Tunnelling, self-trapping and manipulation of higher modes of a BEC in a double well

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For the anecdote

Outline

- Theoretical model
- 2 modes in a double well
  - Tunnelling and self-trapping
- 4 modes modes in a double well
  - Regimes
  - Dynamics
- Conclusion

BEC in a double well

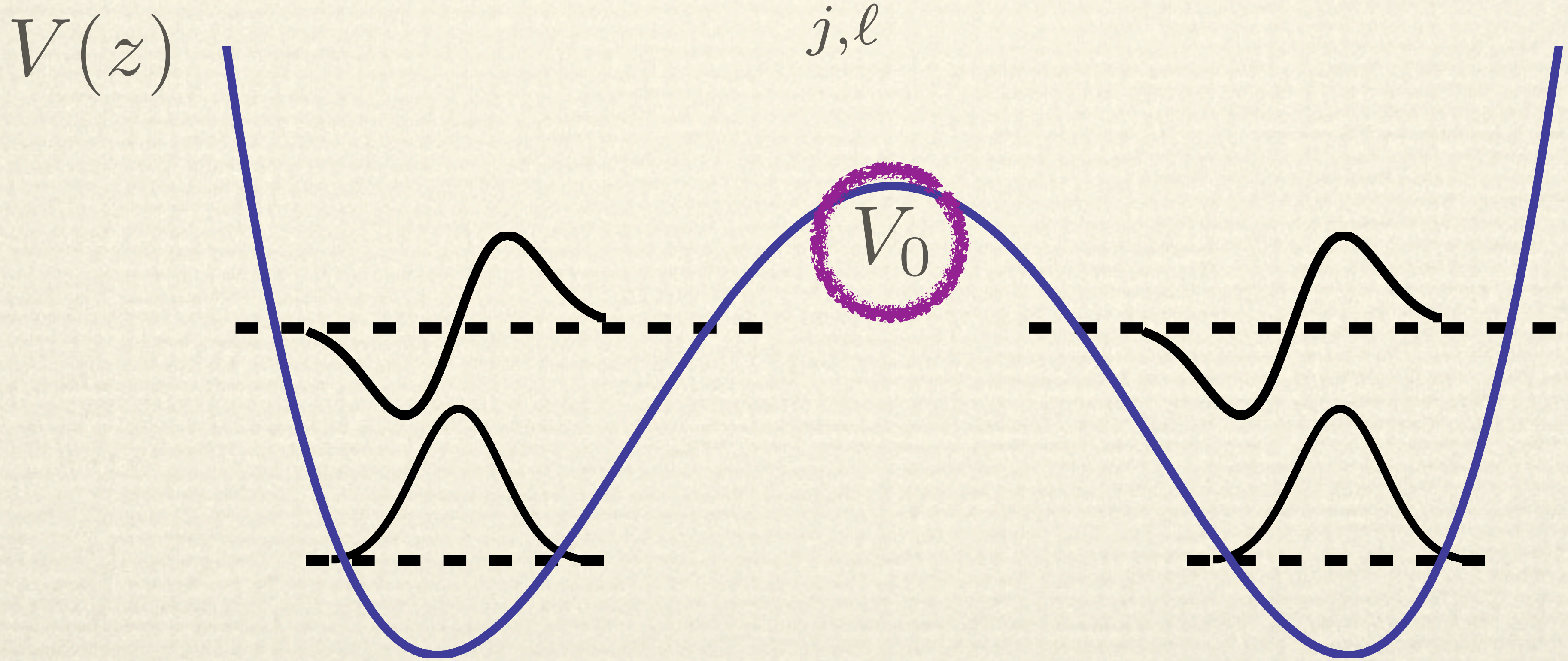
single particle Hamiltonian

two-body interaction

$$H = \int dz \Psi^\dagger(z) \left(-\frac{\hbar^2}{2M}\nabla^2 + V(z) \right) \Psi(z) + \frac{g_1}{2} \int dz \Psi^\dagger(z)\Psi^\dagger(z)\Psi(z)\Psi(z)$$

localised wave-functions

$$\Psi(z) = \sum_{j,\ell} b_{j\ell}\psi_{j\ell}(z)$$

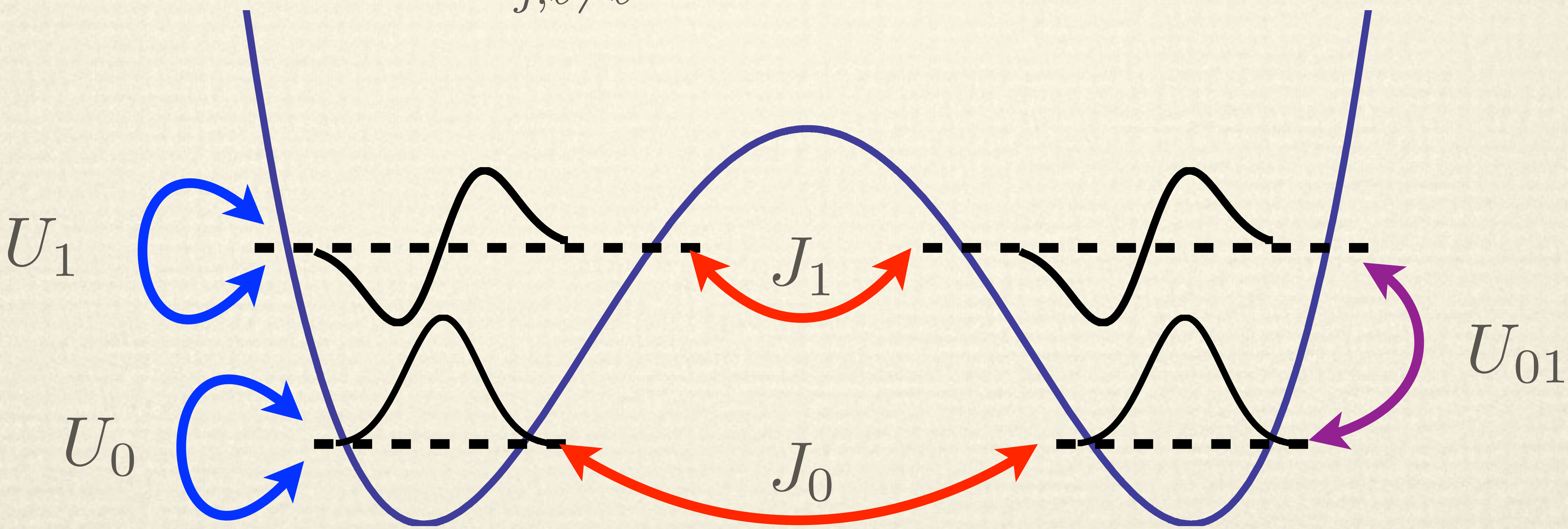


Hamiltonian

$$H = H_0 + H_1 + H_{01}$$

$$H_\ell = E_\ell \left(n_{L\ell} + n_{R\ell}\right) + J_\ell \left(b^\dagger_{L\ell} b_{R\ell} + b^\dagger_{R\ell} b_{L\ell}\right) + U_\ell \sum_j n_{j\ell}\left(n_{j\ell} - 1\right)$$

$$H_{01} = U_{01} \sum_{j,\ell\neq\ell'} \left(2 n_{j\ell} n_{j\ell'} + b^\dagger_{j\ell} b^\dagger_{j\ell} b_{j\ell'} b_{j\ell'}\right)$$



