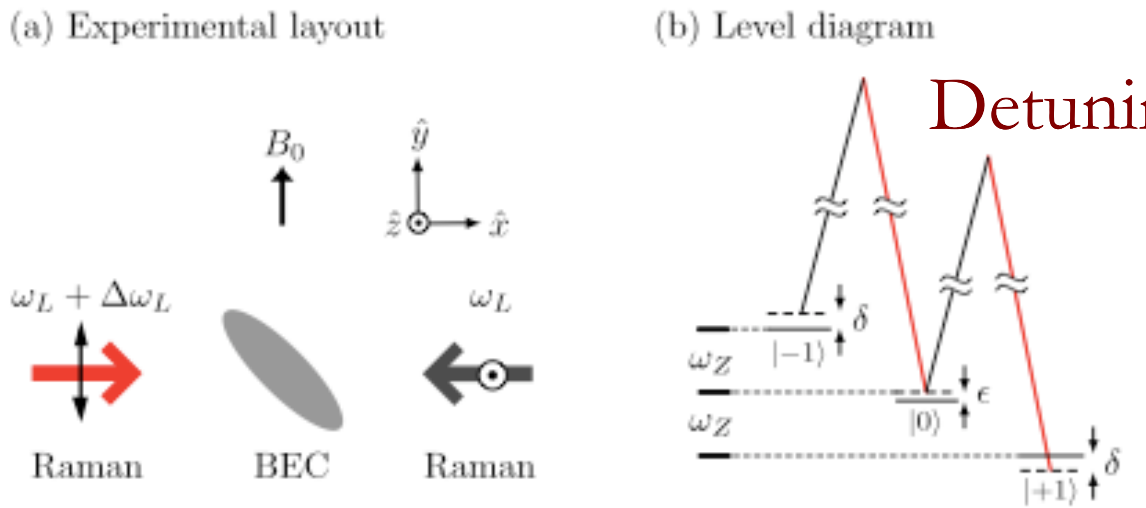
Position-dependent detuning $\delta = \delta(y)$ → $\mathbf{B} \neq 0$

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

Light induced effective magnetic field due to

- ***Spatial dependence*** of atom-light detuning



Detuning $\delta = \delta(y)$

Magnetic flux is again determined by the sample length (**rather than the area**)!

→ One can not create large magnetic flux

Effective gauge potentials – due to position-dependence of **both**

- A) Detuning *and*
  B) Laser amplitudes

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  e.g. **Optical flux lattices**

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- N. R. Cooper, Phys. Rev. Lett. **106**, 175301 (2011)

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Magnetic flux is determined by the <u>**area (!!!)**</u> of atomic cloud

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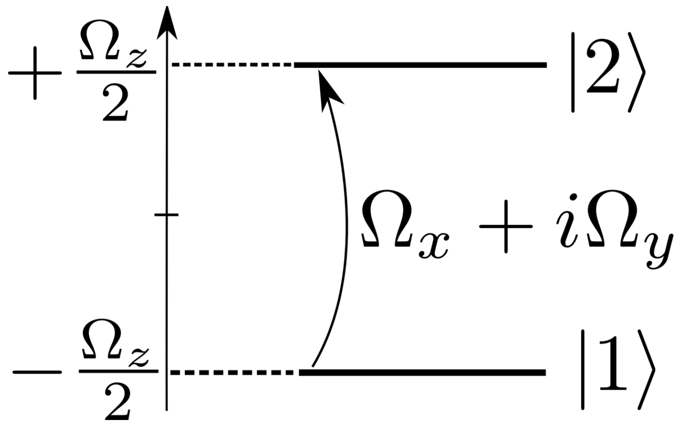
Magnetic flux is determined by the **area (!!!)** of atomic cloud

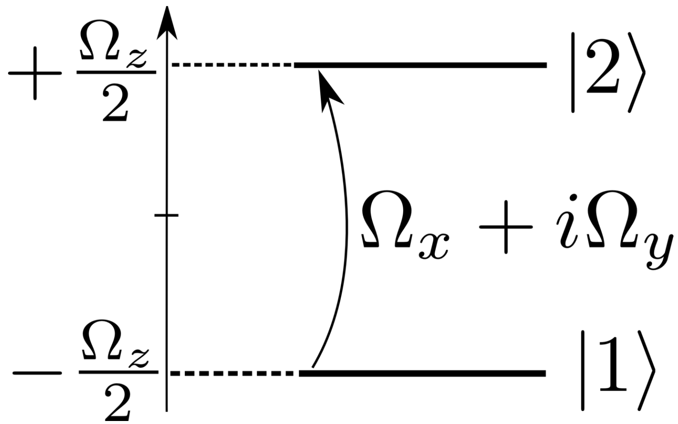
- Related earlier work:
- A. M. Dudarev, R. B. Diener, I. Carusotto, and Q. Niu, Phys. Rev. Lett. 92, 153005 (2004).

Two atomic internal states



- Position-dependent detuning $\Delta(\mathbf{r}) \equiv \Omega_z$
- Position-dependence of the Rabi frequencies of atom-light coupling $\Omega_\pm(\mathbf{r}) \equiv \Omega_x \pm i\Omega_y$

Two atomic internal states



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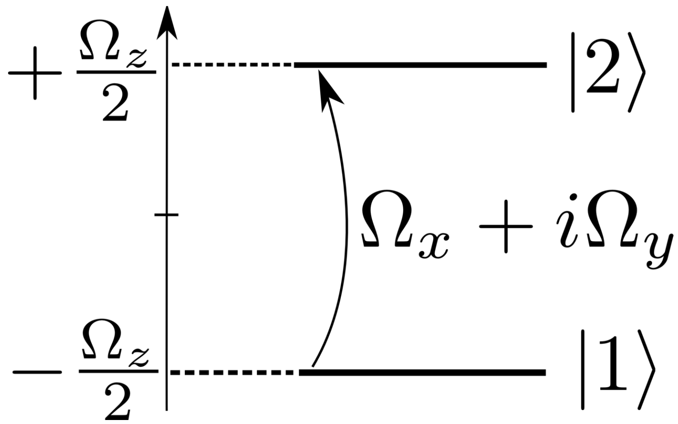
- **Atom-light Hamiltonian:**

$$\hat{H}_0(\mathbf{r}) = -\hbar \begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

