

Novel quantum physics with ultracold fermions

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Outline

- Atomic fermions. Molecular BEC regime
- p -wave resonance for atomic fermions
- Fermionic polar molecules
- Topological $p_x + ip_y$ phase
- Bilayered and multilayered dipolar systems
- Conclusions and outlook

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Experiments

^{40}K ^6Li

Dilute limit $nR_e^3 \ll 1$

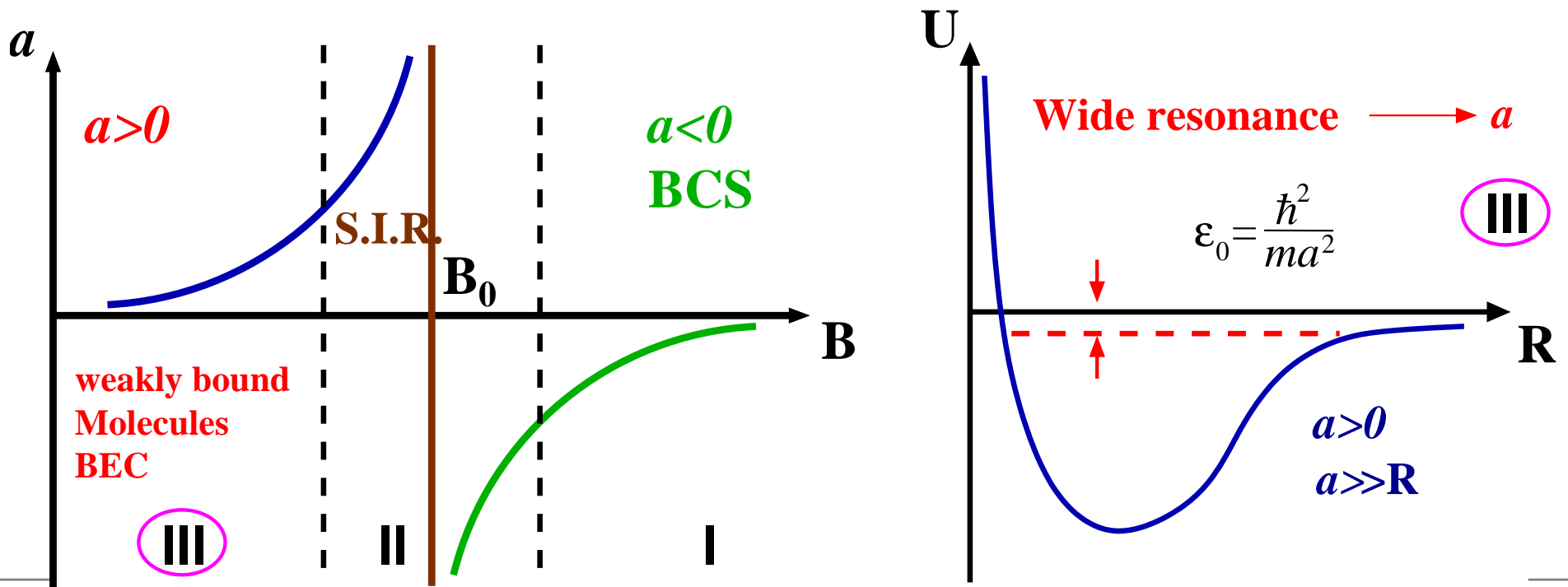
Ultracold limit $\Lambda_T \gg R_e$

Quantum degeneracy \rightarrow JILA 1998 ^{40}K

At present $n \sim 10^{13} - 10^{14} \text{cm}^{-3}$; $T \sim 1\mu\text{K} - 100\text{nK}$

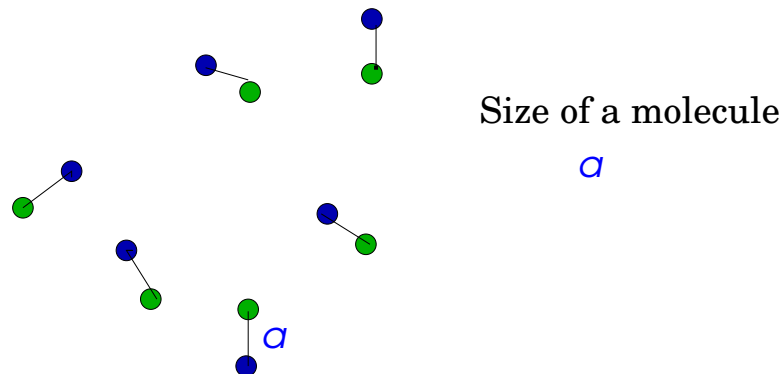
Superfluid BCS transition $\Rightarrow T_C \sim E_F \exp(-\pi/2k_F|a|)$ extremely low for ordinary attractive interaction ($a < 0$; $|a| \lesssim 10\text{nm}$ and $k_F|a| \ll 1$)

FFR resonances JILA, LENS, Innsbruck, MIT, ENS, Rice, Duke, Melbourne, Tokyo, elsewhere