Dynamics

- The annihilation operators equations of motion are

$$i\frac{db_{j\ell}}{dt} = [b_{j\ell}, H]$$

- For a macroscopic BEC, we can write

$$b_{j\ell} = \sqrt{N_{j\ell}}e^{i\phi_{j\ell}}$$

- We have the equations for

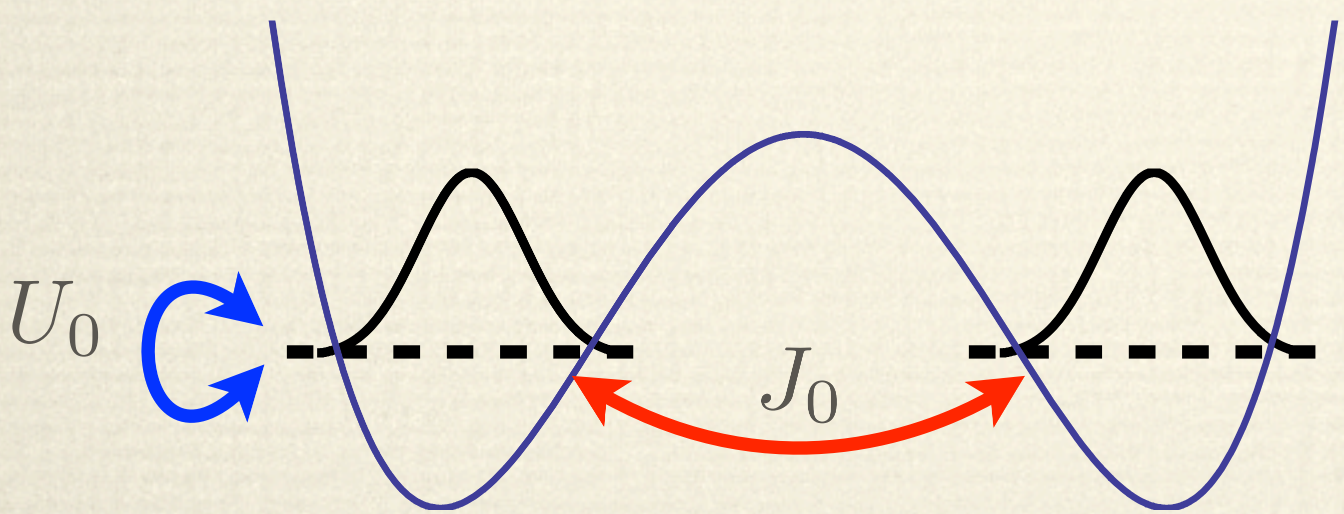
$$\dot{N}_{jl}, \dot{\phi}_{jl}$$

Two-mode model

- Since the total number of atoms is conserved, we can get rid of one pair of conjugate variables
- For two modes,

