DIPOLAR DARK SOLITONS

Krzysztof Pawłowski, Kazimierz Rzążewski

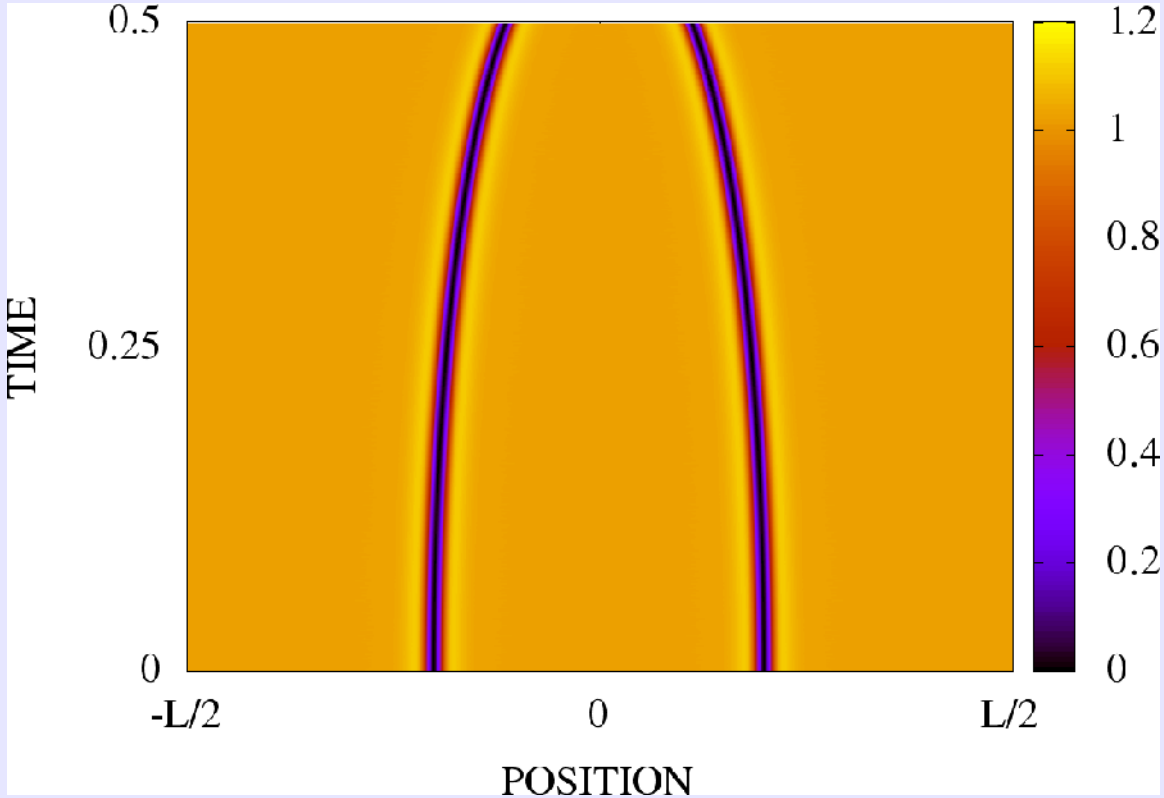
Centrum Fizyki Teoretycznej PAN, Warszawa

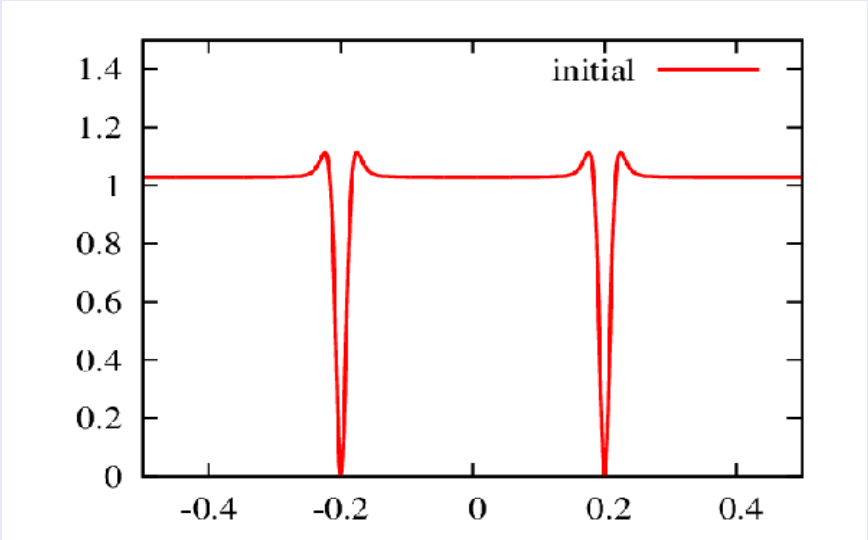
NATIONAL SCIENCE CENTRE
POLAND

Outline

- **Motivation (only contact interaction)**
- **Model**
- **Single soliton**
- **Two solitons**
- **Too many solitons**

EVOLUTION





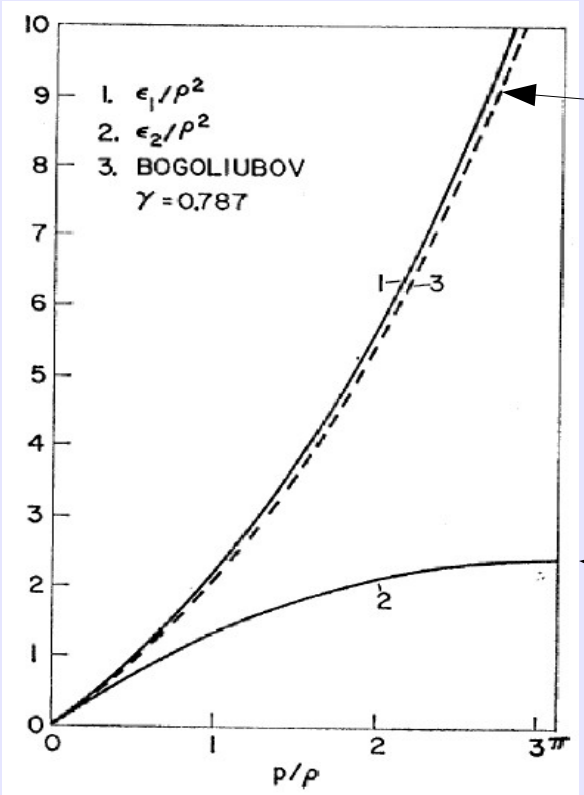
Optical attractive solitons:
- theory: papers of Królikowski
- experiemnt: Nature Physics 2, 769 (2006)

N bosons in 1D box, $V(z_1-z_2) = g\,\delta(z_1-z_2)$

Exact solution for N-body problem

Lieb, Liniger (1963)

Excitation spectrum:



Type I is..
.. Bogolubov

Type II is

??????

Lieb, Phys. Rev. 130, 1616 (1963)

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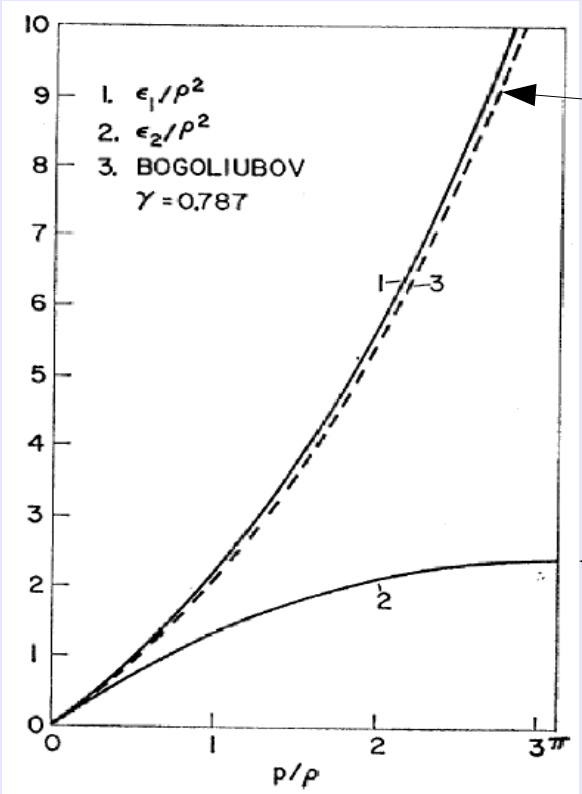

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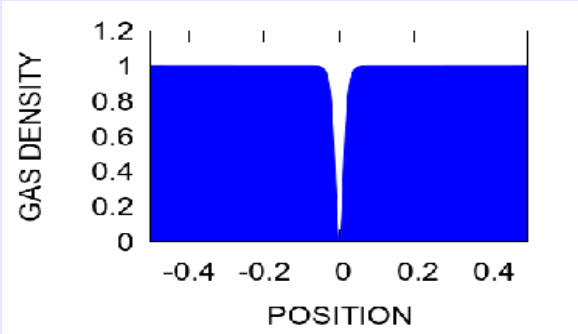
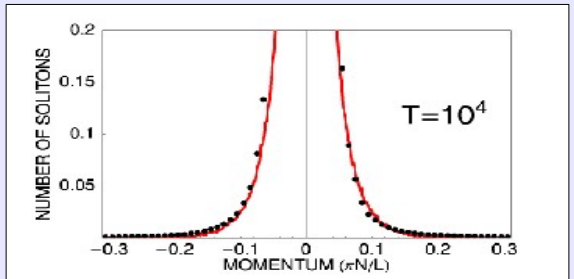
GPE equation (Gross, Pitaevskii 1961)