(2×2 matrix)

Two atomic internal states



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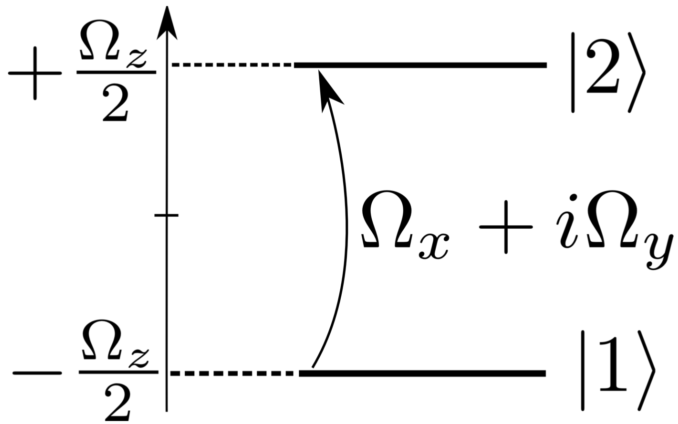
$\Omega_x=\Omega_y=0$, → **No coupling** between the atomic states

→ **Ordinary** atomic **trap** or **lattice**

Two atomic internal states



- Position-dependent detuning $\Delta(\mathbf{r}) \equiv \Omega_z$
- Position-dependence of the Rabi frequencies of atom-light coupling $\Omega_{\pm}(\mathbf{r}) \equiv \Omega_x \pm i\Omega_y$
- **Atom-light Hamiltonian:**

$$\hat{H}_0(\mathbf{r}) = -\hbar \begin{pmatrix} \Omega_z/2 & 0 \\ 0 & -\Omega_z/2 \end{pmatrix}$$

$\Omega_x = \Omega_y = 0$, → **No coupling** between the atomic states

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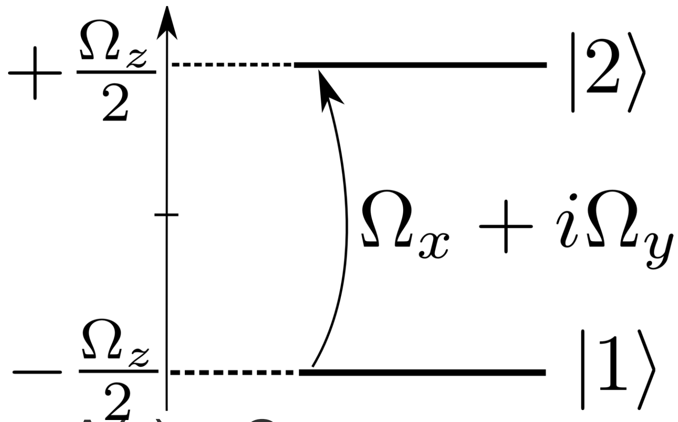
Two atomic internal states



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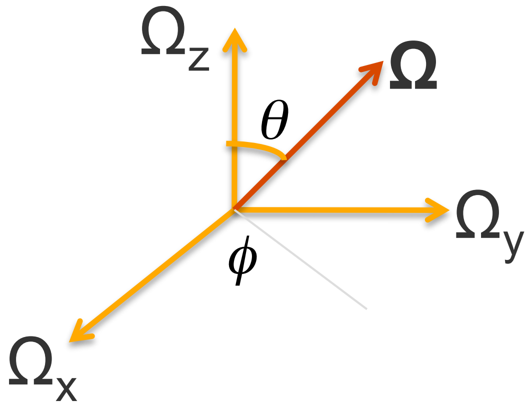
$\Omega_x \neq 0$, $\Omega_y \neq 0$, → **Coupling** between the atomic states →

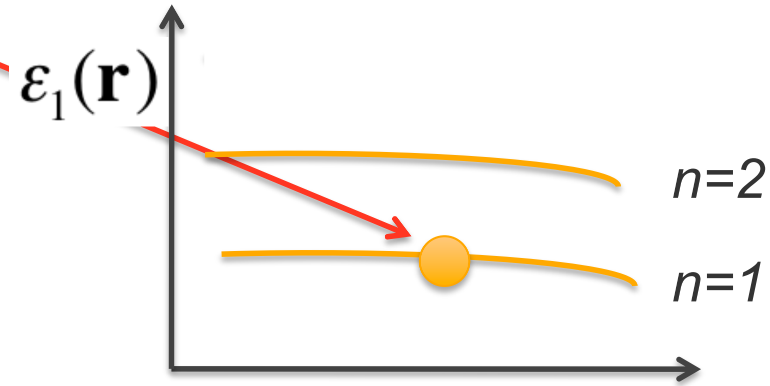
$\hat{H}_0(\mathbf{r})$ has position-dependent eigenstates $|\chi_j(\mathbf{r})\rangle$, j=1,2

$$\hat{H}_0(\mathbf{r})|\chi_j(\mathbf{r})\rangle = \varepsilon_j(\mathbf{r})|\chi_j(\mathbf{r})\rangle \qquad (j=1,2), \qquad \hat{H}_0(\mathbf{r}) = -\hbar\begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