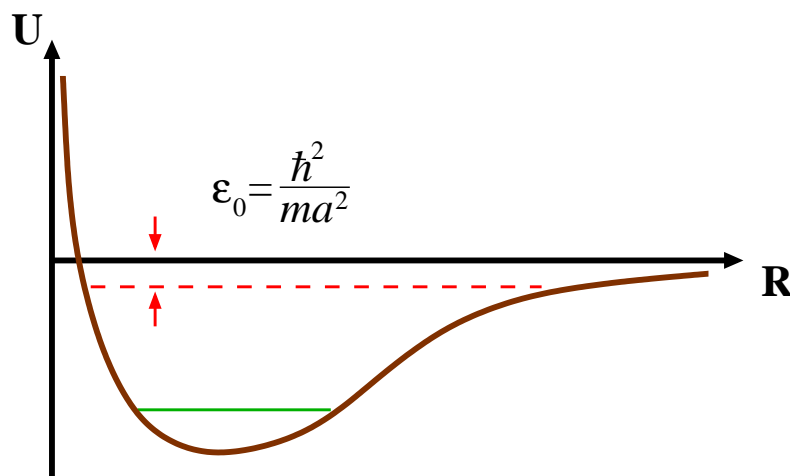
Gas of bosonic molecules (dimers)

Region III ($a > 0$) \Rightarrow gas of weakly bound bosonic molecules



$na^3 \ll 1 \Rightarrow$ weakly interacting Bose gas

Weakly bound dimers \rightarrow Highest rovibrational state \rightarrow Collisional relaxation



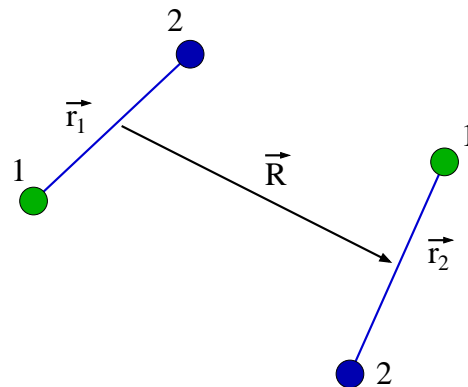
($\tau \sim 1\text{ms}$ for Rb_2 at $n \sim 10^{13}\text{cm}^{-3}$)

Molecule-molecule interaction

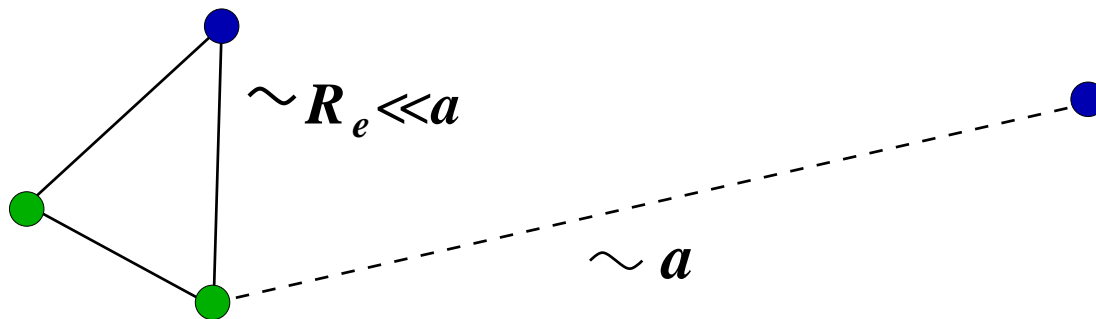
Elastic interaction BEC stability

4-body problem Exact solution for $a \gg R_e$ (Petrov et al 2003)

$$a_{dd} = 0.6a$$



Remarkable collisional stability



$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?} \Rightarrow C(\hbar R_e/m)(R_e/a)^s; \quad s = 2.55$$

$$\tau \sim (\alpha_{rel} n)^{-1} \sim \text{seconds} \quad (\text{Petrov et al 2003})$$

Bose-Einstein condensates of molecules

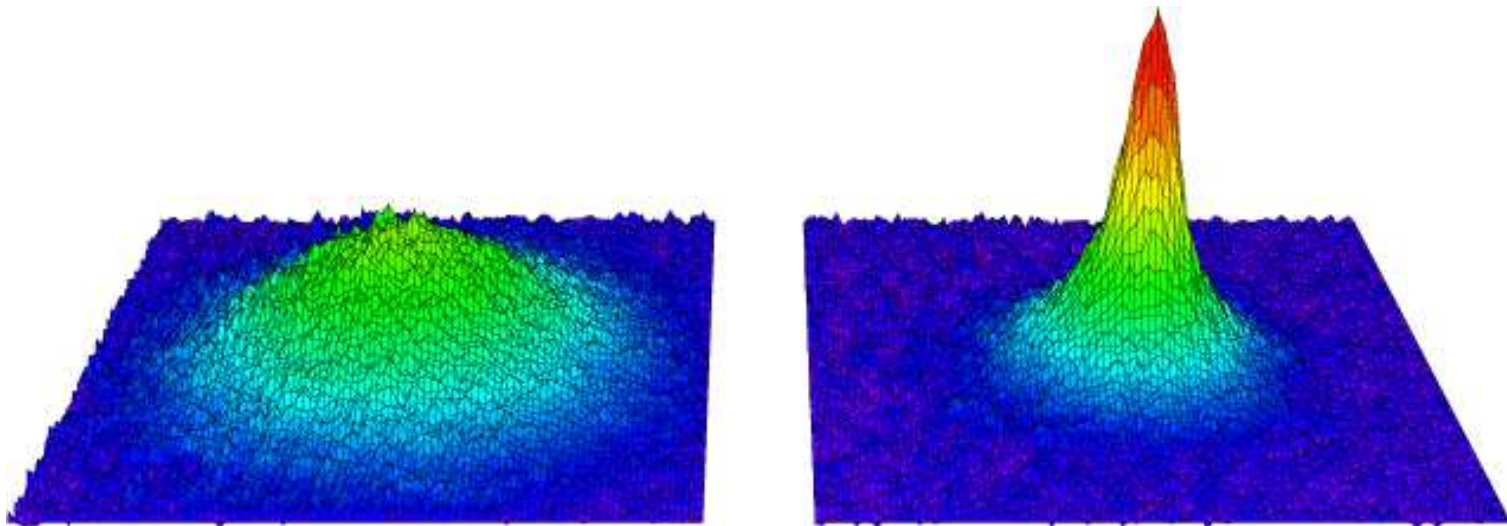
Suppressed relaxation Fast elastic collisions $a_{dd} = 0.6a$

$${}^6\text{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling \rightarrow BEC