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + gN|\psi|^2\right)\psi(z)$$

Solitonic solutions for g>0 (Zakharov 1973)



Thermal samples:
# of solitons versus # of expected excitation II



Phys. Rev. A 91, 013621 (2015)
PRL 109, 205302 (2012)

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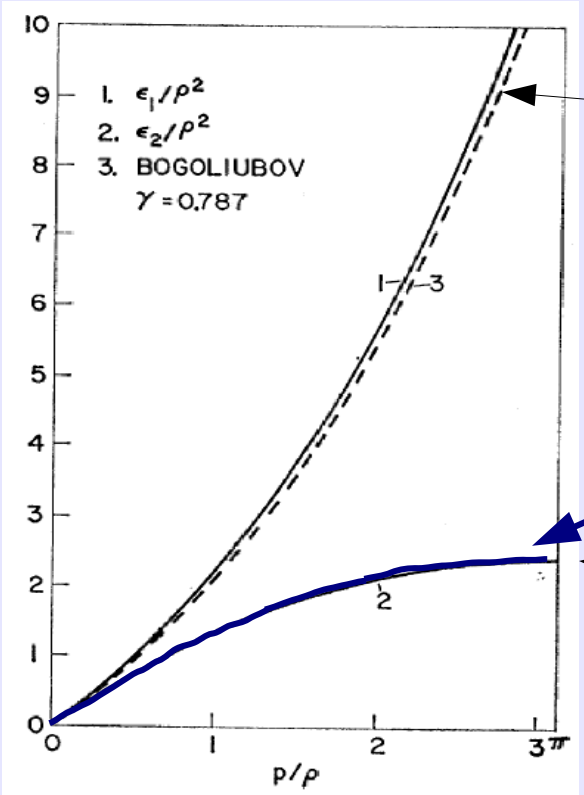

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SOLITON!!

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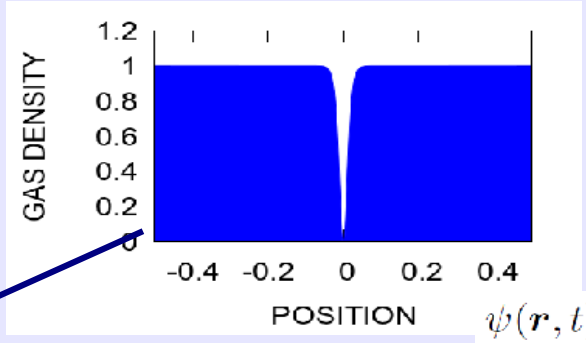

$$\psi(\boldsymbol{r},t) = f(\boldsymbol{r}-\boldsymbol{v}t)$$

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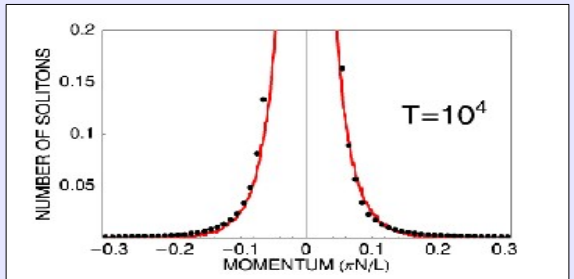
# of solitons versus # of expected excitation II



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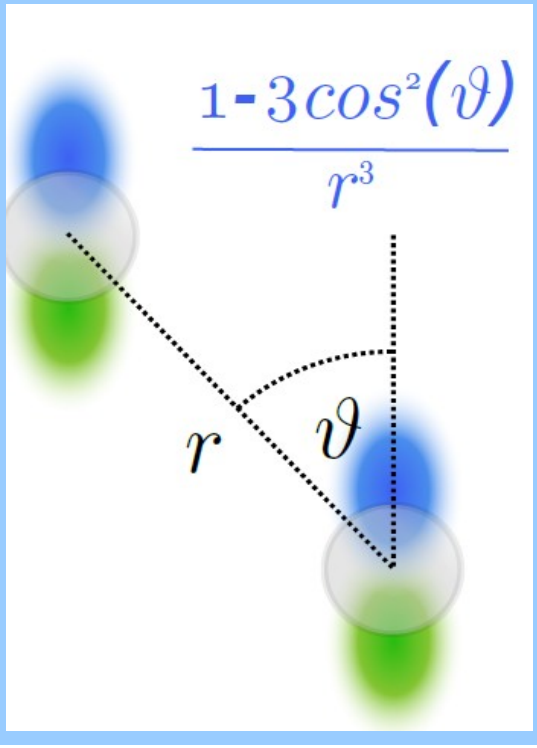
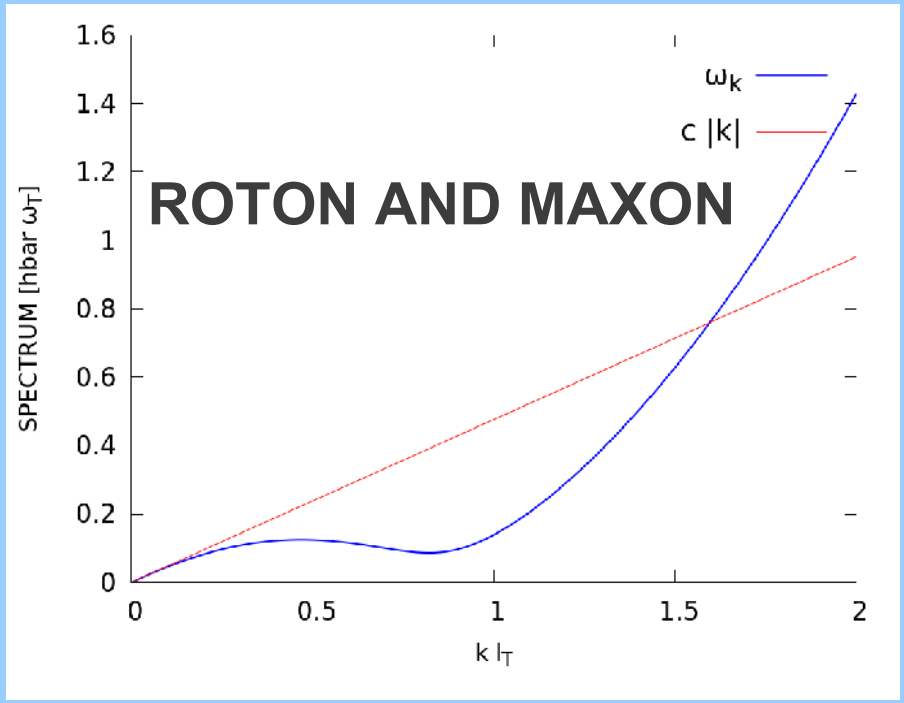
N dipolar bosons q1D ?

dipole-dipole interaction:



$$V_{dip} = \frac{3\hbar^2 a_{dd}}{m} \frac{1-3\left(\hat{d}\hat{r}\right)^2}{r^3}$$

Bogolubov spectrum



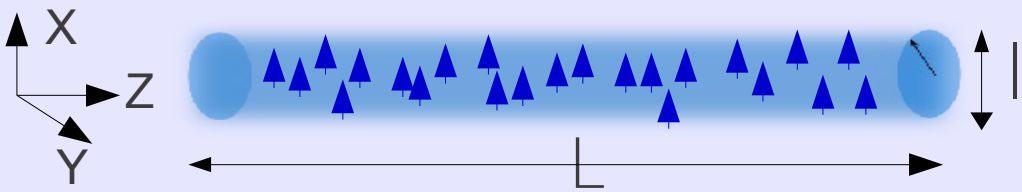
Basic properties of solitons ?

Do they appear at finite temperatures?

B Malomed, L. Santons, P. Pedri, D J Frantzeskakis, S.H. Adhikari

MODEL

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2\Delta}{2m} + \frac{1}{2}m\omega_{\mathrm{T}}^2\left(x^2+y^2\right) + gN|\psi|^2 + \int dr'|\psi(\boldsymbol{r}')|^2 \mathrm{V}_{\mathrm{dip}}(\boldsymbol{r}-\boldsymbol{r}')\right)\psi(\boldsymbol{r})$$