- Effective vector potential for atomic motion in the lower dressed state $|\chi_1(\mathbf{r})\rangle$:

$$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$



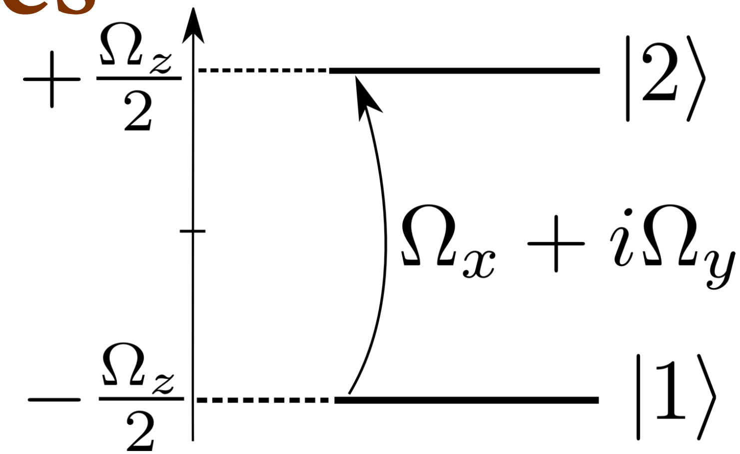


$$\mathbf{A}(\mathbf{r}) = \frac{\hbar}{2}(\cos\theta - 1)\nabla\phi$$

See, e.g.: J.Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg. To appear in Rev. Mod. Phys. (ArXiv1008.5378).

→ Optical flux lattices

Two-level system:



$$\hat{H}_0(\mathbf{r}) = -\hbar \begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x - i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

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[N. R. Cooper, PRL **106**, 175301 (2011)]

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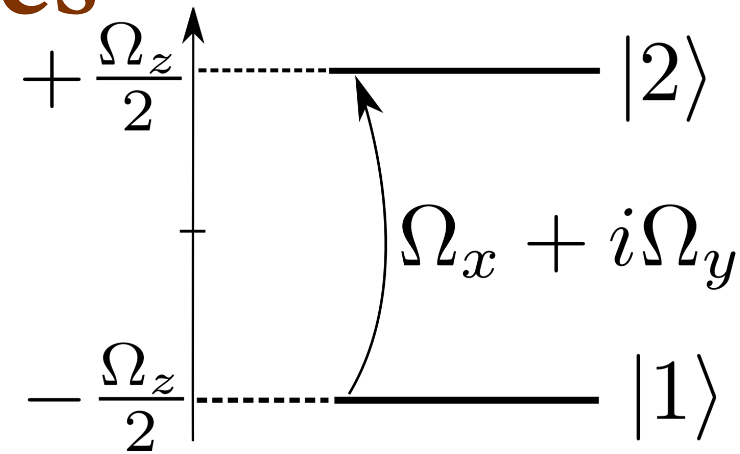

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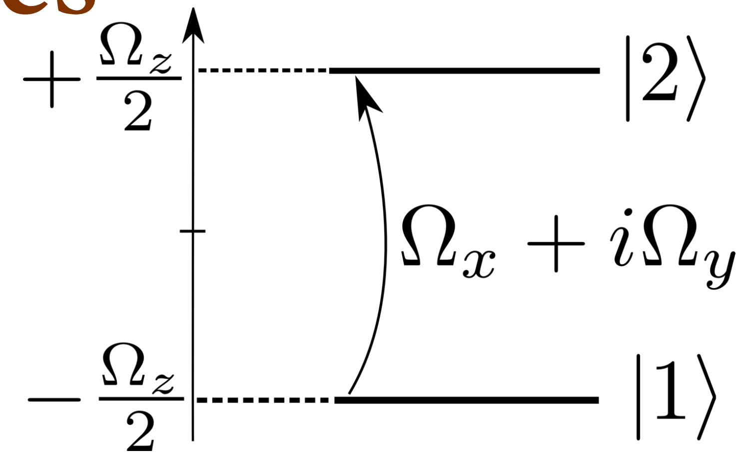

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Periodic coupling operator $\hat{H}_0(\mathbf{r})$ with a periodicity $2a$
(*Unconventional optical lattice*)

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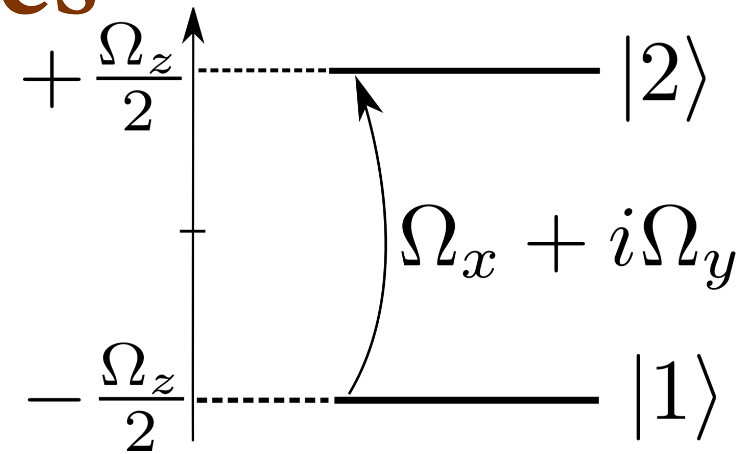

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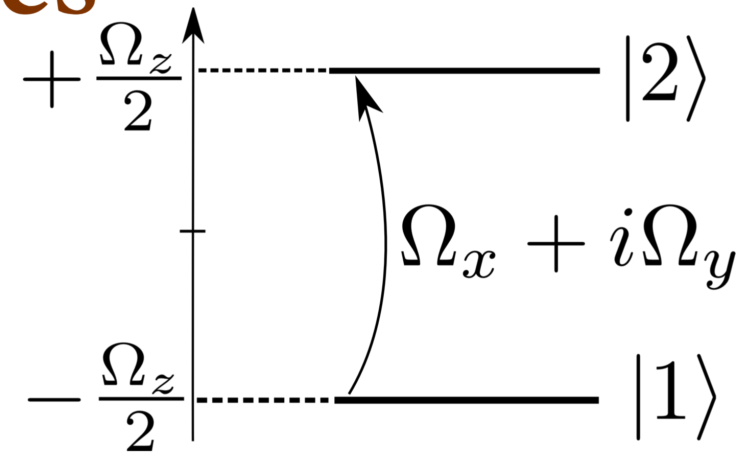