JILA, Innsbruck, MIT, ENS, Rice, Duke

Largest diatomic molecules in the world $\sim 3000 \text{ \AA}$



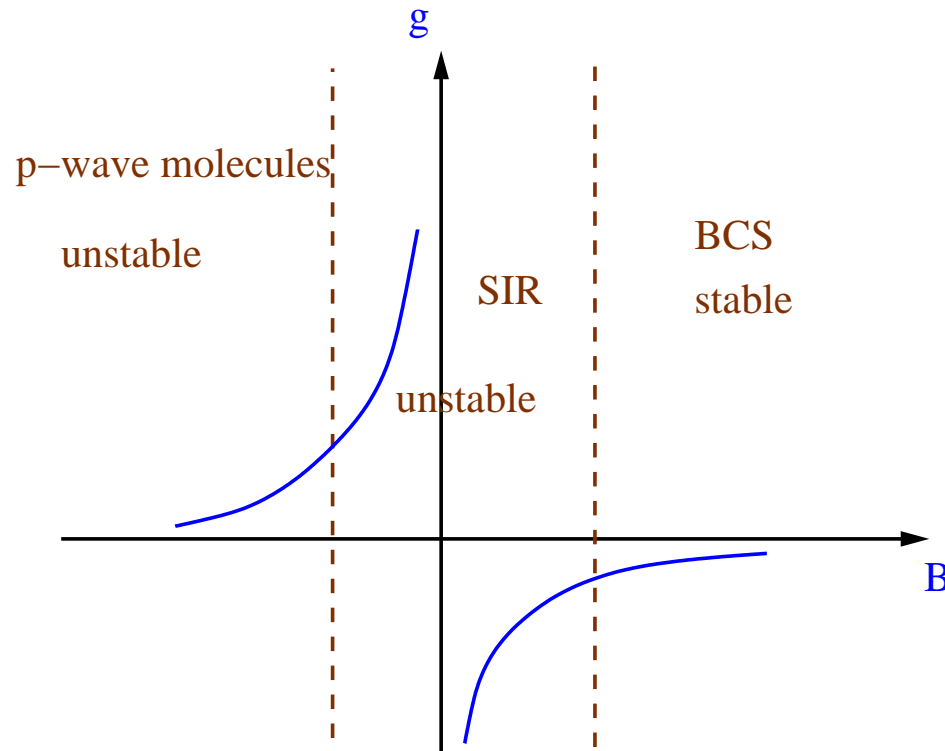
p-wave resonance for fermionic atoms

p-wave resonance Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

$$\text{BCS} \Rightarrow T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right) \text{ practically zero}$$

Molecular and strongly interacting regimes \Rightarrow rather high T_c , but collisional instability

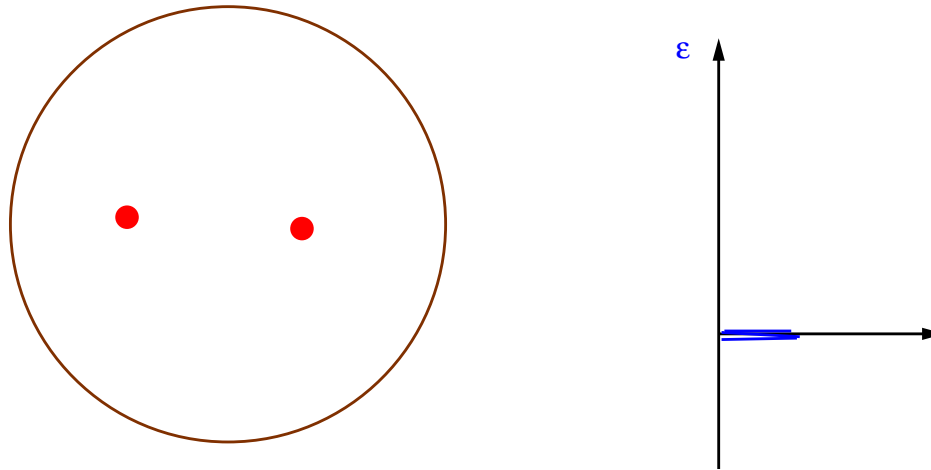
Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio



Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)



The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics \Rightarrow Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing

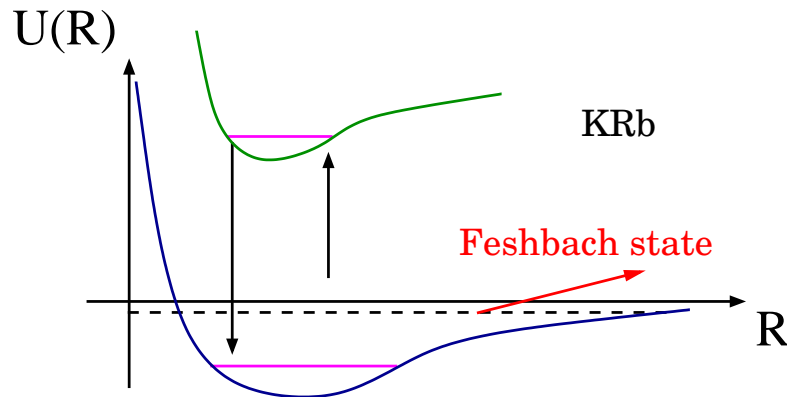
Polar molecules. Creation of ultracold clouds