$$\hbar\omega_{\mathrm{T}} \gg k_B T,\ gn,\ g_{\mathrm{dd}} n$$

Feschbach resonances – to avoid any influence of contact interaction terms

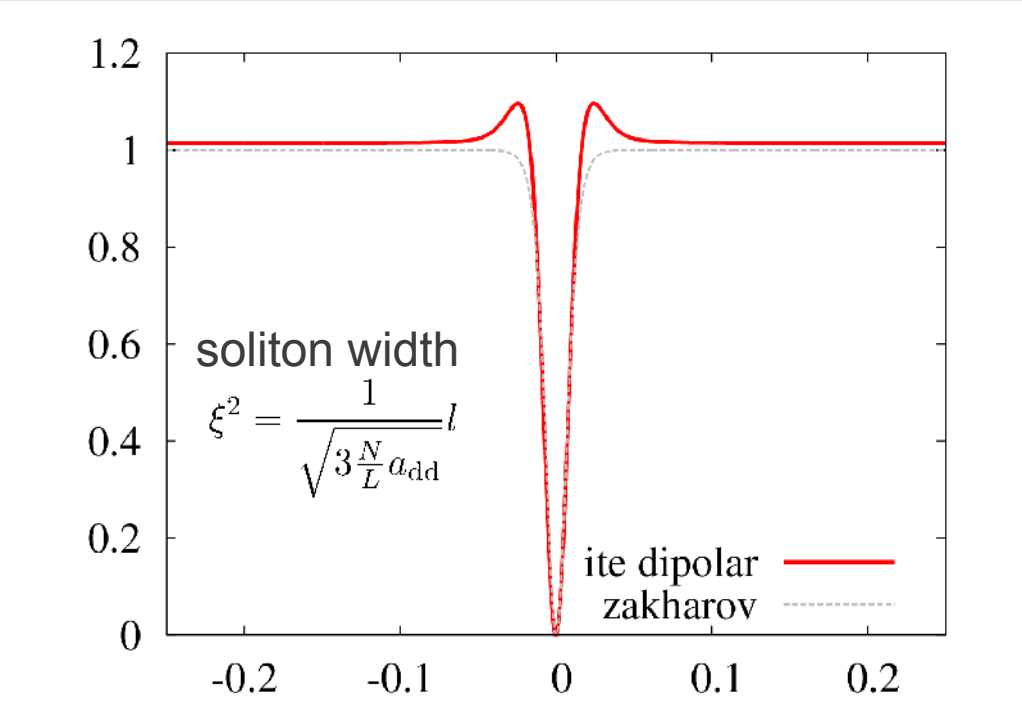
$$i\hbar\partial_t\psi_{1D} = \left(-\frac{\hbar^2\partial_z}{2m} + N\int dz' U_{1D}(z-z')|\psi_{1D}(z')|^2\right)\psi_{1D}(z)$$

aspect ratio: $\lambda = l/L$

$\mathbf{a_{dd}}$ ”strength” of dipolar interaction

SINGLE SOLITON

States with the lowest energy .. but having jump of phase π



(rotating frame, such that periodic boundary conditions are matching)

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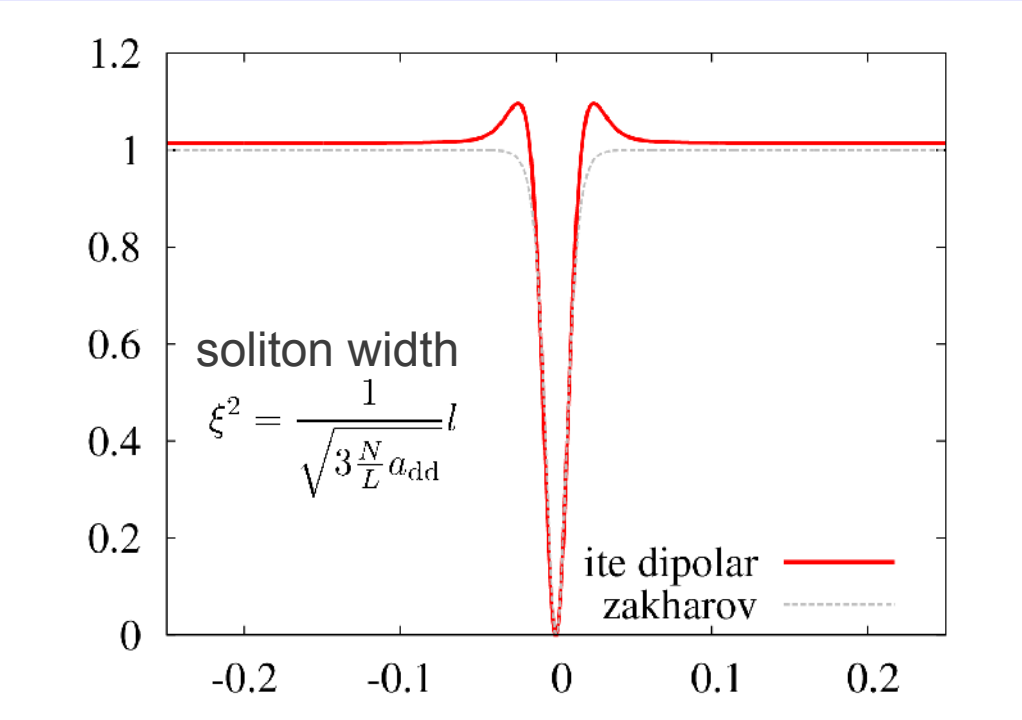

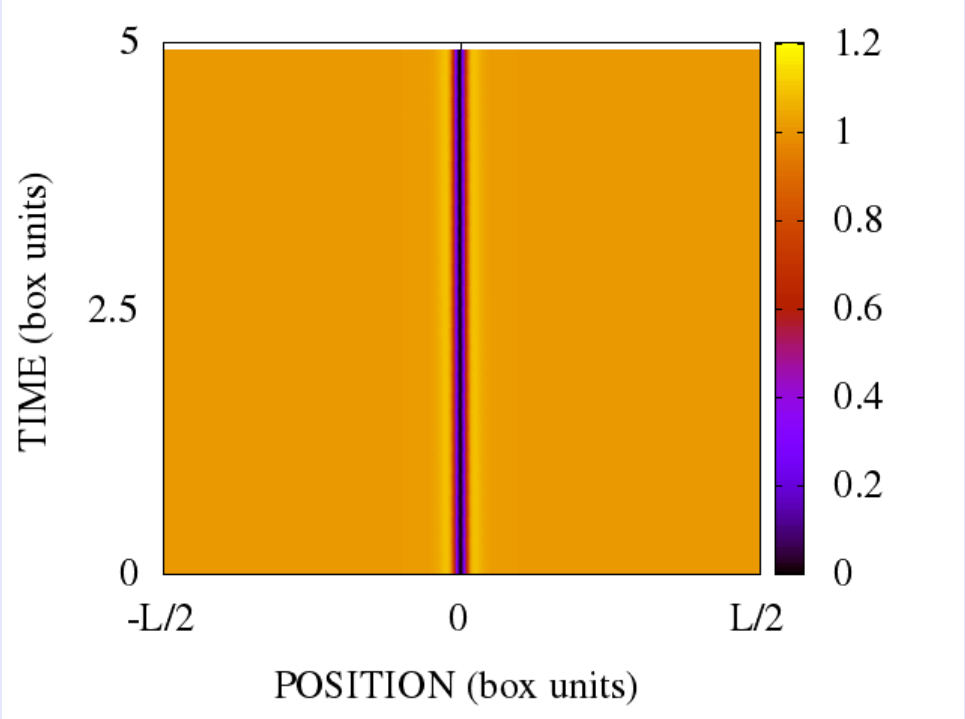

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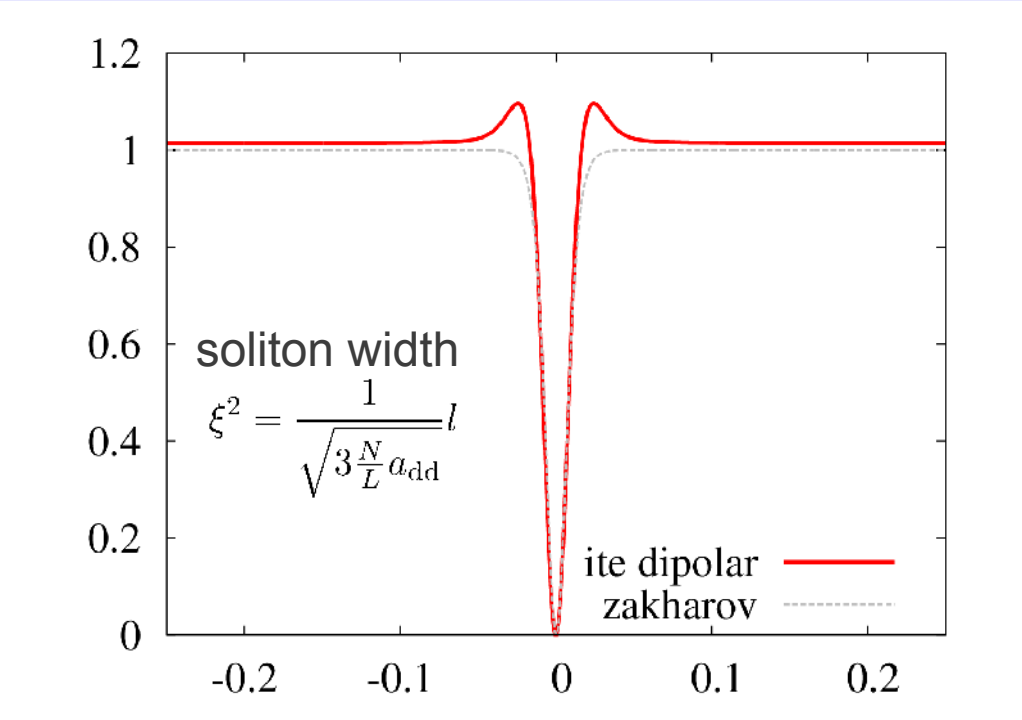
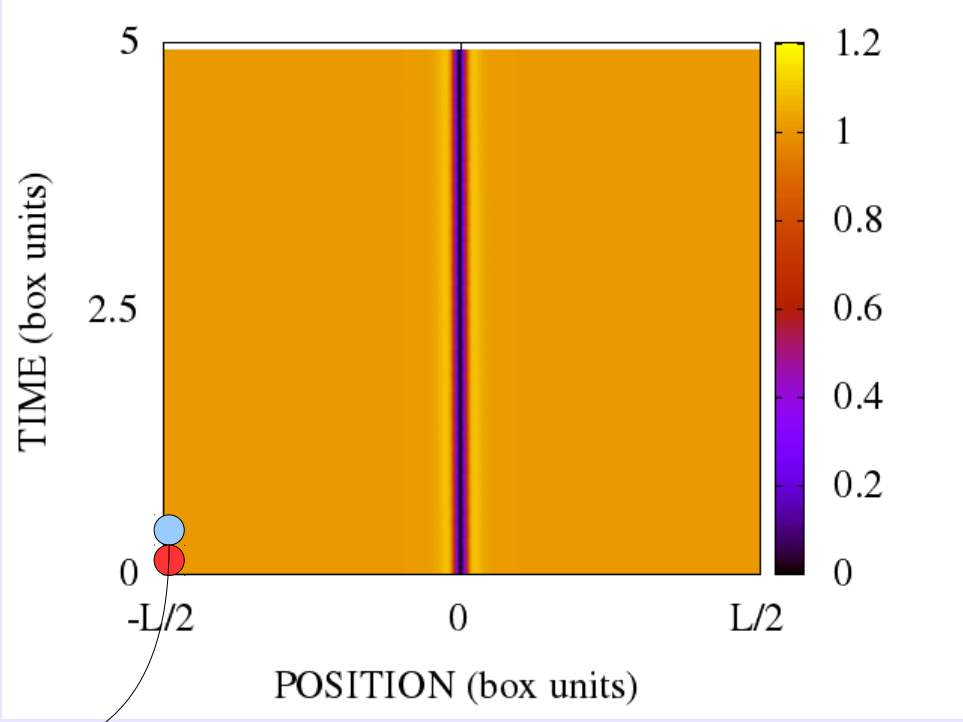