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Zero magnetic flux over the elementary cell:

$\Phi = \oint \mathbf{A}\cdot d\mathbf{l} = 0$ (for adiabatic motion of atoms)

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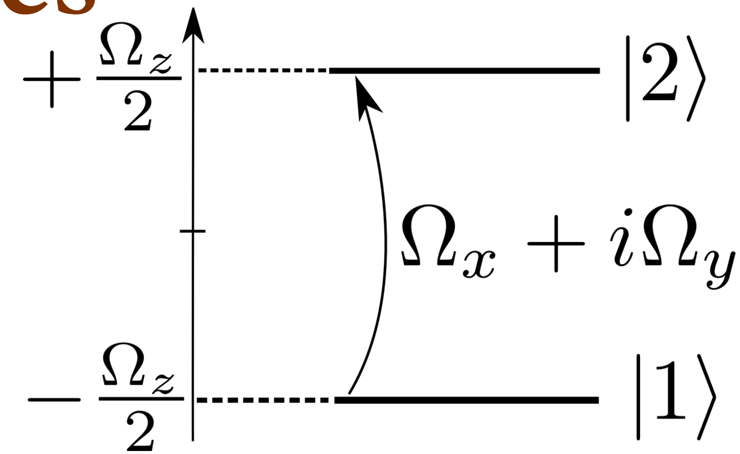

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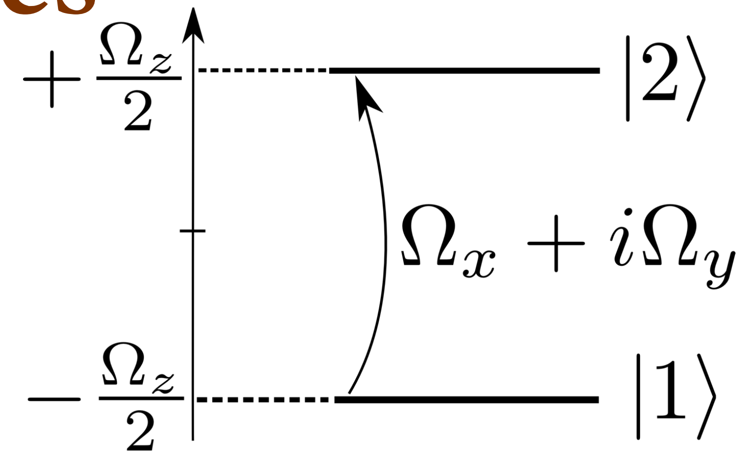

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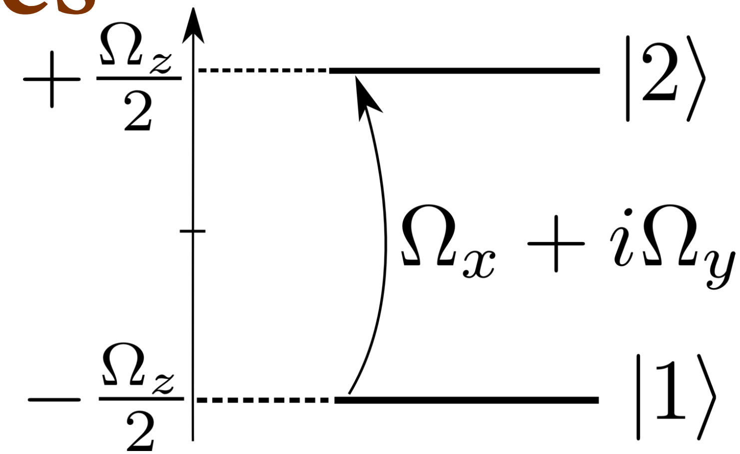

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AB = Aharonov - Bohm

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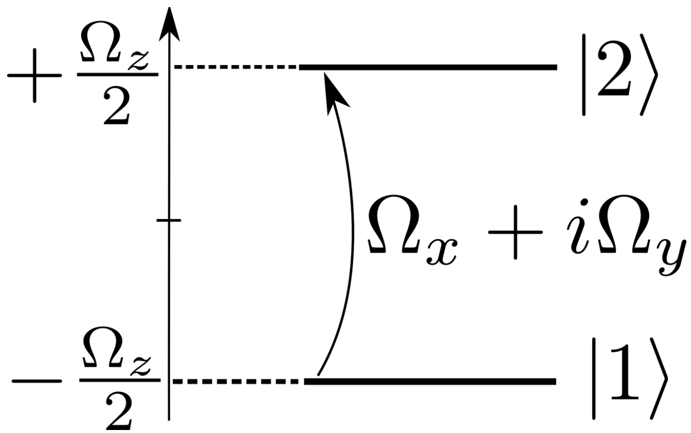
Optical flux lattices



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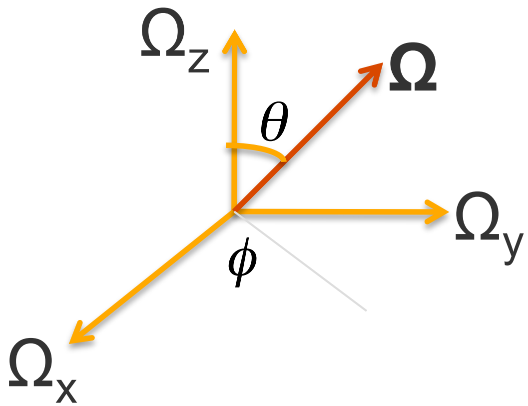