$$z = \frac{N_{L0} - N_{R0}}{N}$$

$$\theta = \phi_{L0} - \phi_{R0}$$

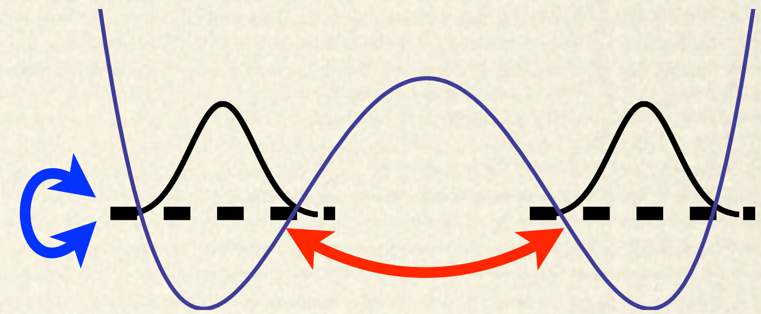


$$\dot{z} = -J_0 \sin\theta \sqrt{1 - z^2}$$

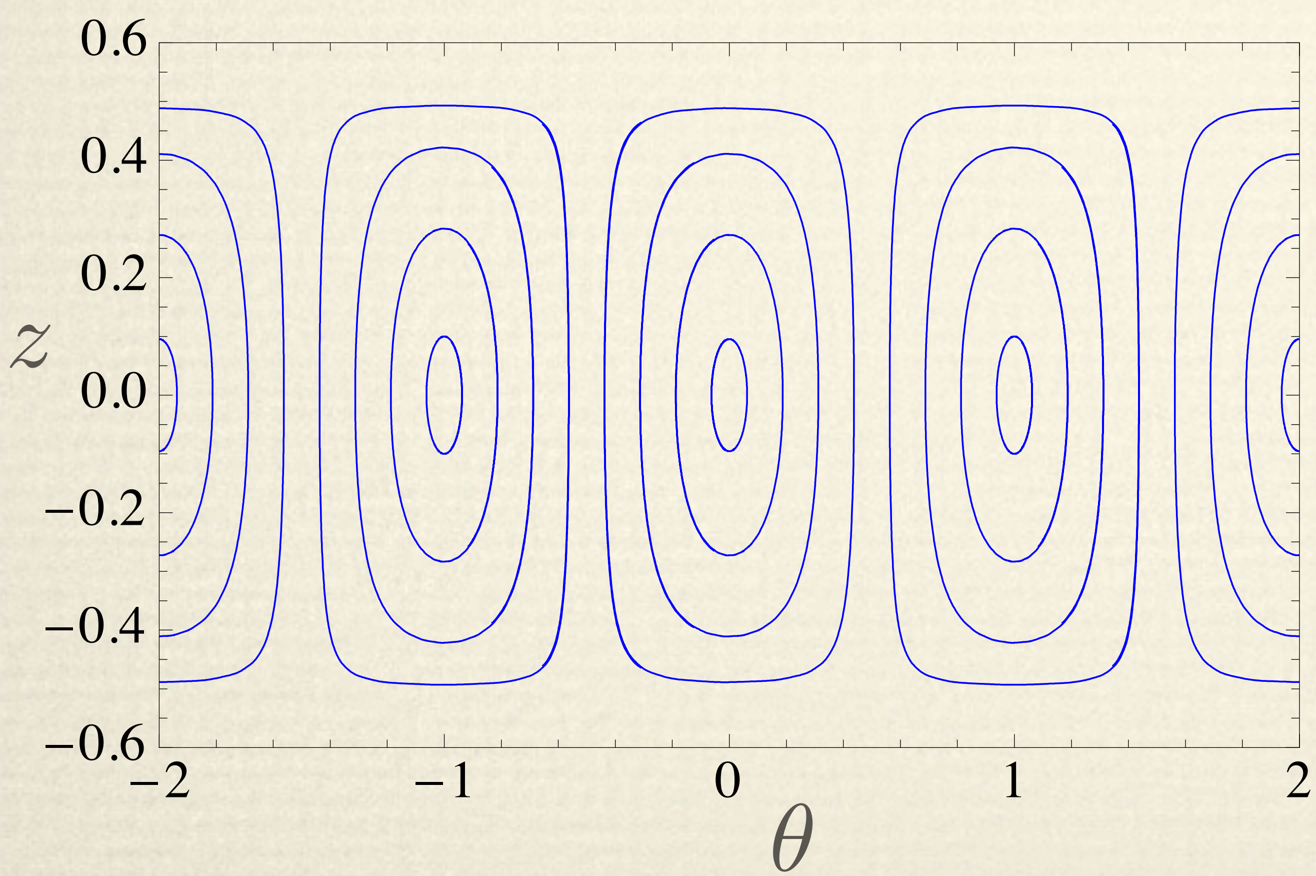
$$\dot{\theta} = 2NU_0 z + \frac{4J_0 z}{\sqrt{1 - z^2}} \cos\theta$$