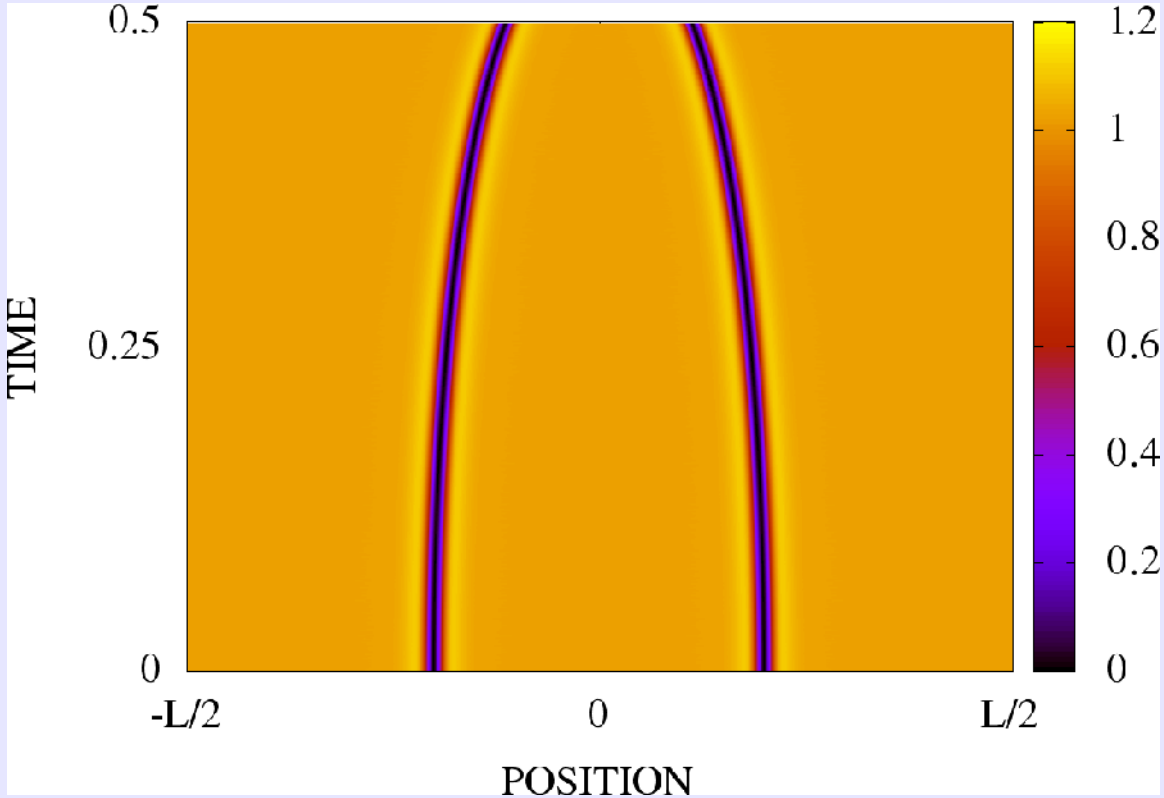
$1/\omega = (\lambda)_{\text{box units}} = 0.01$

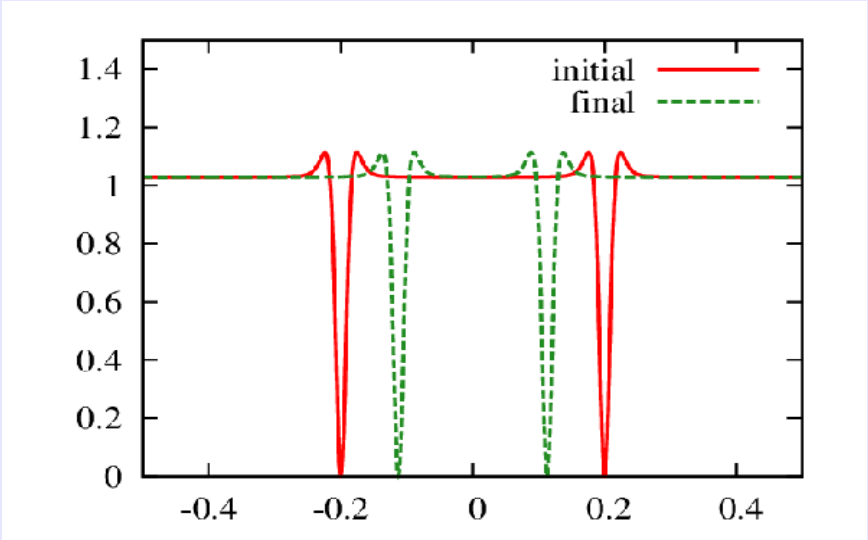
$t_{phon} = L / c = 0.16$

Dysprosium
$\omega_T = 2\pi$ (5 kHz)
N = 600
t_{phon} = 50ms

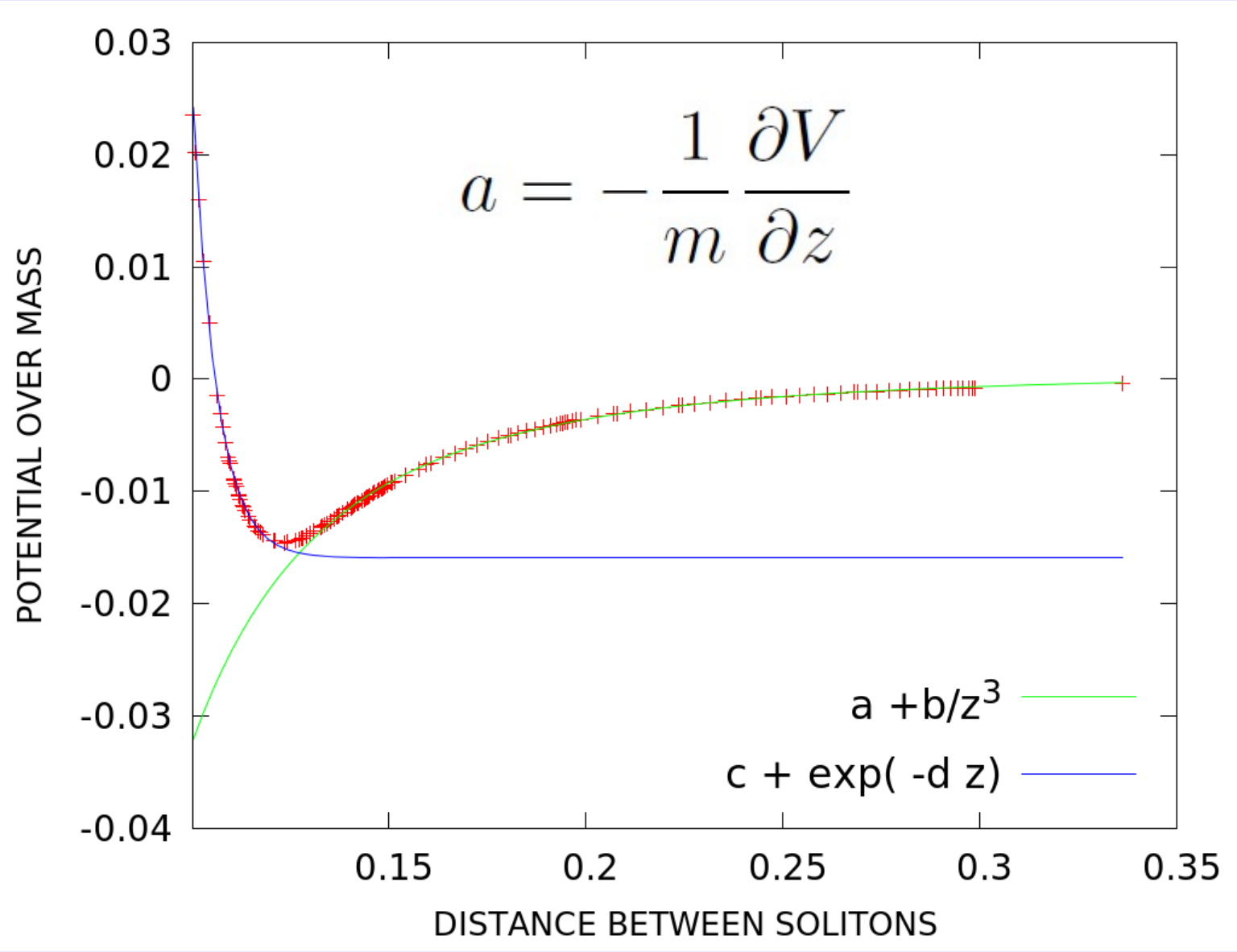
(rotating frame, such that periodic boundary conditions are matching)