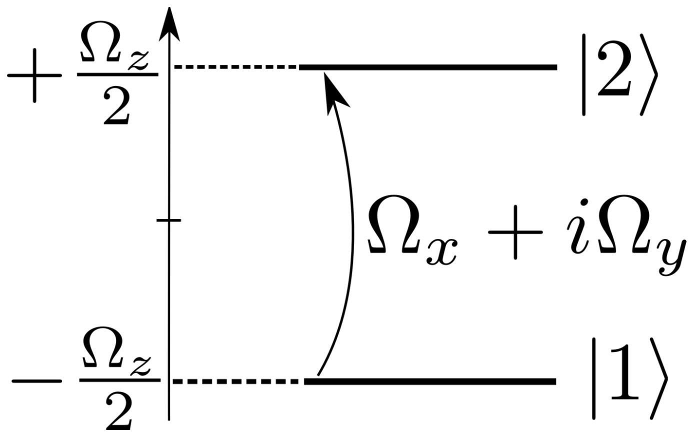
$$\mathbf{A}(\mathbf{r}) = \frac{\hbar}{2}(\cos\theta - 1)\nabla\phi$$

$\mathbf{A}(\mathbf{r})$ contains **an array of AB tubes** for $\Omega_x, \Omega_y \to 0$ **&** $\Omega_z \to -\Omega$

where $\mathbf{A}(\mathbf{r}) \approx -\hbar\nabla\phi$

AB vector potential

Optical flux lattices



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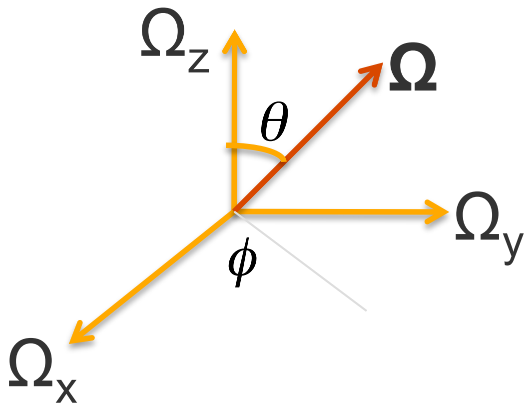

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Each **AB tube** - **a single Dirac flux** quantum **(non-mesurable)**

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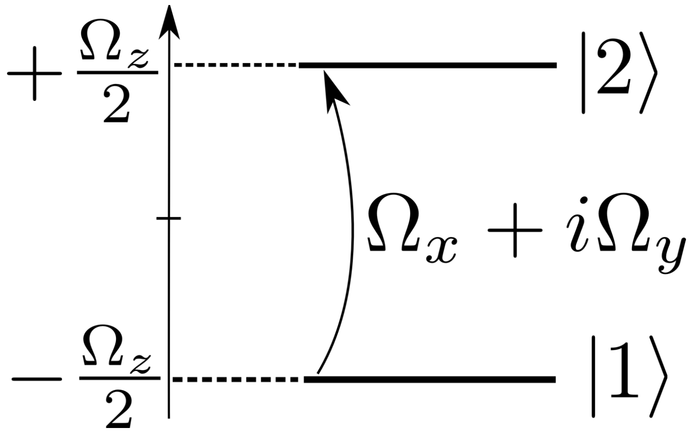
Optical flux lattices



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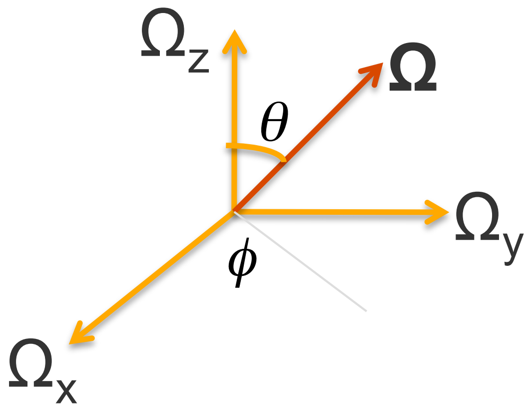

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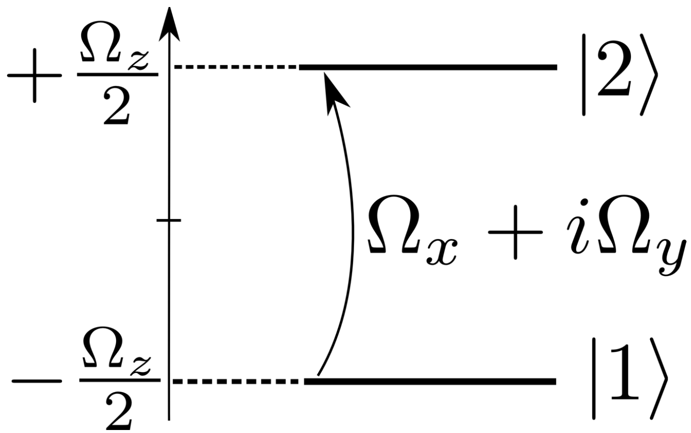
Two AB flux tubes of the **<u>same sign</u>** per elementary cell

Optical flux lattices



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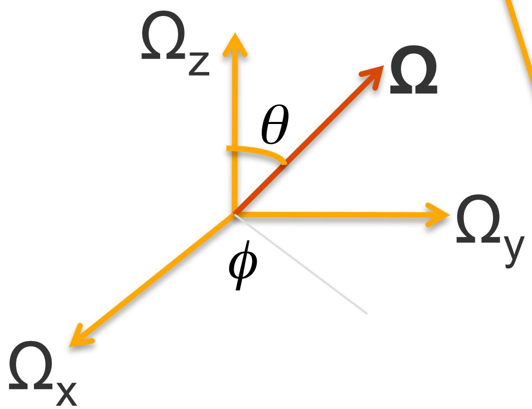

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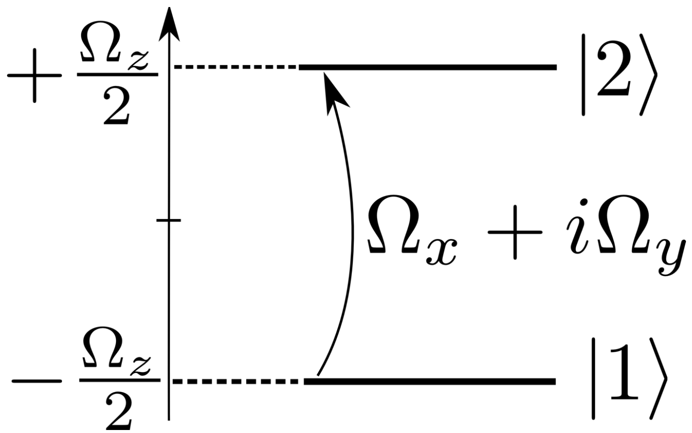