Two-mode dynamics

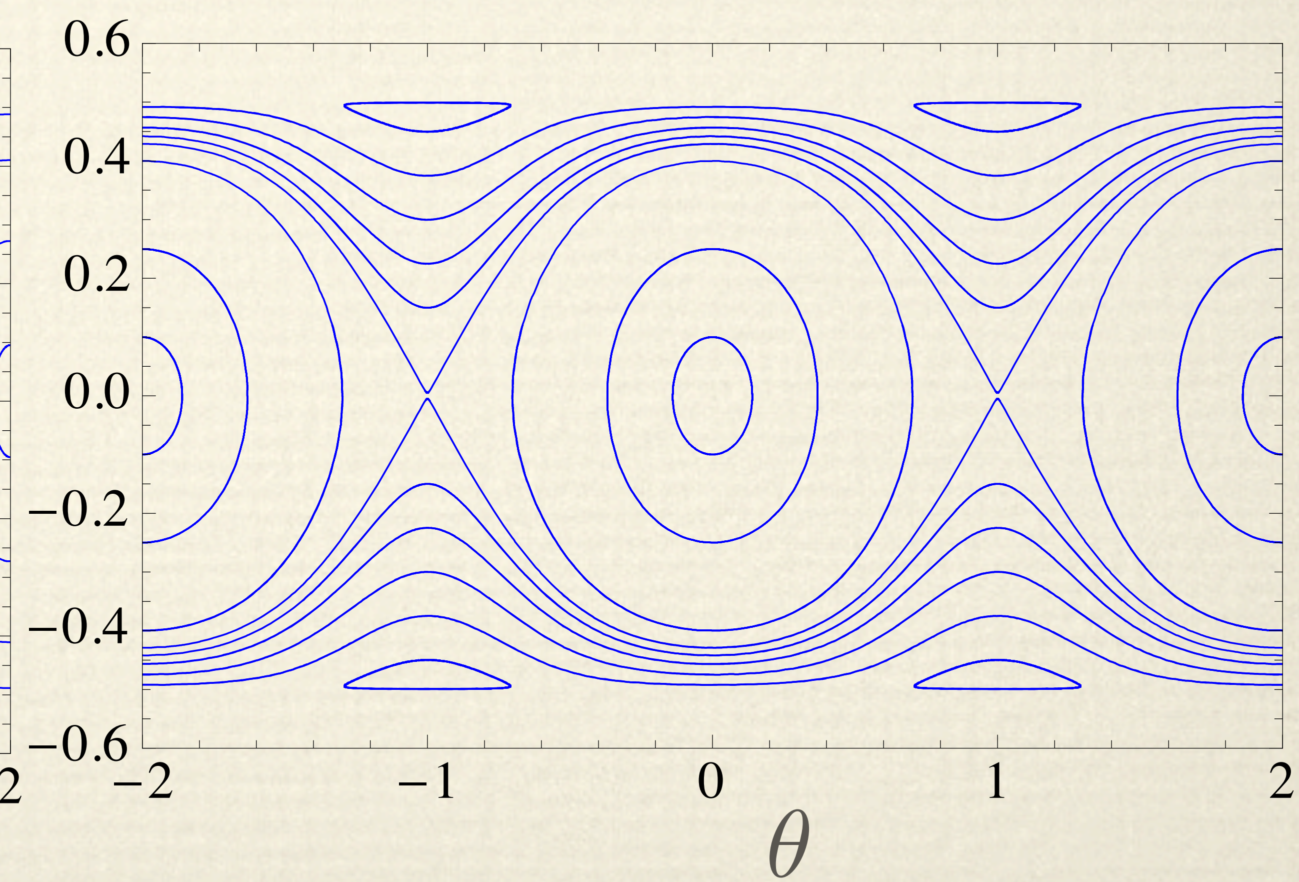
- Phase space plans



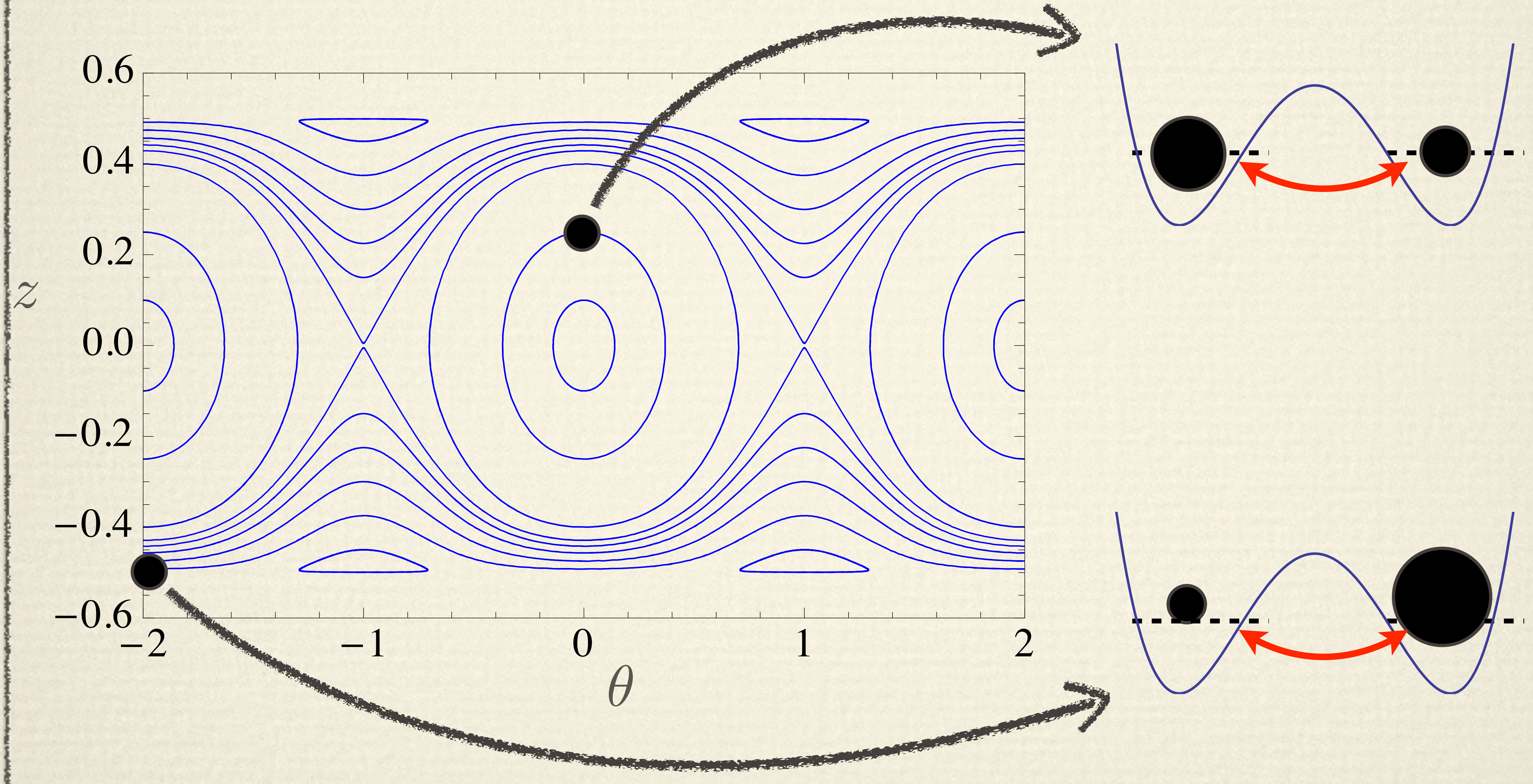
$NU_0 < 2J_0$



$NU_0 > 2J_0$



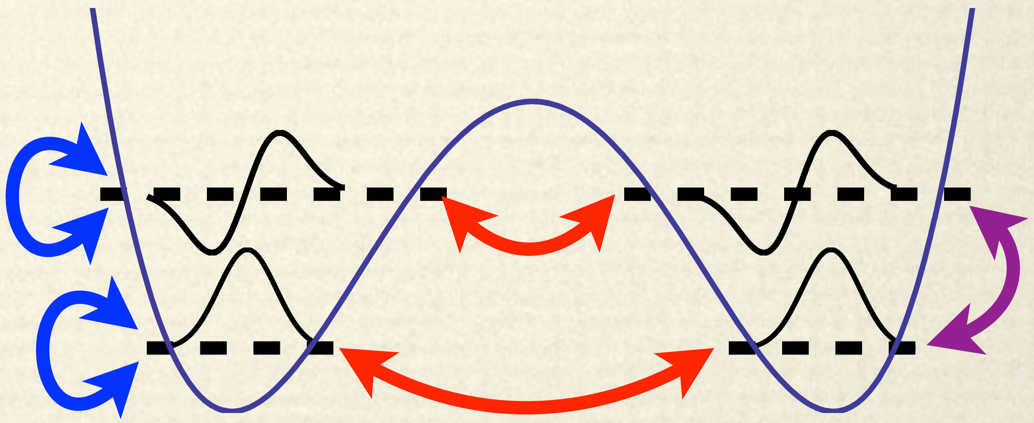
Phase space diagram



Four-mode model