EVOLUTION

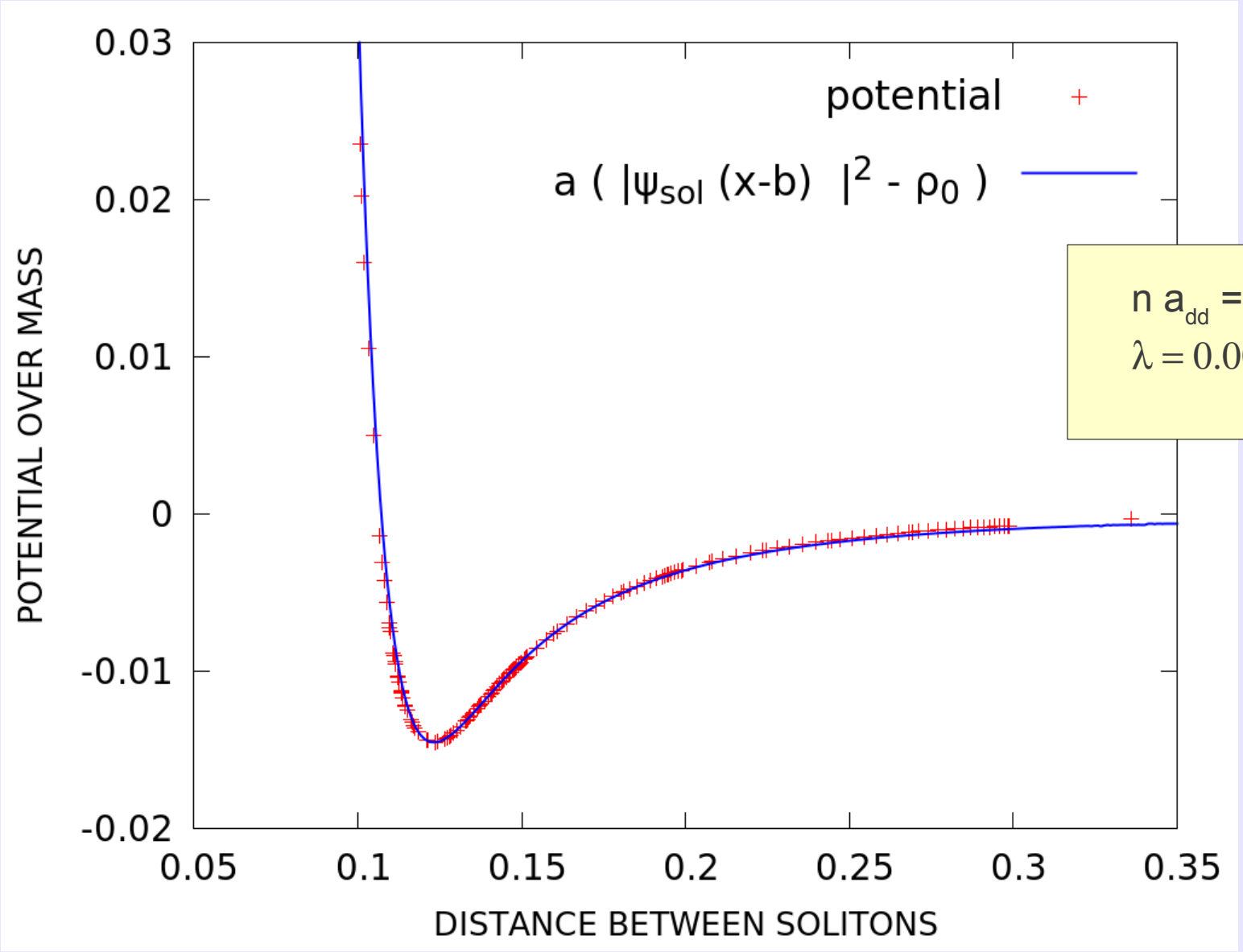

0.5
0.25
0
TIME
-L/2
0
L/2
POSITION
1.2
1
0.8
0.6
0.4
0.2
0


initial
final
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.4
-0.2
0
0.2
0.4

Intersolitonic potential: scaling with distance



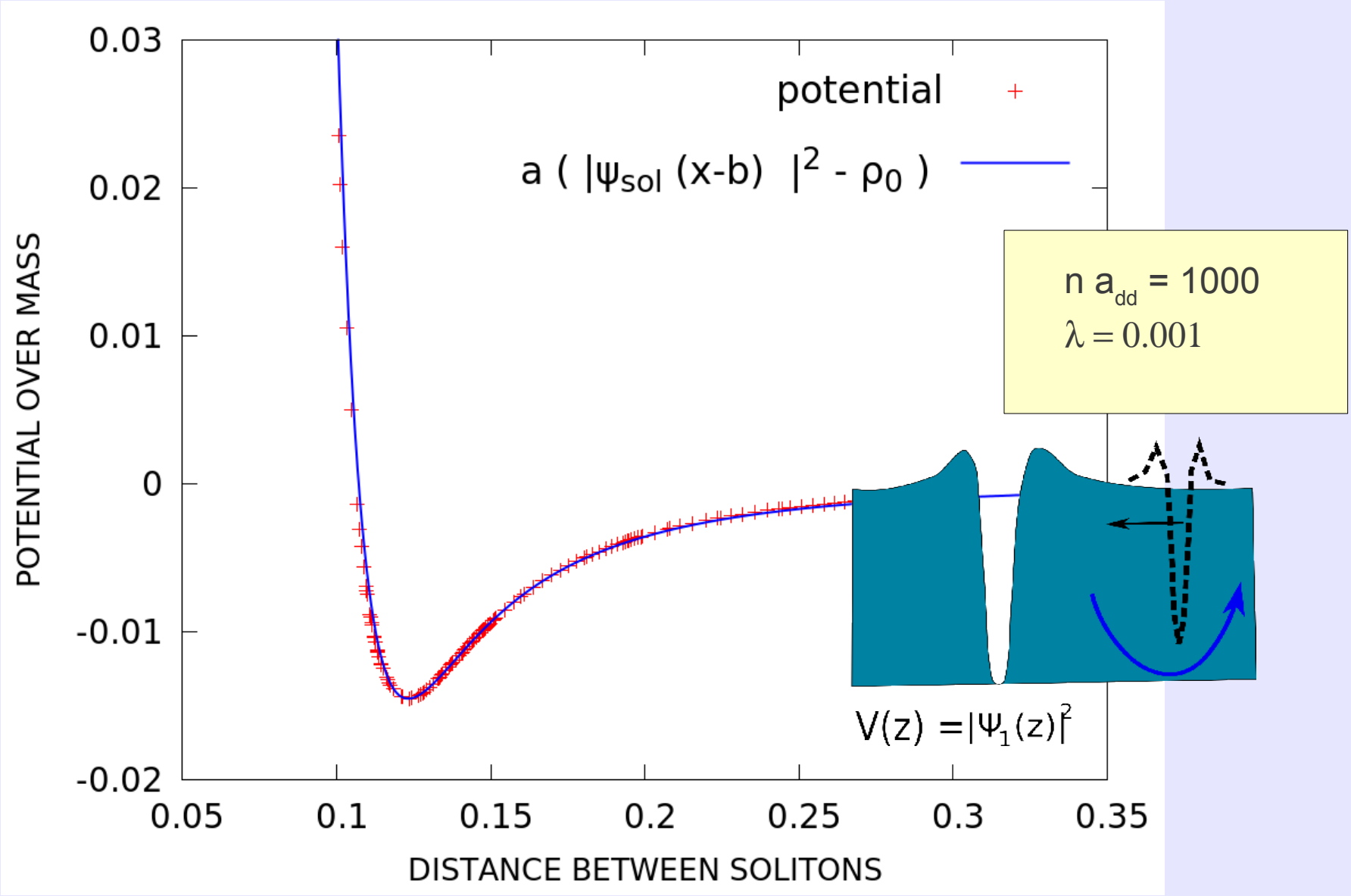
Intersolitonic potential : comparison with soliton density