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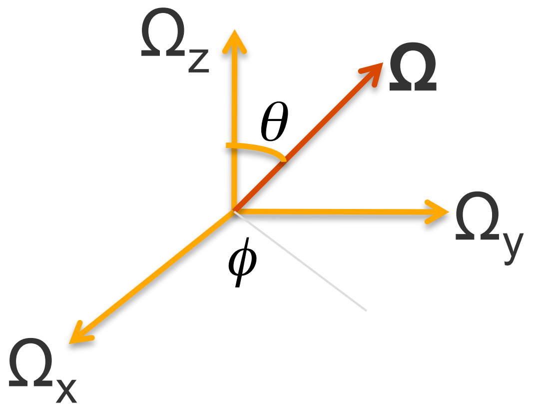

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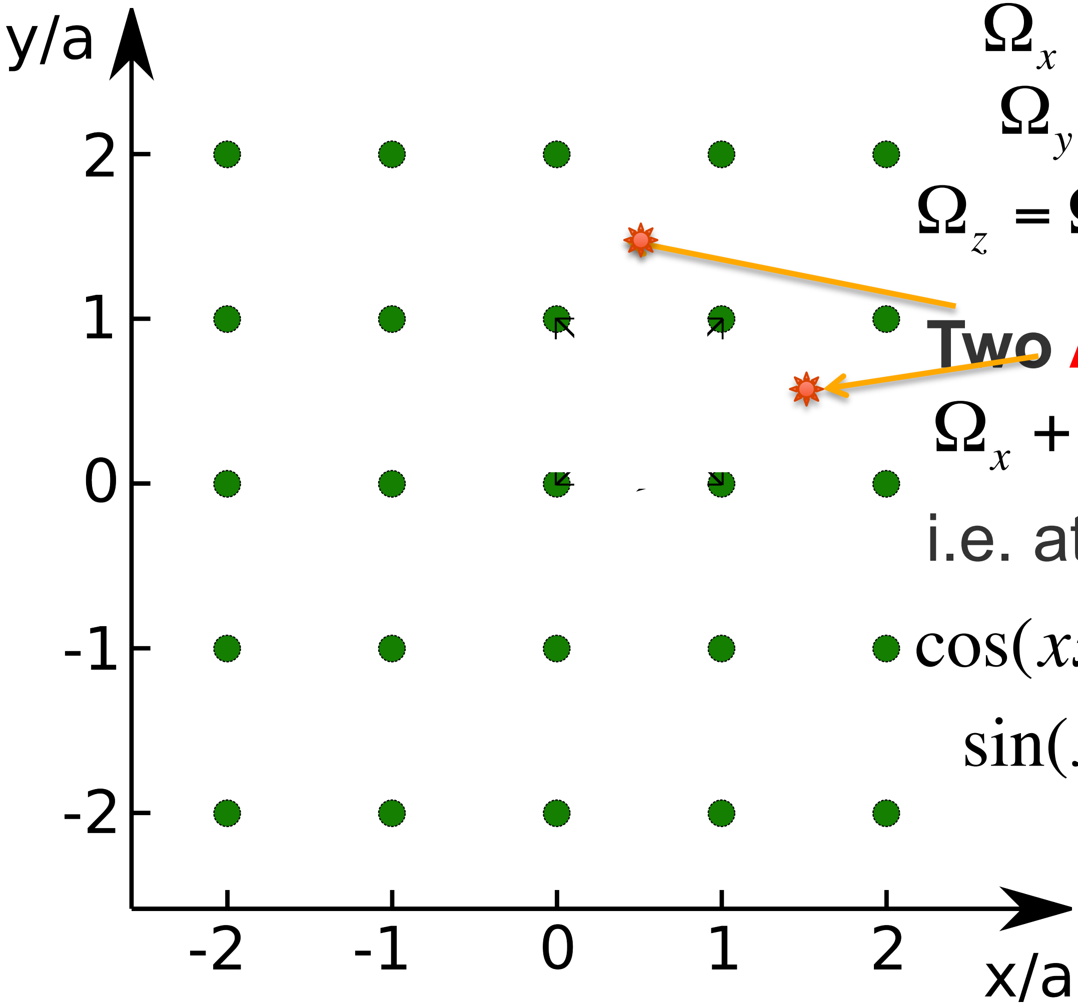
Background flux to compensate the AB tubes $\left(\Phi = \oint \mathbf{A}\cdot d\mathbf{l} = 0\right)$

→ Optical flux lattices (square)