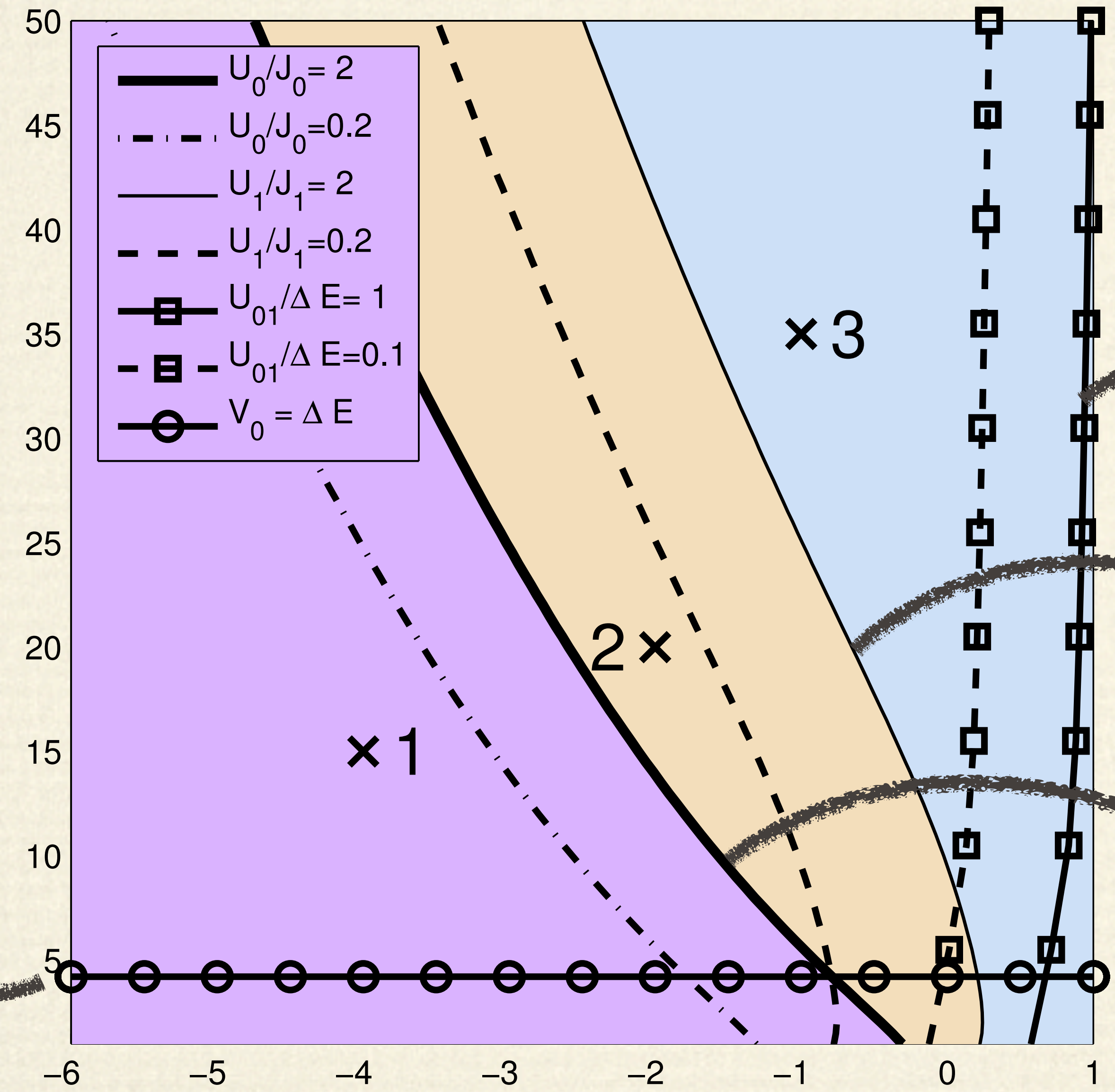
- Once again, we can get rid of one pair of conjugate variables since N is constant
- There are many possible choices for a new basis
- To highlight self-trapping in each level, we choose