- Buffer gas cooling (Harvard, J. Doyle)
- Stark deceleration (Berlin, JILA)
- Optical collisions (photoassociation, D. DMille group, JILA, elsewhere)

Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb molecules to the ground rovibrational state
JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13} \text{ cm}^{-3}$$
$$T \approx 200 \text{ nK} \sim E_F$$

Ground-state LiCs molecules at Heidelberg at low density

Ultracold chemical reactions $\text{KRb} + \text{KRb} \Rightarrow \text{K}_2 + \text{Rb}_2$

New trends in ultracold chemistry

Suppress instability \rightarrow induce intermolecular repulsion

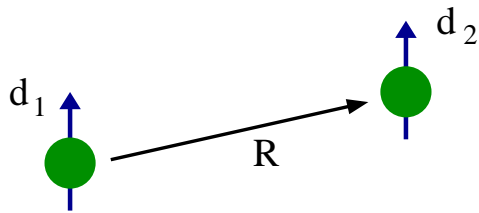
For example, 2D geometry with dipoles perpendicular to the plane

Select non-reactive molecules, like NaK

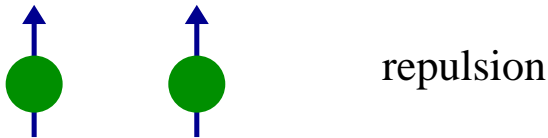
What are prospects for novel physics ?

Polar molecules. New object - Dipolar gas

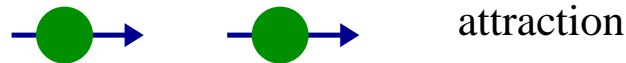
Dipole-dipole interaction $V_d = \frac{\vec{d}_1 \vec{d}_2 R^2 - 3(\vec{d}_1 \vec{R})(\vec{d}_2 \vec{R})}{R^5} \sim \frac{1}{R^3}$



long-range, anisotropic



repulsion

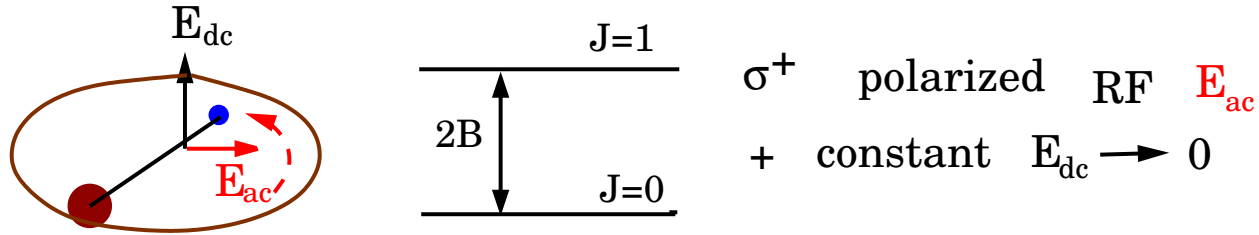


attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6 D$ for KRb to $5.5 D$ for LiCs

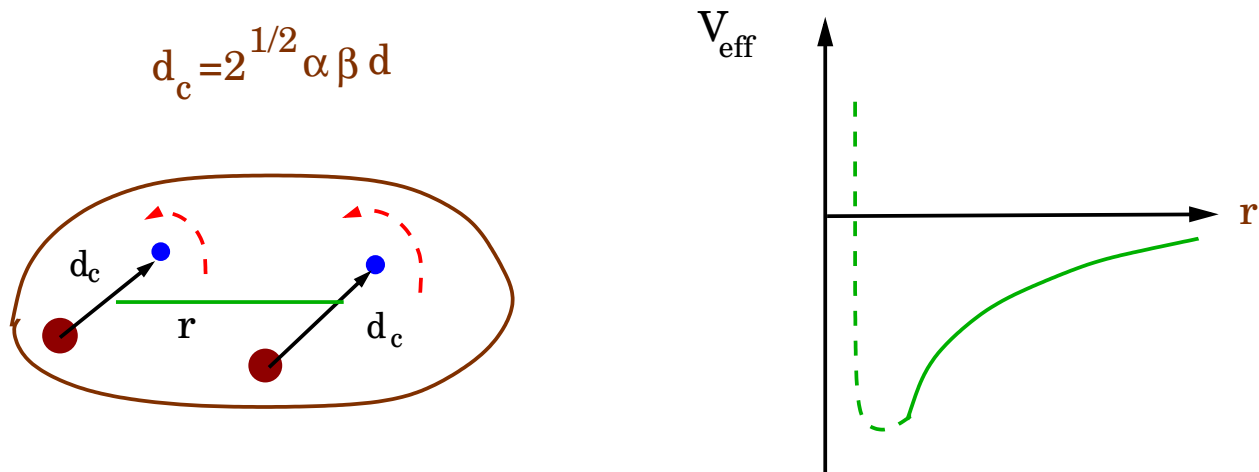
RF-dressed polar molecules in 2D; Gorshkov et al (2008)



Dressed states $|+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle$; $|-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2 + \Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2 + \Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency



Large $r \rightarrow V_{eff} = \langle (1 - 3 \cos^2 \phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = md_c^2/2\hbar^2$

Fermionic RFD molecules. Superfluid transition; Cooper/G.S. (2009)