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Two **AB tubes** at

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i.e. at

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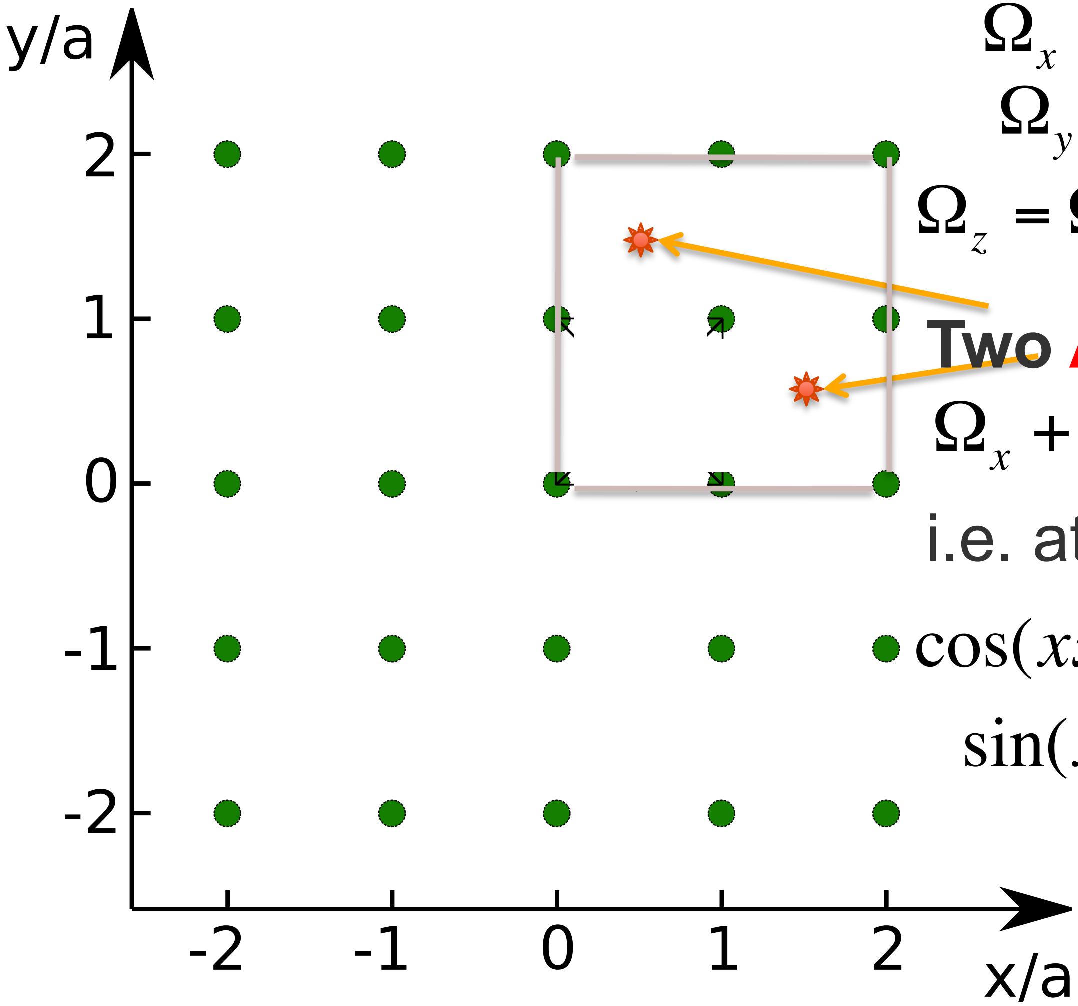

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Non-zero background magnetic flux over an elementary cell

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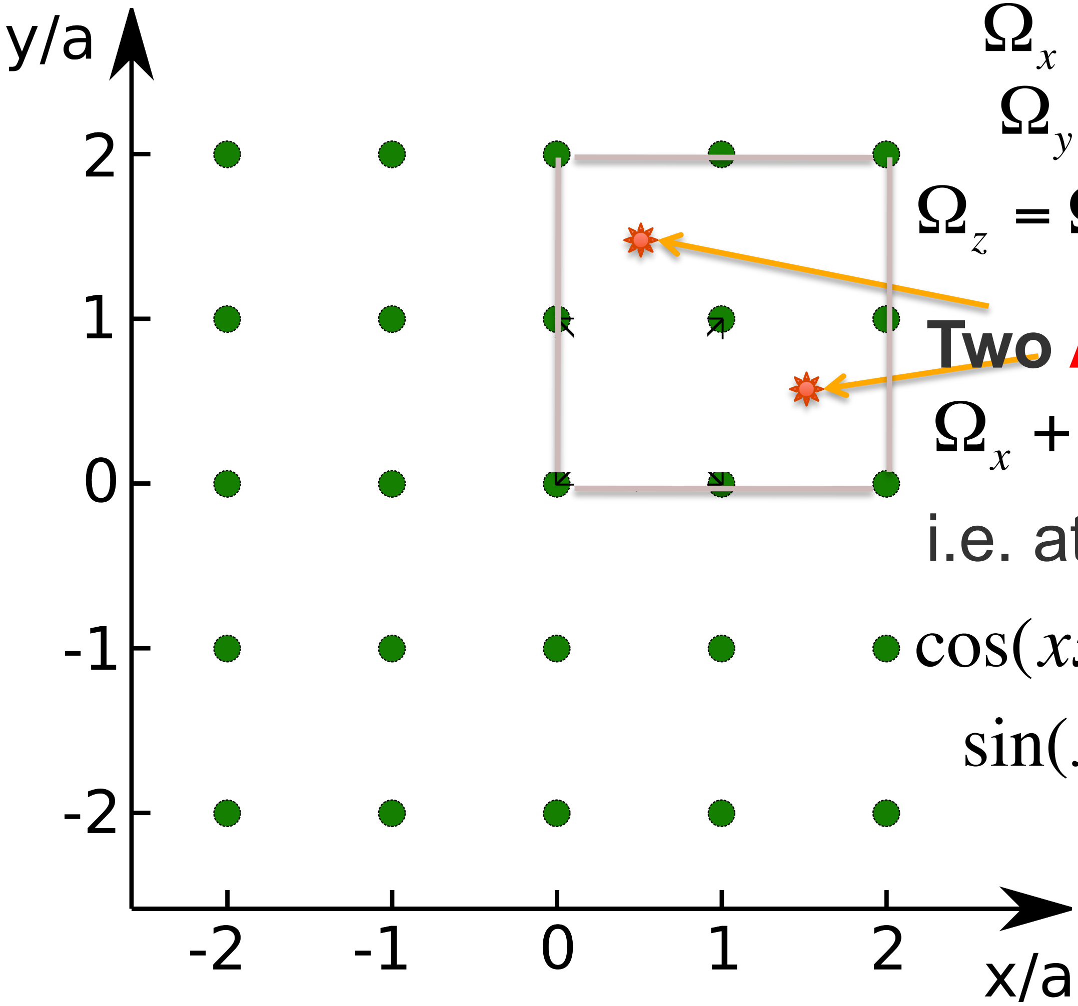