$$Nz_0 = N_{L0} - N_{R0}$$
$$Nz_1 = N_{L1} - N_{R1}$$
$$Nz_2 = (N_{L0} + N_{R0}) - (N_{L1} + N_{R1})$$



Regimes

V_0



$\log(Ng_1/2)$

$NU_{01} = \Delta E$

$NU_1 = 2J_1$

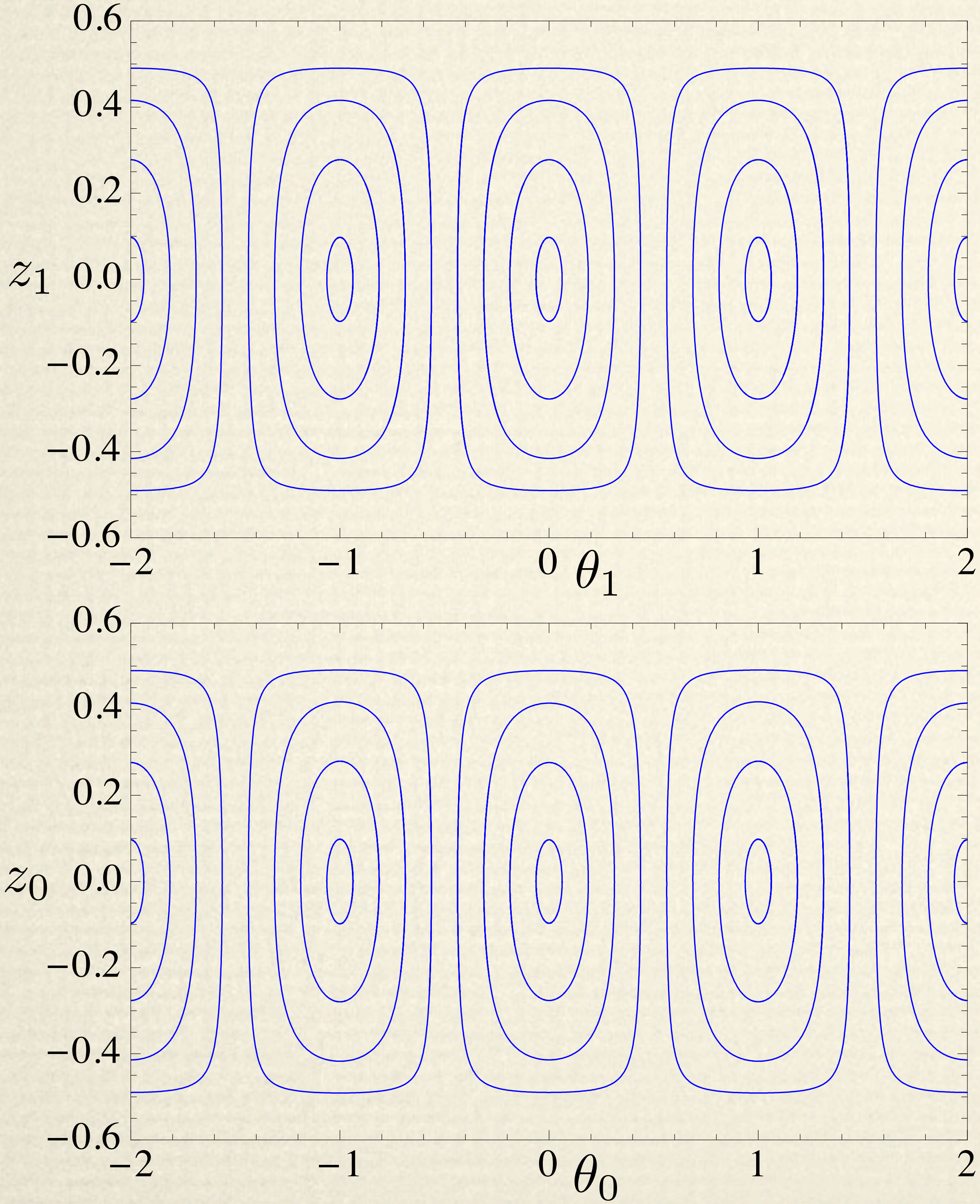
$NU_0 = 2J_0$

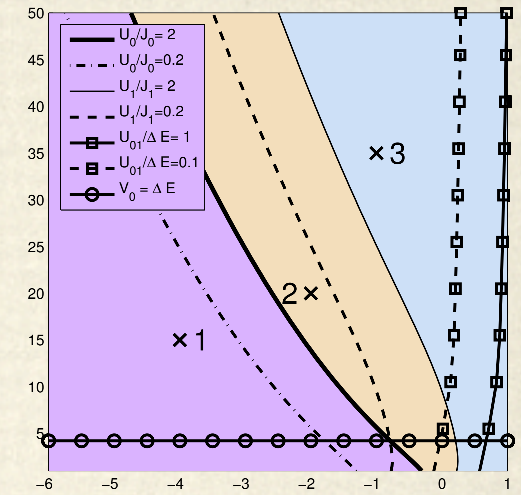
$V_0 = E_1$

- In all three cases

$\boxed{\dot{z}_2 \simeq 0}$

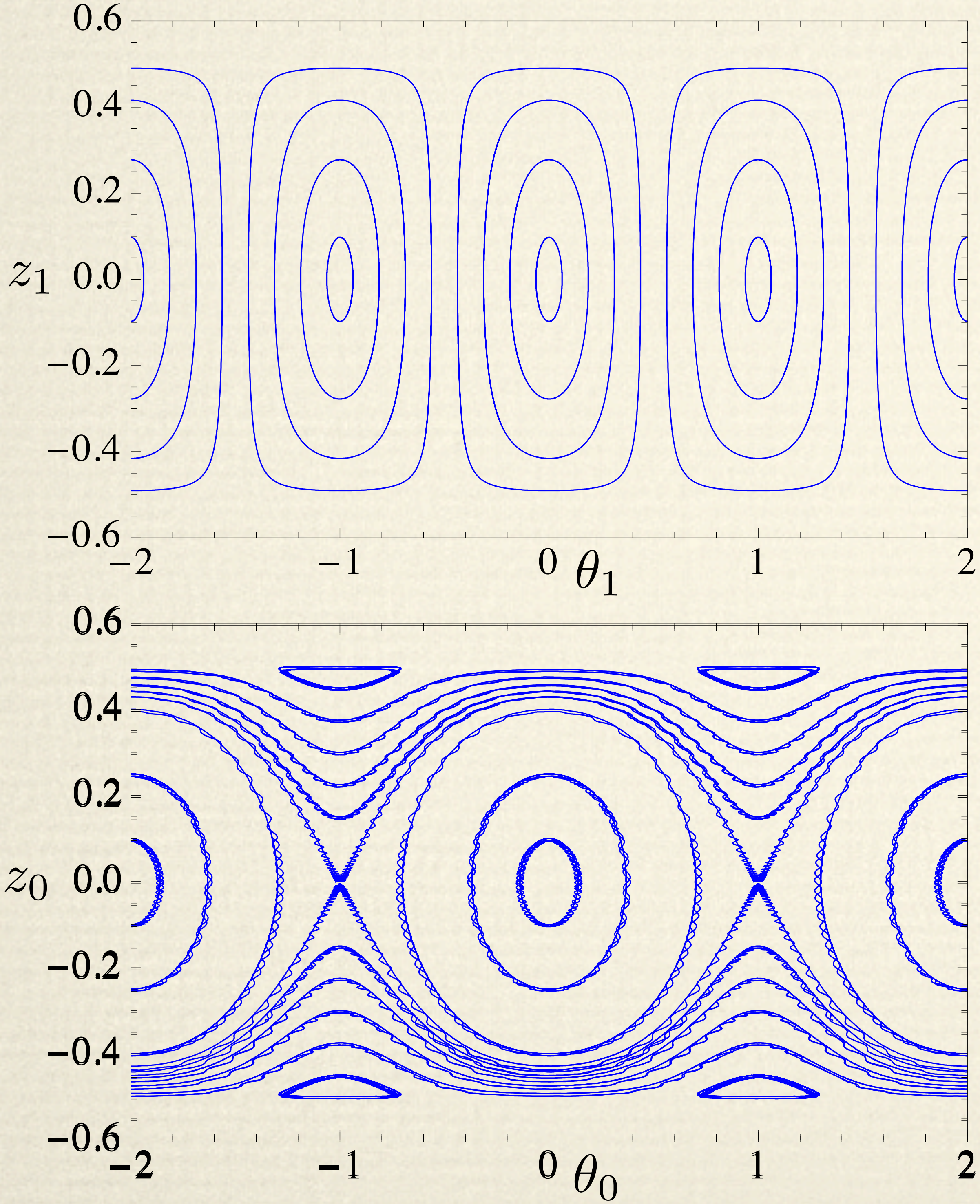
Situation 1

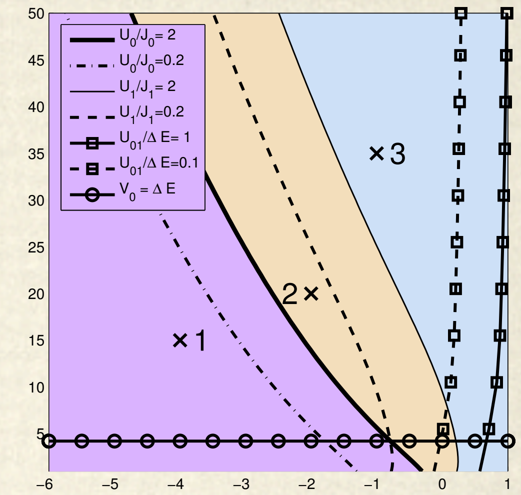




- Pure tunnelling
- Identical behaviour for both energy levels
- Different Rabi frequencies
- 50% of atoms in each level

