DIPOLAR DARK SOLITONS

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Outline

- Motivation (only contact interaction)
- Model
  - Single soliton
  - Two solitons
  - Too many solitons
Optical attractive solitons:
- theory: papers of Królikowski
N bosons in 1D box, \( V(z_1 - z_2) = g \delta(z_1 - z_2) \)

Exact solution for N-body problem

Lieb, Liniger (1963)

Excitation spectrum:

Type I is.. Bogolubov

Type II is..

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**GPE equation (Gross, Pitaevskii 1961)**

\[ i\hbar \partial_t \psi = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + gN|\psi|^2 \right) \psi(z) \]

**Solitonic solutions for g>0 (Zakharov 1973)**

**Excitation spectrum:**

- \( \varepsilon_1/\rho^2 \)
- \( \varepsilon_2/\rho^2 \)
- Bogoliubov \( \gamma = 0.787 \)

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**Thermal samples:**

- # of solitons versus # of expected excitation II

PRL 109, 205302 (2012)
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\]

**Excitation spectrum:**

1. \( \epsilon_1 / \rho^2 \)
2. \( \epsilon_2 / \rho^2 \)
3. BOGOLIUBOV \( \gamma = 0.787 \)

Type I is.. .. Bogolubov

Type II is SOLITON!!

**Solitonic solutions for g>0 (Zakharov 1973)**

**Thermal samples:**

# of solitons versus # of expected excitation II

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PRL 109, 205302 (2012)
N dipolar bosons q1D

dipole-dipole interaction:

\[ V_{\text{dip}} = \frac{3\hbar^2 a_{dd}}{m} \frac{1 - 3\cos^2(\vartheta)}{r^3} \]

Bogolubov spectrum

Basic properties of solitons?
Do they appear at finite temperatures?

B Malomed, L. Santons, P. Pedri, D J Frantzeskakis, S.H. Adhikari
\[ i\hbar \partial_t \psi = \left( -\frac{\hbar^2 \Delta}{2m} + \frac{1}{2} m\omega_T^2 (x^2 + y^2) + gN|\psi|^2 + \int \mathrm{d}r' |\psi(r')|^2 V_{\text{dip}}(r-r') \right) \psi(r) \]

Aspect ratio: \( \lambda = \frac{l}{L} \)

Feschbach resonances – to avoid any influence of contact interaction terms

\[ \hbar \omega_T \gg k_B T, gn, g_{\text{dd}} n \]

Dipolar interaction strength: \( a_{\text{dd}} \)
SINGLE SOLITON

States with the lowest energy .. but having jump of phase $\pi$

\[
\xi^2 = \frac{1}{\sqrt{3\frac{N}{L} a_{dd}}} l
\]

(rotating frame, such that periodic boundary conditions are matching)
SINGLE SOLITON

States with the lowest energy .. but having jump of phase $\pi$

\[ \xi^2 = \frac{1}{\sqrt{3} \frac{N}{L} a_{dd}} \]

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SINGLE SOLITON

States with the lowest energy .. but having jump of phase $\pi$

\[ \xi^2 = \frac{1}{\sqrt{3 \frac{N}{L} a_{dd}}} \]

Soliton width

Dysprosium
$\omega_T = 2 \pi$ (5 kHz)
$N = 600$
$t_{phon} = 50 \text{ ms}$

(rotating frame, such that periodic boundary conditions are matching)
Intersolitonic potential: scaling with distance

\[ a = - \frac{1}{m} \frac{\partial V}{\partial z} \]

\[ a + b/z^3 \]

\[ c + \exp(-d \cdot z) \]
Intersolitonic potential: comparison with soliton density

\[ a \left( |\psi_{\text{sol}}(x-b)|^2 - \rho_0 \right) \]

Potential

\[ \text{n } a_{dd} = 1000 \]
\[ \lambda = 0.001 \]
Intersolitonic potential: comparison with soliton density

\[ n a_{dd} = 1000 \]
\[ \lambda = 0.001 \]

\[ V(z) = |\psi_1(z)|^2 \]

\[ a ( |\psi_{sol}(x-b)|^2 - \rho_0 ) \]
Other parameters: testing

\[ n \alpha_{dd} = 0.1 \]
\[ \lambda = 0.01 \]
Other parameters: testing

- $n_{dd} = 0.1$
- $\lambda = 0.01$
Other parameters: testing

- inelastic collisions → radiation → gray solitons → acceleration
- solitonic height: $< 10^{-7}$
- $0.007$
Two solitons: long evolution

minimla density

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

0 10 20 30 40 50

time [box units]
Two solitons: long evolution

Phenomenological picture:
- N atoms,
- initially in an excited level,
- interacting with small environment.

„zoom” onto evolution around t=40
between t=40 and t=41
FINITE TEMPERATURES: naïve approach

\[ T = 10^4 \text{ box u.} \]

\[ T = 5 \times 10^4 \text{ box u.} \]

\[ T = 10^5 \text{ box u.} \]

\[ N = 1000 \]

\[ n a_{dd} = 2 \]

\[ \lambda = 0.01 \]
Summary

Shapes of solitons

Shallow Inter-solitonic potential

Non-elastic collision → acceleration of solitons

In progress:
The solitons are present in thermal samples

STABILITY OF SOLITONS (radiation)

Solitons vs rotons