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p -wave scattering ($l = \pm 1$)

$$\hat{H} = \int d^2r \hat{\Psi}^\dagger(\mathbf{r}) \left\{ -(\hbar^2/2m)\Delta + \int d^2r' \hat{\Psi}^\dagger(\mathbf{r}') V_{eff}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') - \mu \right\} \hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r} - \mathbf{r}') = \langle V_{eff}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}) \hat{\Psi}(\mathbf{r}') \rangle$$

Gap equation
$$\Delta(\mathbf{k}) = - \int \frac{d^2k'}{(2\pi)^2} V_{eff}(\mathbf{k} - \mathbf{k}') \Delta(\mathbf{k}') \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$$

$$\epsilon(k) = \sqrt{(\hbar^2 k^2 / 2m - \mu)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F$$

$$T_c \approx E_F \exp(-3\pi/4k_F r_*)$$

$$\Delta(\mathbf{k}) = \Delta \exp(i\phi_k) \quad p_x + ip_y \text{ state } (l = \pm 1)$$

Superfluid transition. Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$

This is the case for the atoms

Anomalous scattering in $1/r^3$ potential \rightarrow Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \quad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \quad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

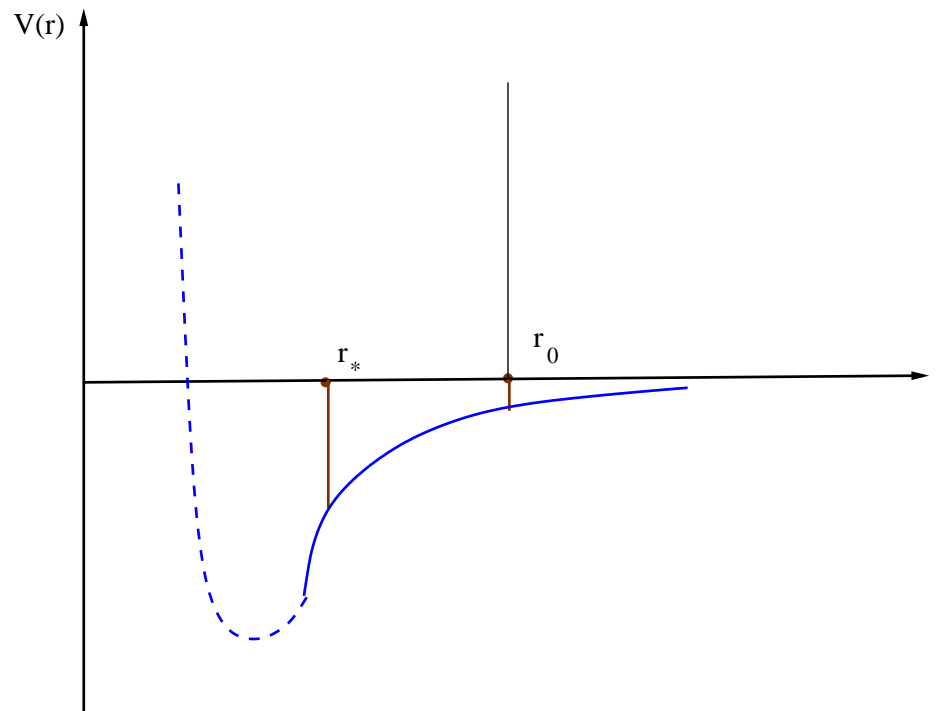
Transition temperature

Do better than simple BCS. Reveal the role of short-range physics

Scattering amplitude. Asymptotic method for slow scattering ($kr_* \ll 1$)

Divide the range of distances into two parts, $r < r_0$ and $r > r_0$

The distance r_0 is such that $r_0 \gg r_*$, but $kr_0 \ll 1$



$r < r_0$ Match exact zero-energy with free finite- k solution at $r = r_0$: $f \Rightarrow (\pi/2)d^2 r_* k^2 \ln k$

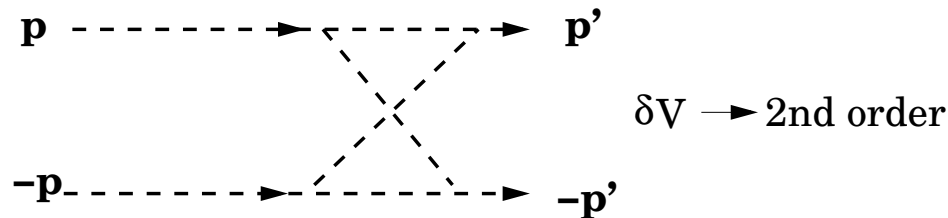
$r > r_0$ interaction as perturbation: $f = -(8\pi/3)d^2 k + (\pi/2)d^2 r_* k^2 \ln k$

Related results for the off-shell scattering amplitude

Manipulate T_c ?

$$T_c = \frac{2e^C}{\pi} E_F \exp \left\{ -\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64} \ln[k_F r_* u] \right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



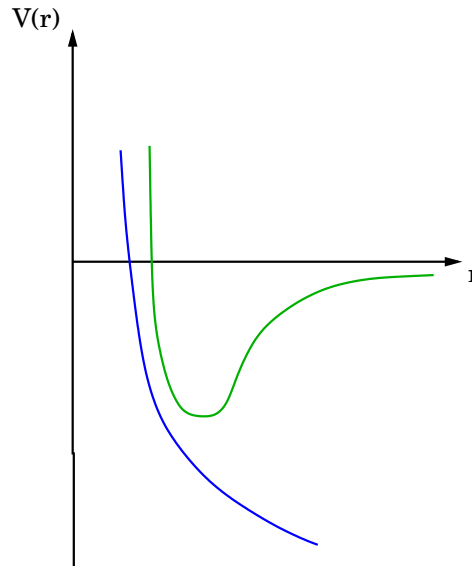
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp \left\{ -\frac{3\pi}{4k_F r_*} \right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p -wave atomic superfluids: BCS $\Rightarrow T_c \rightarrow 0$ Resonance \Rightarrow collisional instability