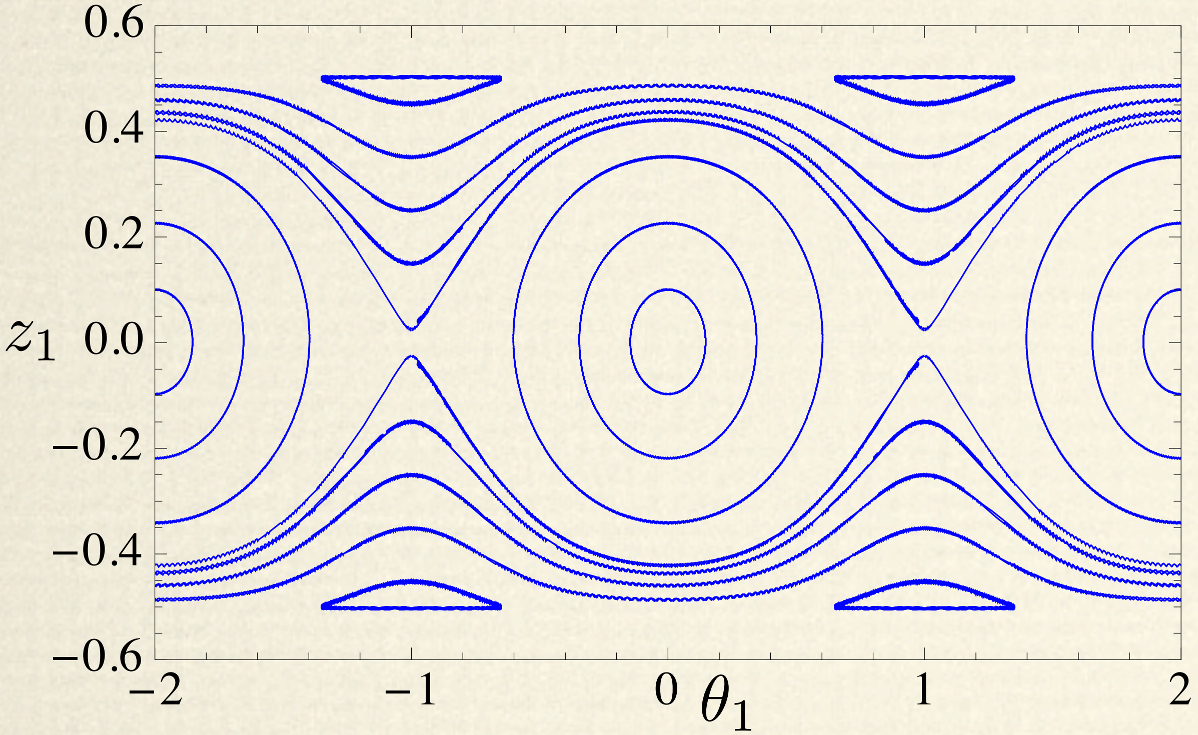
Situation 2

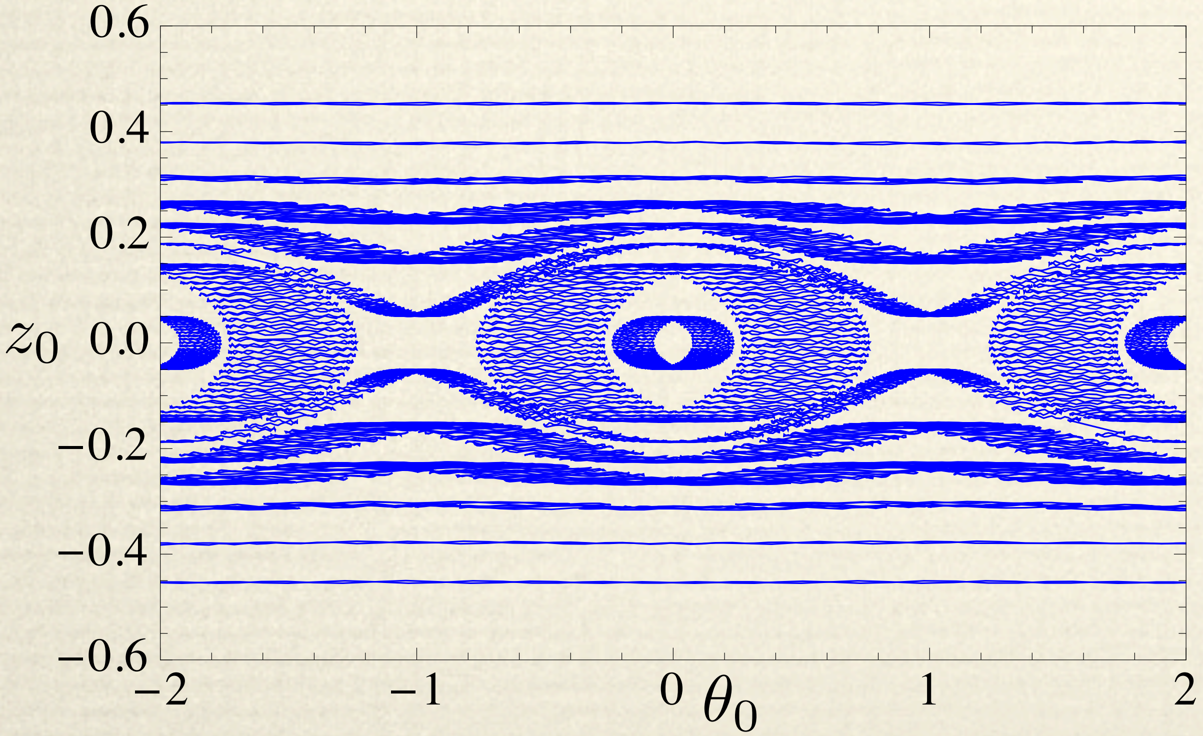


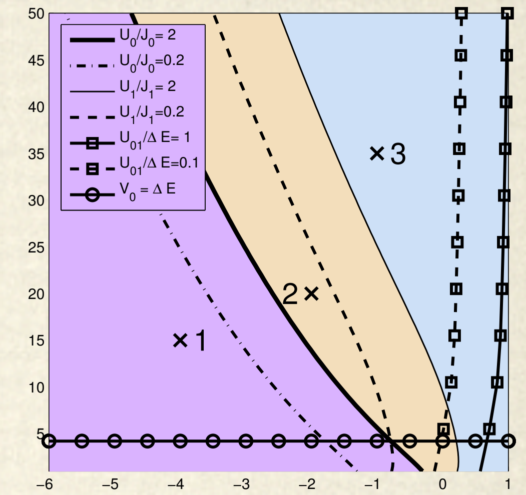


- Strong U_0, negligible U_1
- Tunnelling in the excited modes
- Self-trapping in the ground modes
- Initial conditions in stationary states