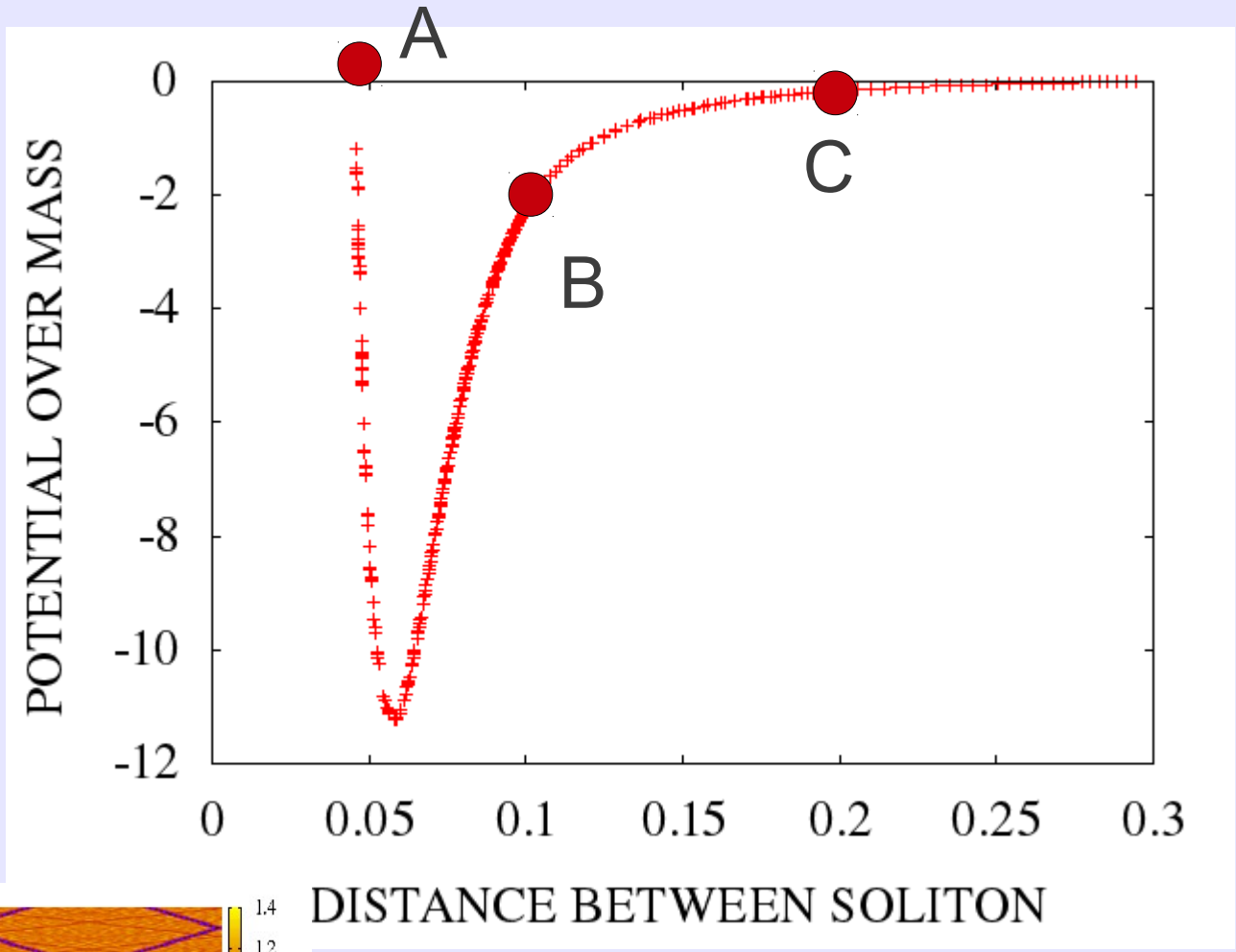
$n\, a_{dd} = 1000$

$\lambda = 0.001$

Intersolitonic potential : comparison with soliton density

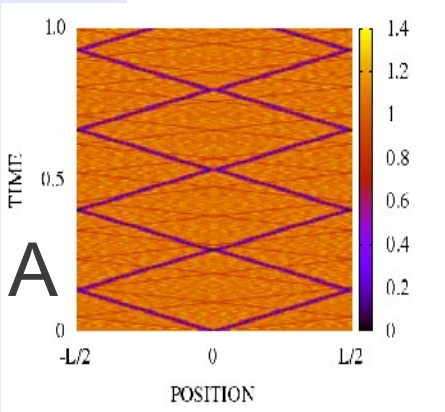


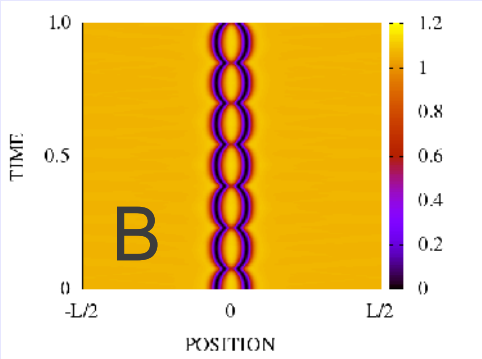
Other parameters: testing



$n\, a_{dd} = 0.1$

$\lambda = 0.01$





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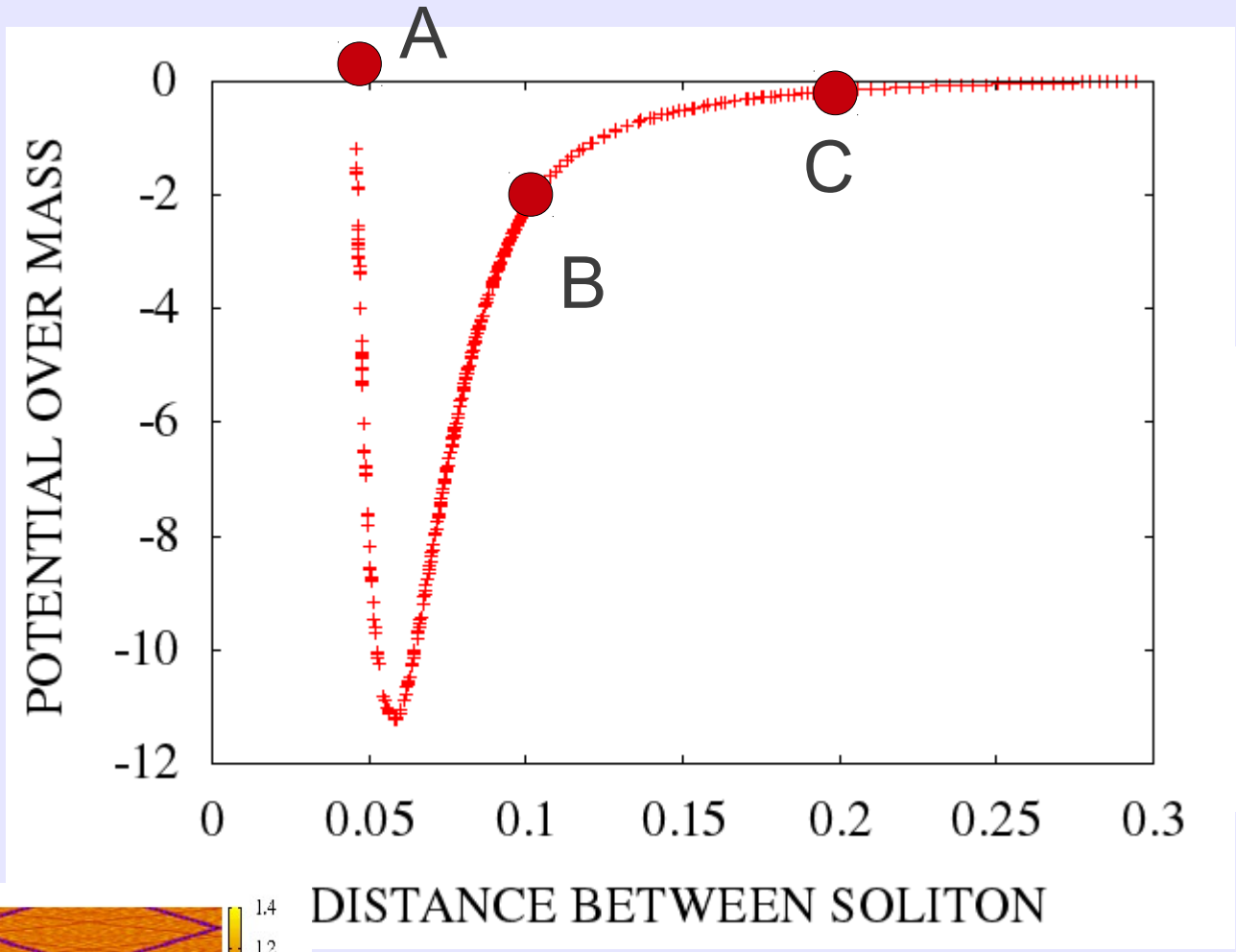