Polar molecules \Rightarrow sufficiently large T_c and collisional stability

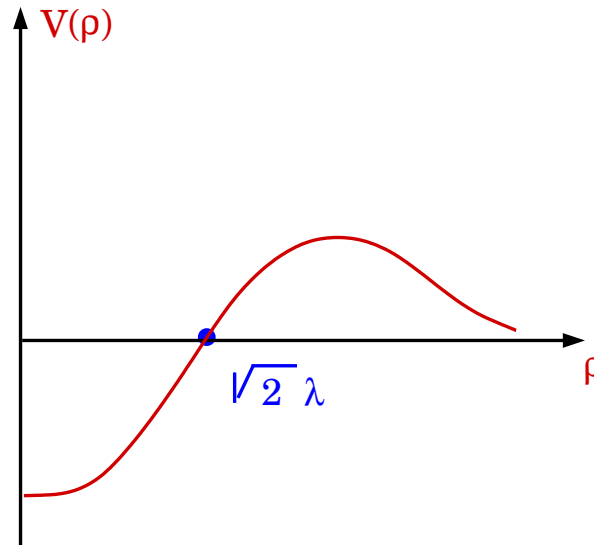
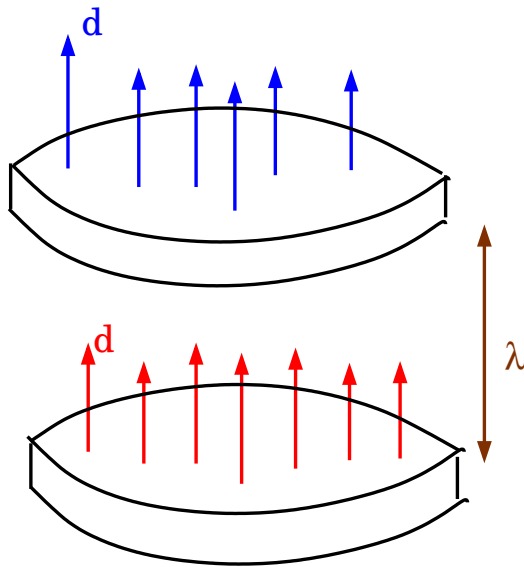


$$\alpha_{in} = A \frac{\hbar}{m} (kr_*)^2; \quad A \Rightarrow 10^{-3} - 10^{-4} \quad \alpha_{in} \rightarrow (10^{-8} - 10^{-9}) \text{ cm}^2/\text{s}$$

LiK molecules $\rightarrow d \simeq 3.5 \text{ D}$ $r_* \approx 4000a_0$

$$n = 2 \times 10^8 \text{ cm}^{-2} \Rightarrow E_F = 2\pi\hbar^2 n/m = 120 \text{ nK} \quad T_c \approx 10 \text{ nK}; \quad \tau \sim 2\text{s}$$

Bilayered dipolar fermionic systems. BCS-BEC crossover



$$V(\rho) = d^2 \left\{ \frac{1}{(\rho^2 + \lambda^2)^{3/2}} - \frac{3\lambda^2}{(\rho^2 + \lambda^2)^{5/2}} \right\} \quad \text{Always a bound state of } \uparrow \text{ and } \uparrow \text{ dipoles}$$

Dipole-dipole length $r_* = md^2/\hbar^2$ Dipole-dipole strength $\beta = r_*/\lambda$.

$$r_* \leq b \Rightarrow \epsilon_b \simeq \frac{\hbar^2}{4mb^2} \exp \left[-\frac{8\lambda^2}{r_*^2} \left(1 - \frac{r_*}{\lambda} \right) - (5 + 2\gamma) \right]$$