Situation 3

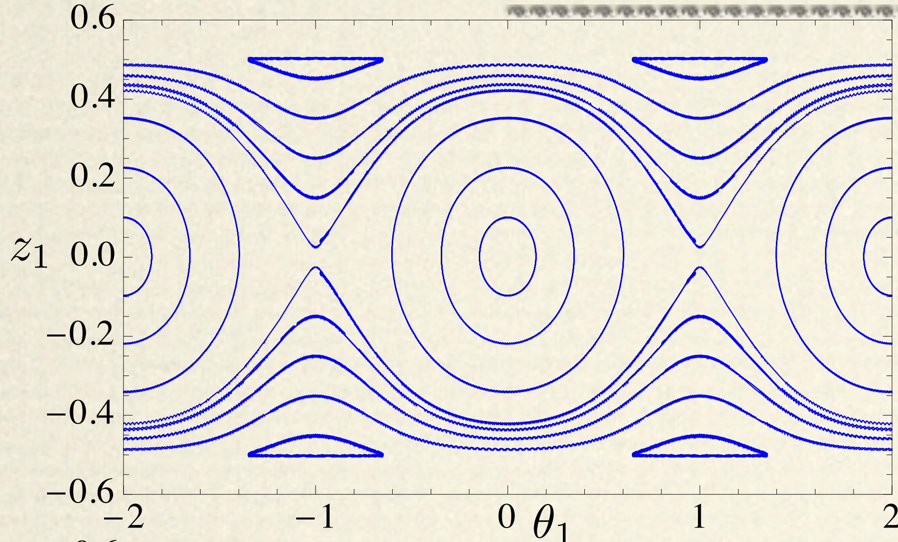




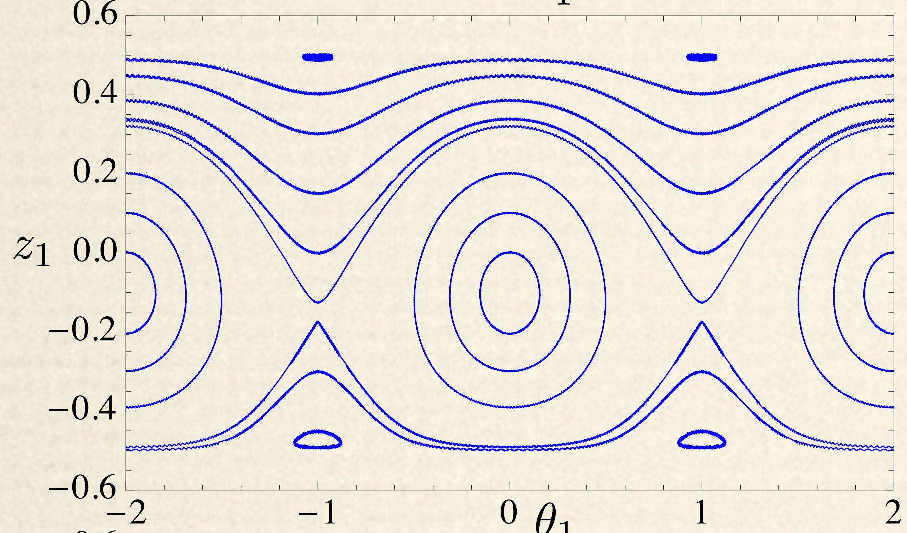


- Strong U_1, stronger U_0
- Self-trapping in the excited modes
- Strong self-trapping in the ground modes
- Initial conditions in stationary states

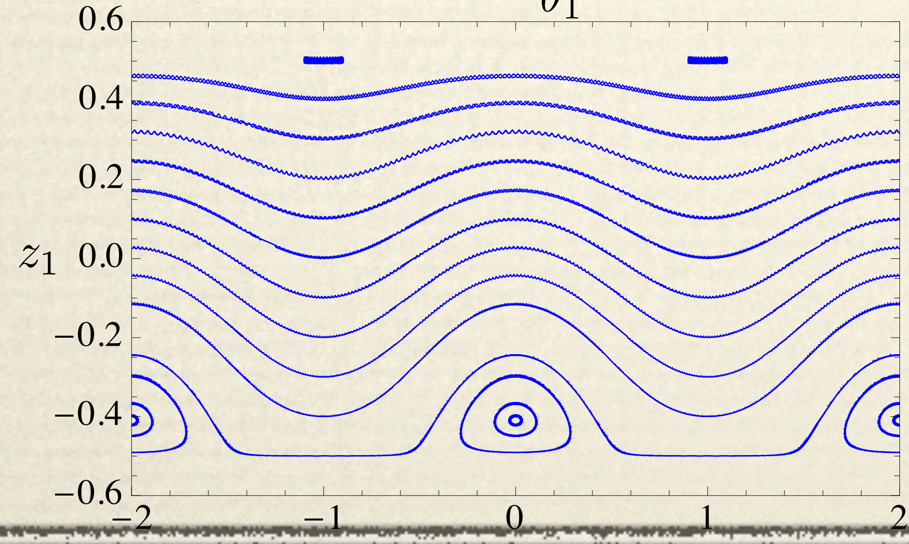
Mode manipulation



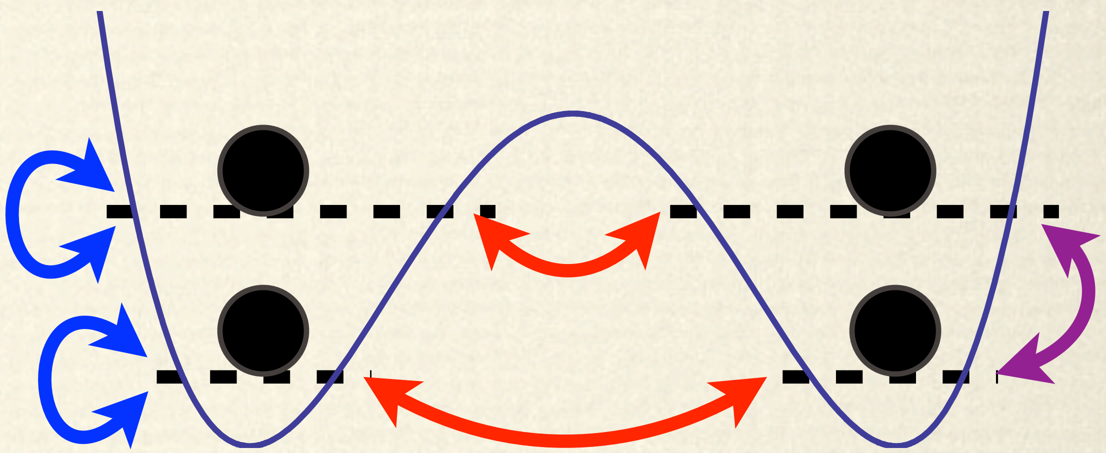
$z_0(0) = 0$



$z_0(0) = 0.1$



$z_0(0) = 0.45$



The population in the ground modes affects the excited modes

Conclusion

- Tunnelling and self-trapping can be observed in higher modes
- U_{01} influences the dynamics even when no atoms jump energy levels
- The initial population in the ground modes can strongly influence the excited modes