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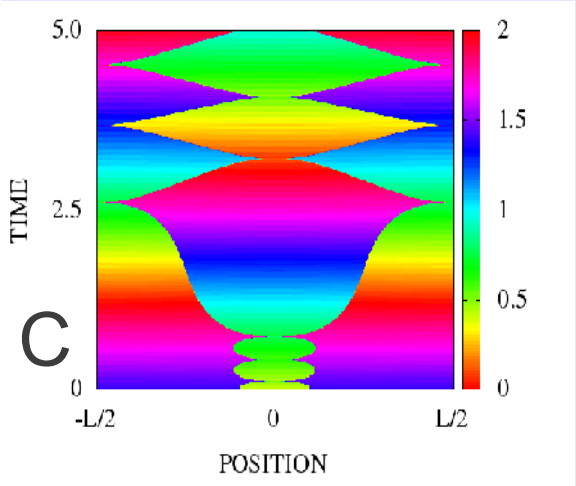

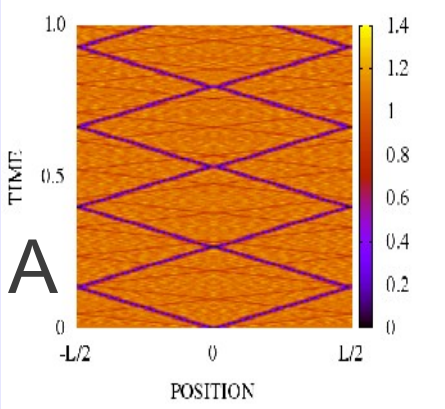

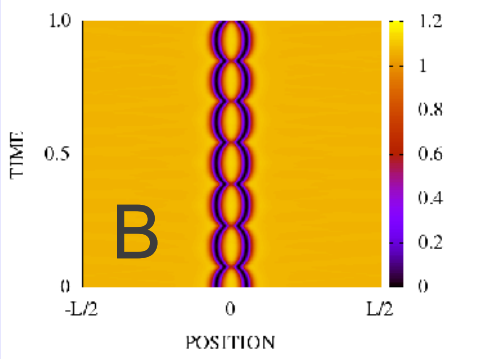

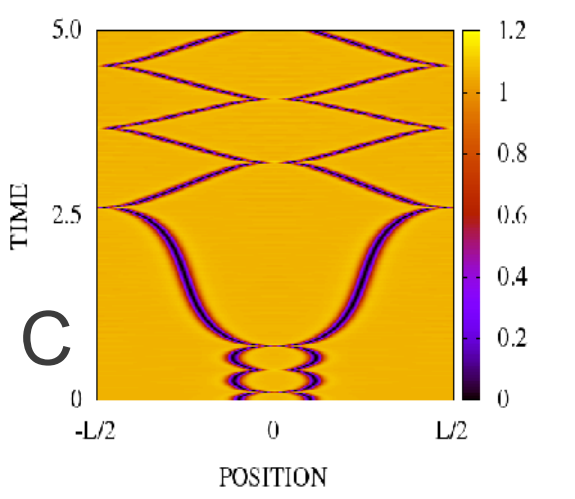

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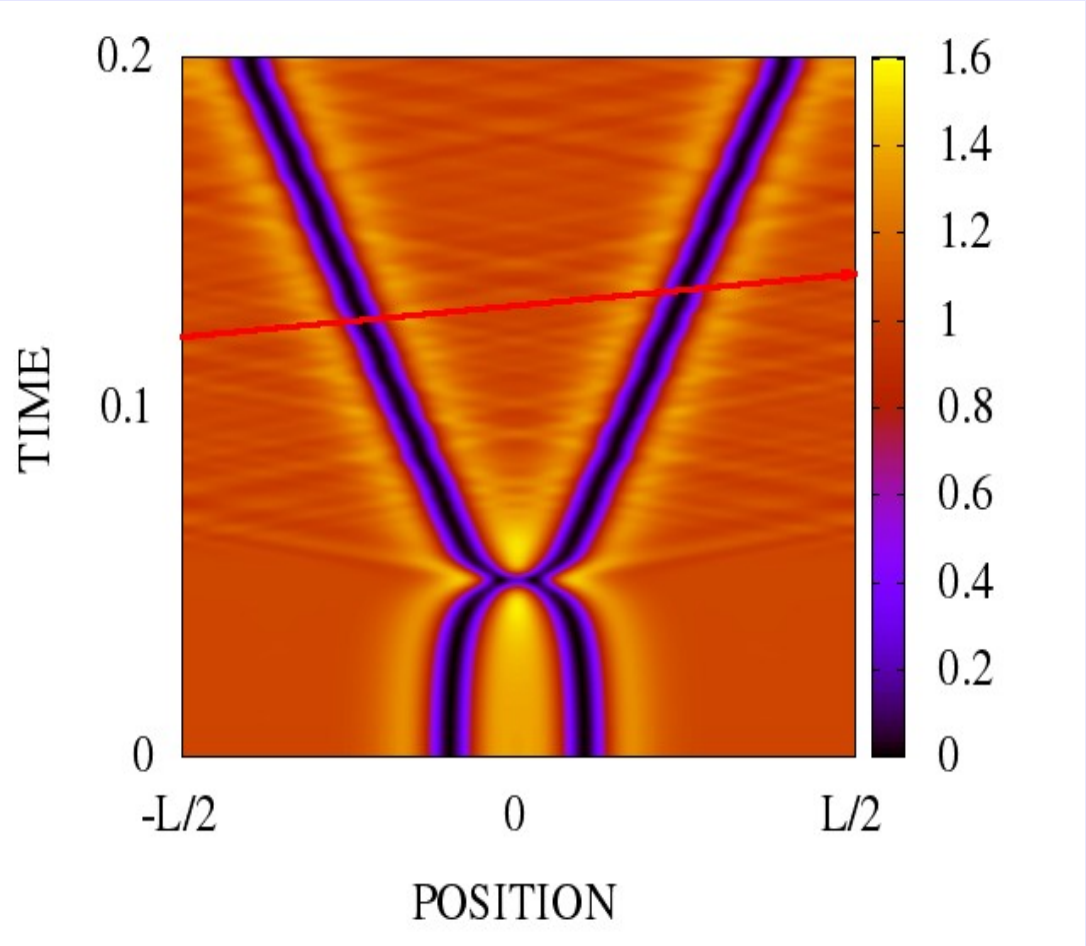
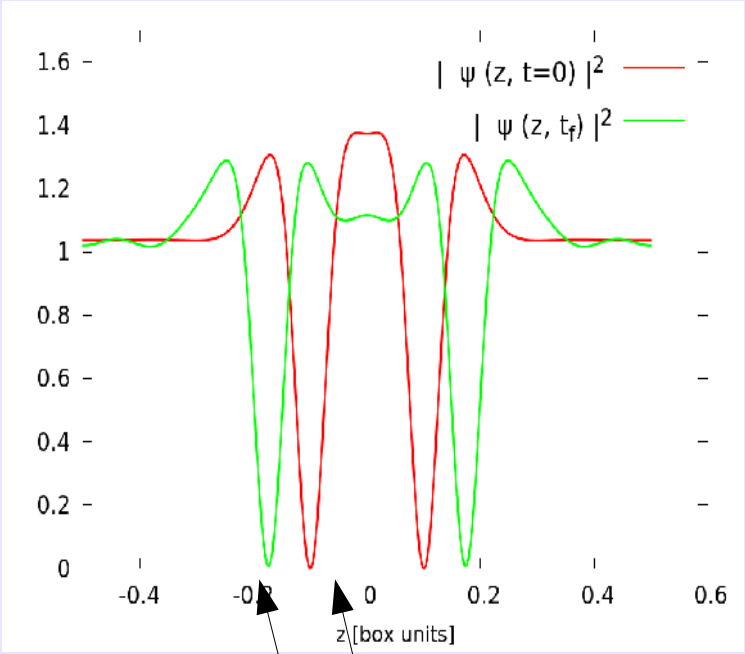