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Magnetic flux is determined by the **area (!!!)** of atomic cloud

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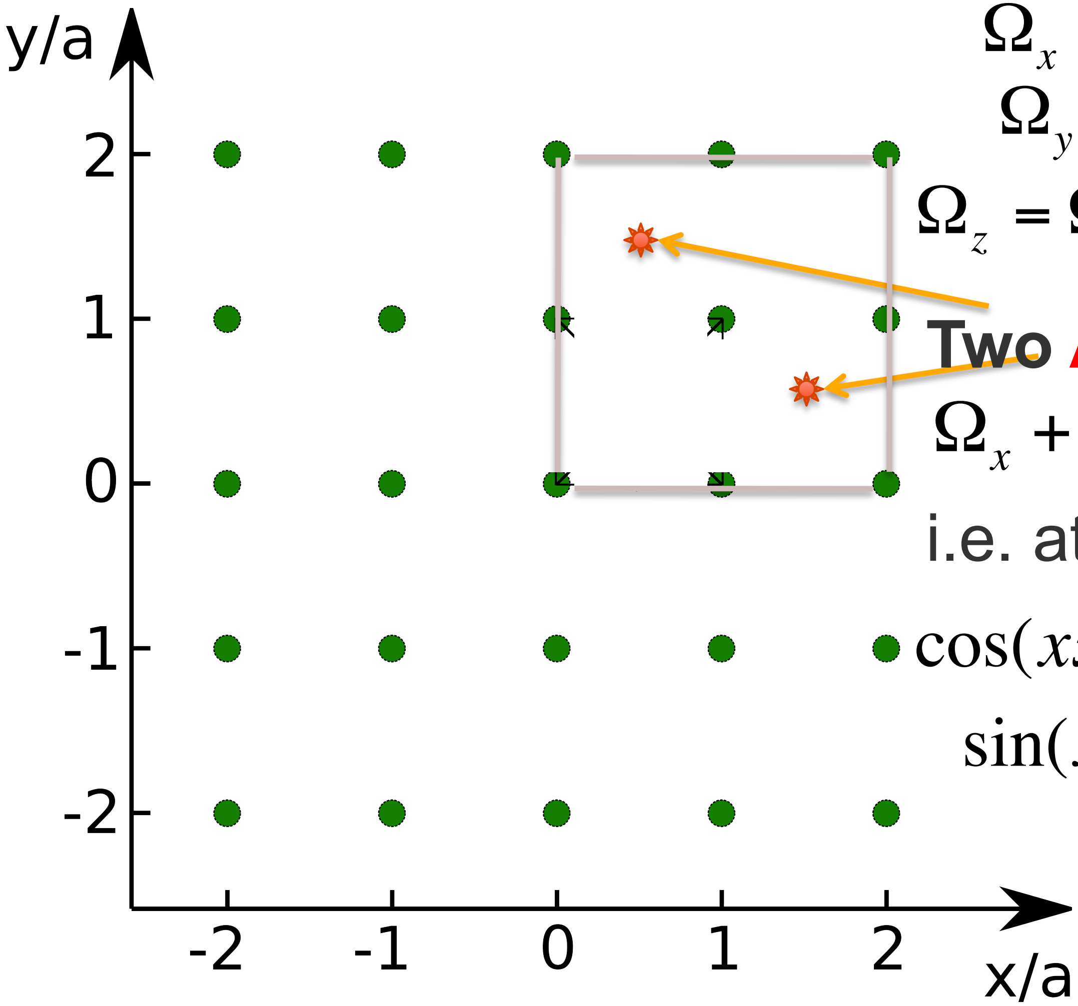

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Magnetic flux is determined by the **area (!!!)** of atomic cloud

Additionally periodic potential (minima shown in green).

OFL can be produced

- Using optical transitions between two atomic long-lived internal state with opposite polarisability (anti-magic wave-length)

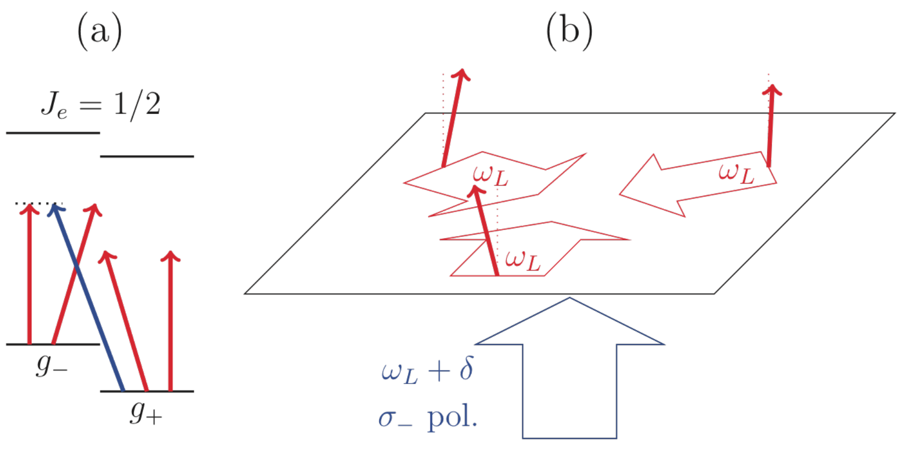
N. R. Cooper, PRL **106**, 175301 (2011)

$$\hat{H}_0(\mathbf{r}) = -\hbar \begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x - i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

OFL can be produced

- Using Raman transitions between the hyperfine states of alkali atoms (and specially shaped laser fields)

<u>Triangular</u> optical flux lattice
N. R. Cooper and J. Dalibard, arXiv:1106.0820.



<u>Square</u> optical flux lattice:
G. Juzeliunas and I.B. Spielman, in preparation

Characteristic features of light-induced gauge potentials

- No rotation of atomic gas
- No lattice is necessary
- Effective magnetic field can be shaped by choosing proper laser beams
- The magnetic flux can be made proportional to the area using the optical flux lattices
- Extension to the non-Abelian case

Thank you!