Interlayer superfluids

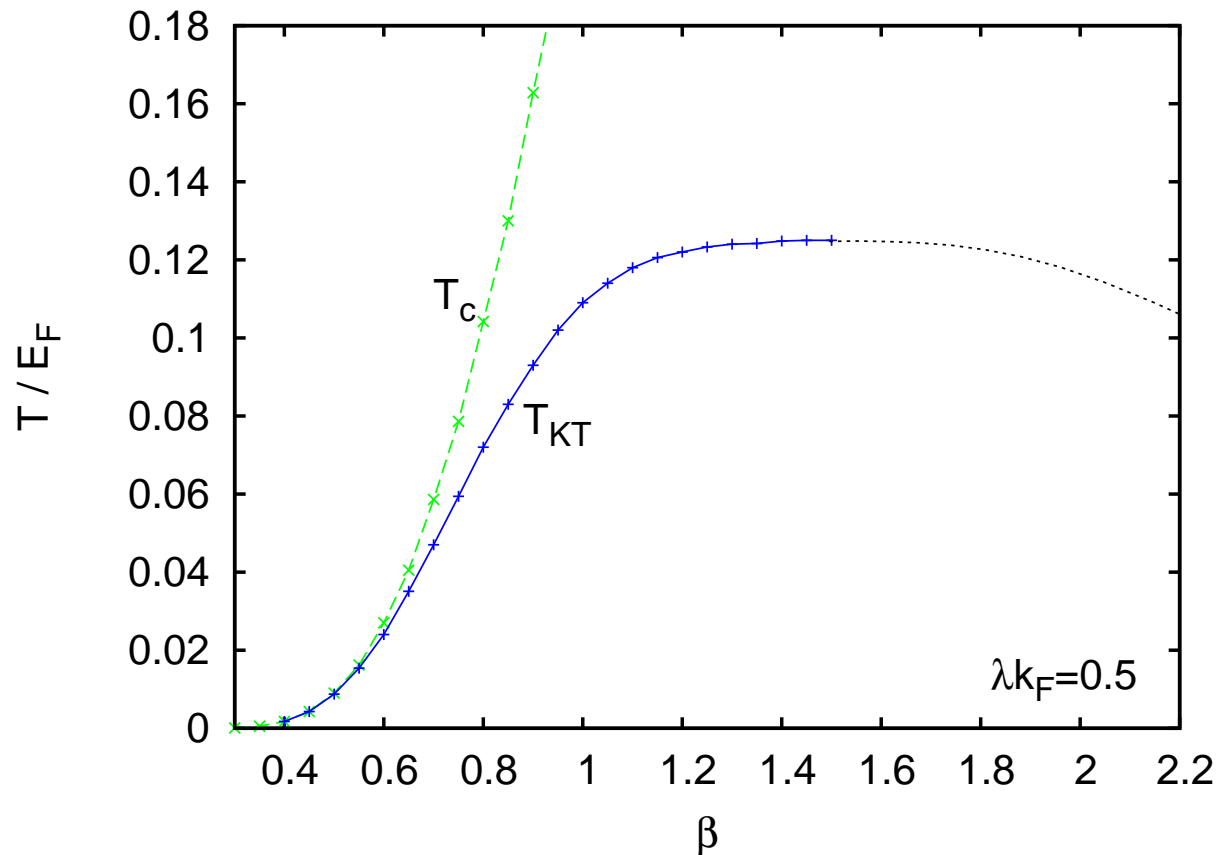
$\epsilon_b \ll E_F \Rightarrow f < 0 \rightarrow$ *s*-wave BCS pairing

$\epsilon_b \gg E_F \Rightarrow$ Molecules of \uparrow and \uparrow dipoles. Molecular BEC

Transition temperature

New BCS-BEC crossover (Pikovski, Klawunn, Santos, GS)

Innsbruck group of P. Zoller



LiCs and KRb molecules $\lambda \simeq 250$ nm, $n \simeq 5 \cdot 10^8$ cm⁻², $k_F \lambda \simeq 2$, $E_F \simeq 110$ nk
 $\Rightarrow T_{KT}$ of a few nanokelvin

Interlayer superfluids

Multilayer system → Harvard group of E. Demler

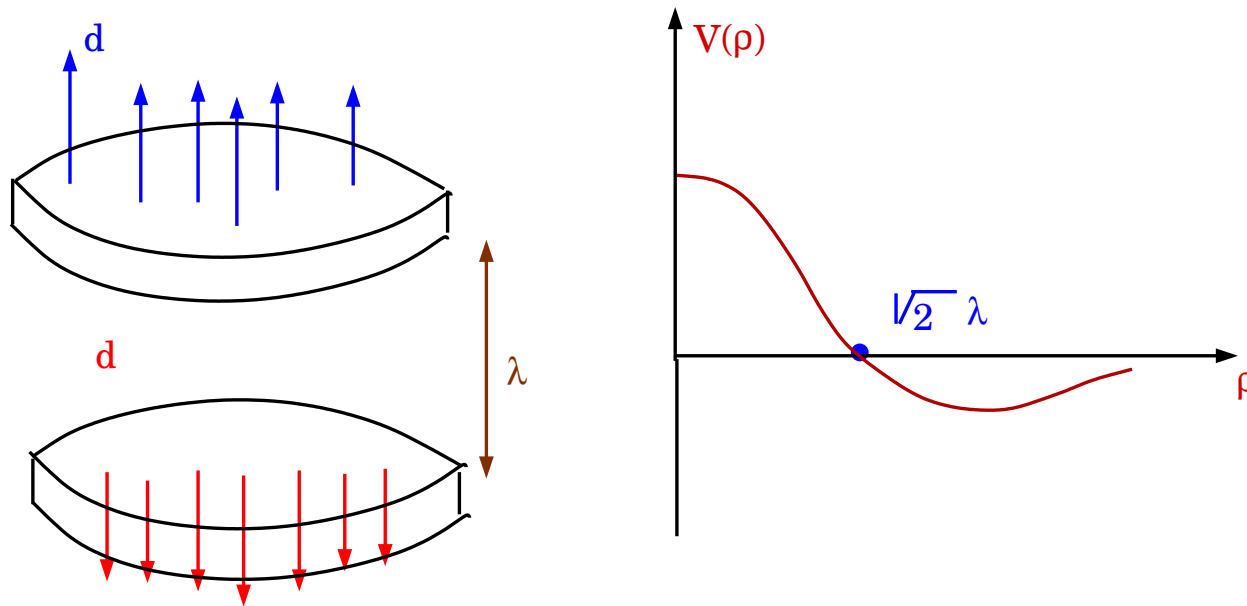
Bilayer systems of \uparrow and \downarrow dipoles

Bilayer system of \uparrow and \downarrow dipoles

Put $J = 0$ molecules in one layer and $J = 1$ in the other

Apply an electric field perpendicularly to the layers

Slightly non-uniform to prevent resonant dipolar flips leading to a rapid decay



Always a bound state of \uparrow and \downarrow dipoles

$$\beta \lesssim 1 \Rightarrow \epsilon_b \simeq \frac{\hbar^2}{m\lambda^2} \exp[-8/\beta^2 - 8/\beta - (5 + 2C - 2 \ln 2)]$$

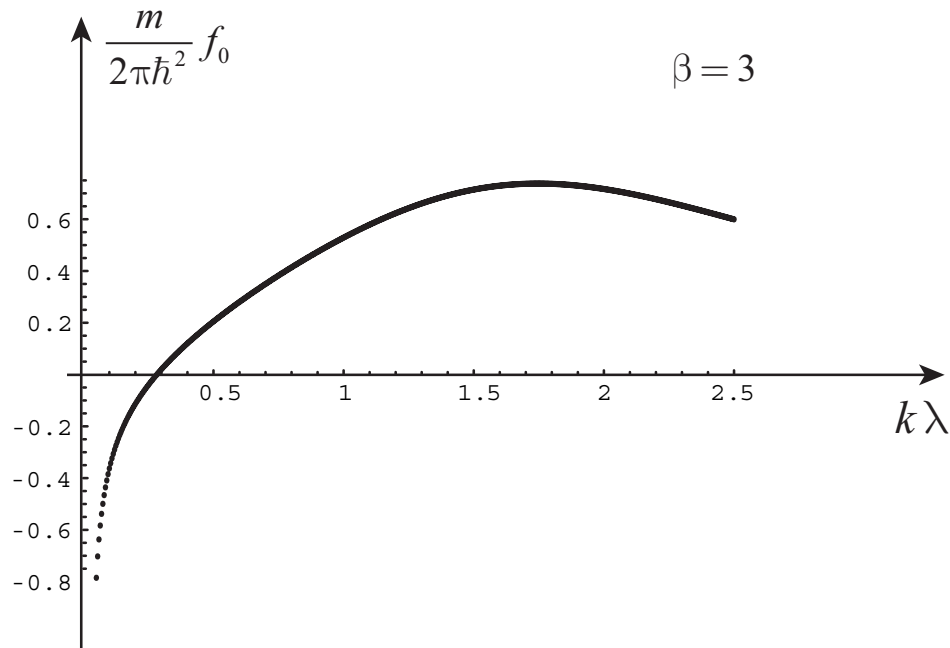
$$\beta = r_*/\lambda$$

Interlayer interaction. Scattering amplitudes

$$s\text{-wave amplitude } k \rightarrow 0 \quad f_0(k) = \frac{4\pi\hbar^2}{m \ln(\epsilon_b/\epsilon)} + \frac{8\hbar^2}{m} kr_*$$

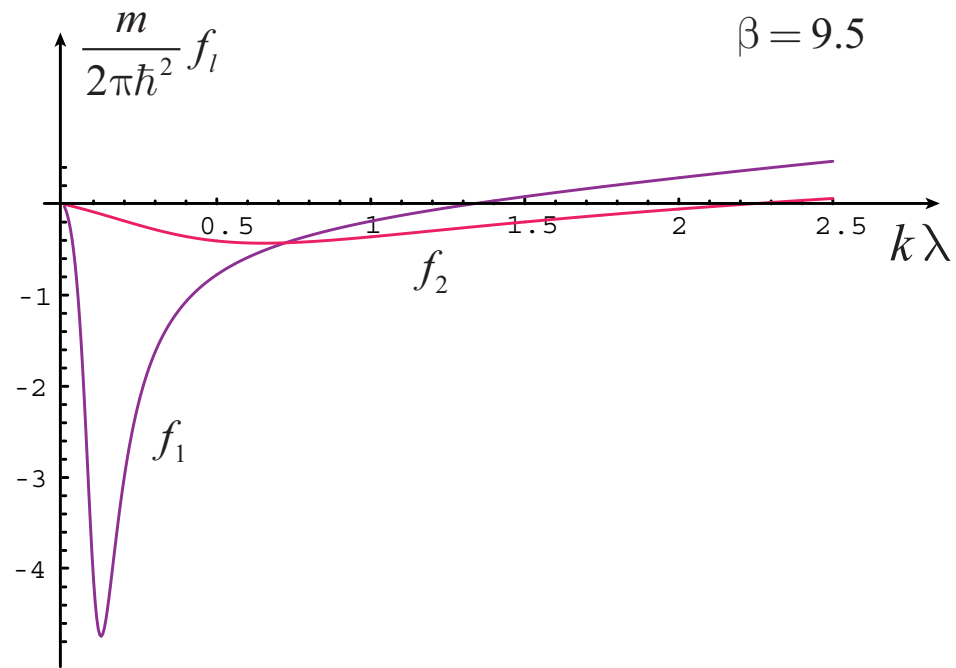
$$\epsilon = \hbar^2 k^2 / m \quad r_* = md^2 / \hbar^2$$

$f_0 > 0$ for reasonable k . No interlayer superfluid pairing



Interlayer interaction. Scattering amplitudes

p -wave and d -wave amplitudes are < 0



Interlayer *p*-wave and *d*-wave pairing

For $k_F r_* \gtrsim 1$ the effective mass significantly decreases

$$\text{Transition temperature } T_c \sim E_F^* \exp\left(\frac{2\pi\hbar^2}{m_* |f(k_F)|}\right)$$

The quasiparticle Fermi energy increases

Compensate the decrease of m_* in the exponent by increasing d^2 and, hence, f

p-wave interlayer superfluid with $T_c \sim$ tens of nK

d-wave superfluids with $T_c \sim$ nK. Analogy with high-temperature superconductors

LiCs with $n > 10^9 \text{ cm}^{-2}$

Conclusions

Remarkable results for atomic Fermi gases

Creation of ultracold polar molecules opens wide avenues to make new quantum states

- $p_x + ip_y$ topological state for identical fermions
- Interlayer superfluids in bilayer and multilayer fermionic dipolar systems