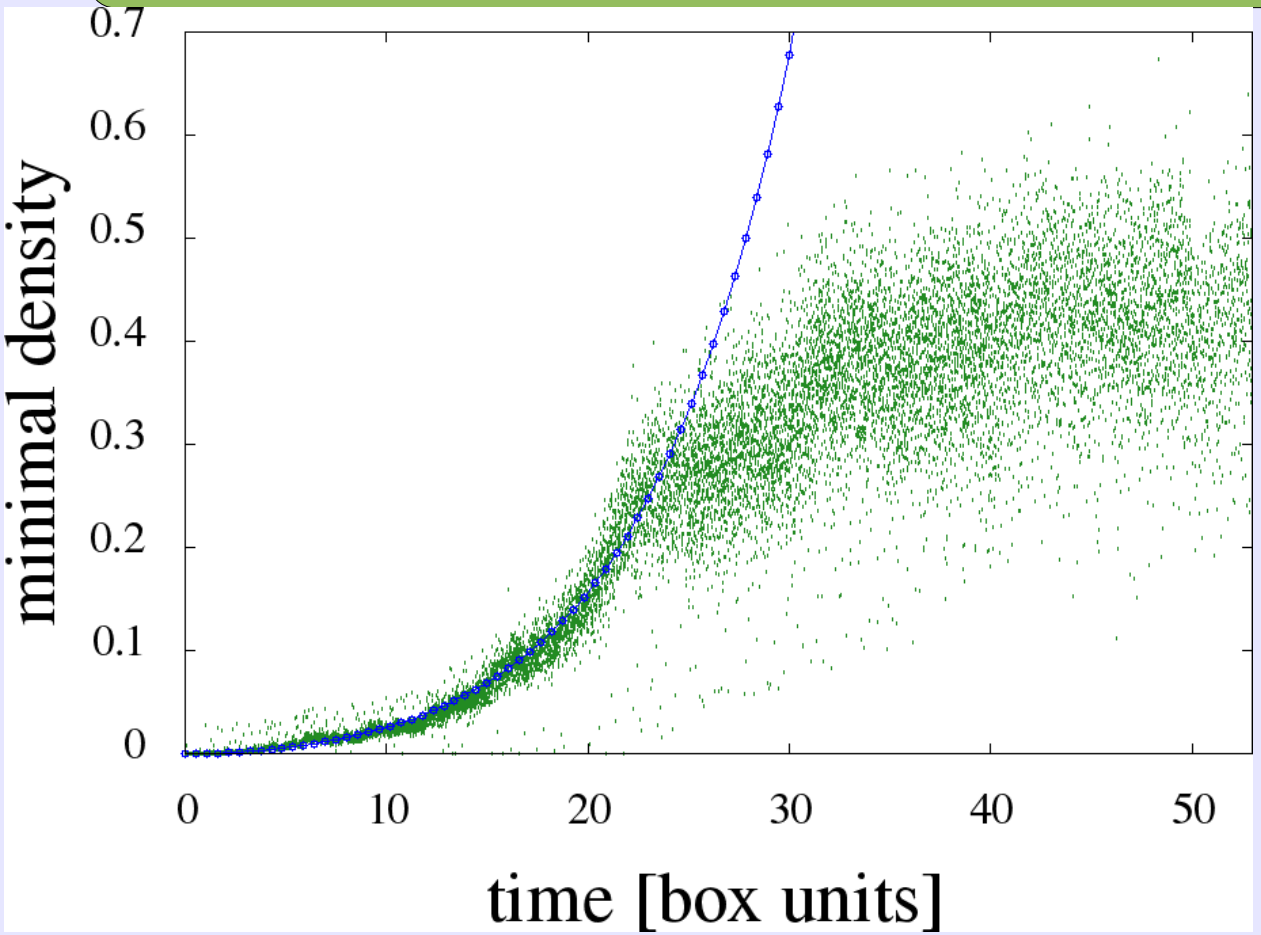
$< 10^{-7}$

0.007

solitonic height

inelastic collisions → radiation
→ gray solitons → accelaration

Two solitons : long evolution



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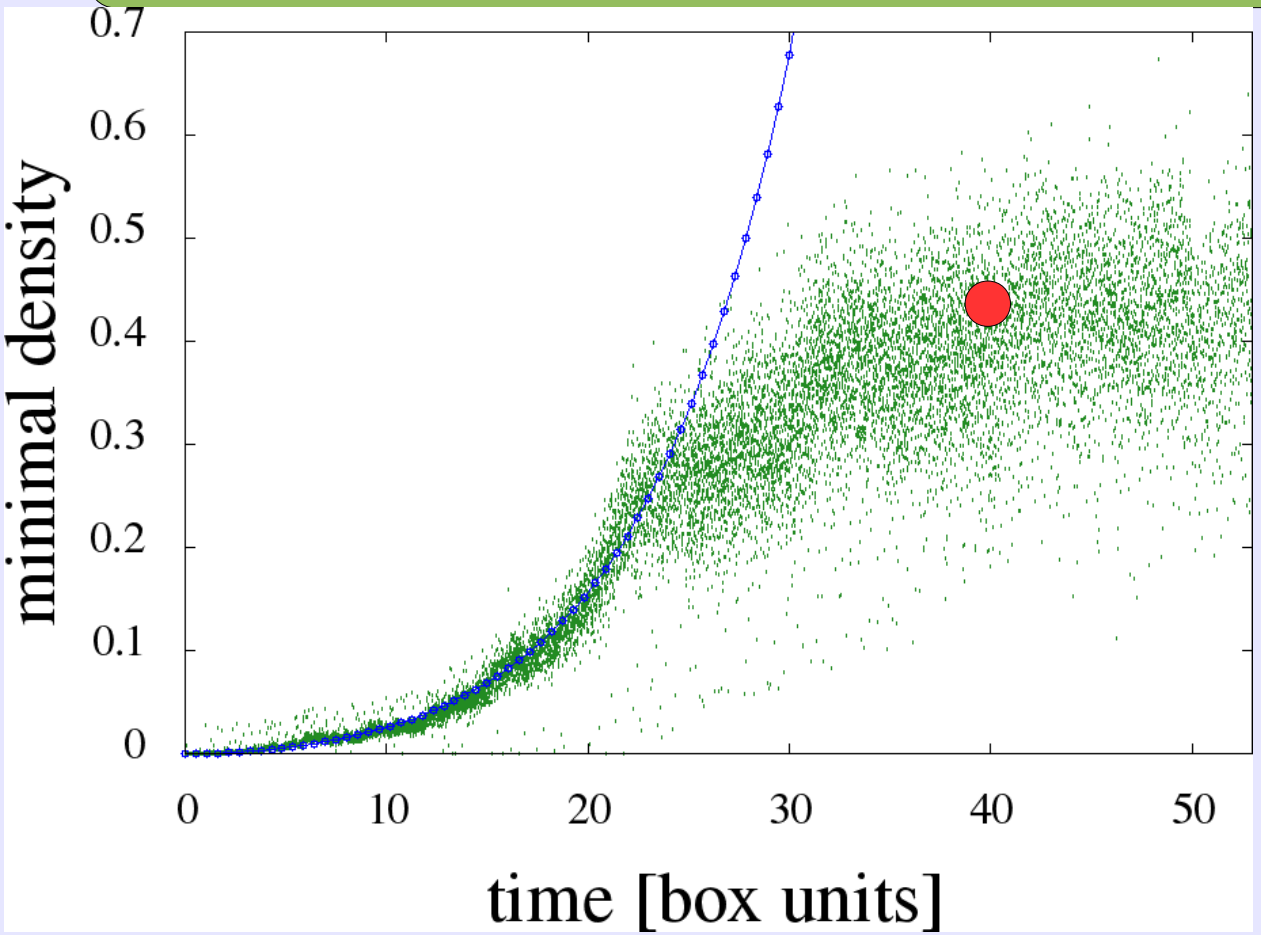
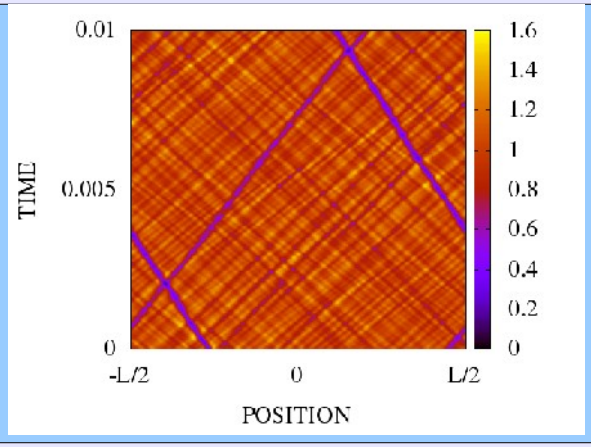


Phenomenological picture:

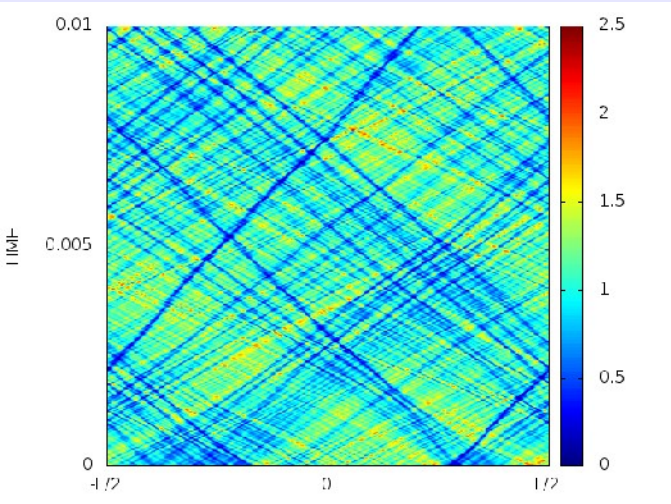
- N atoms,
- initially in an excited level,
- interacting with small environment.

,,zoom" onto evolution around t=40

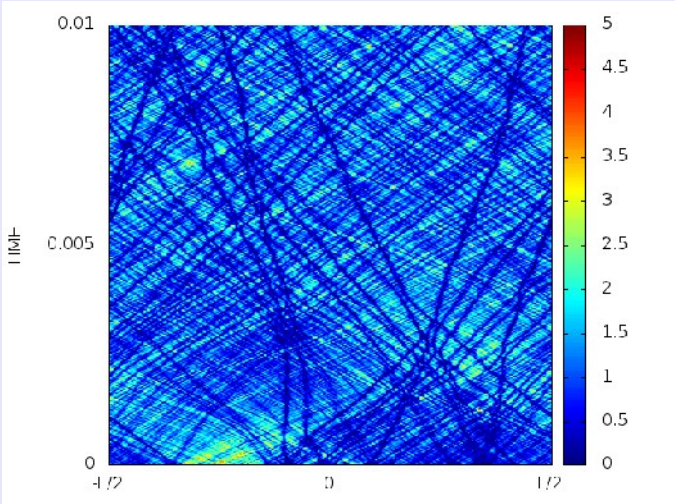
between t=40 and t=41



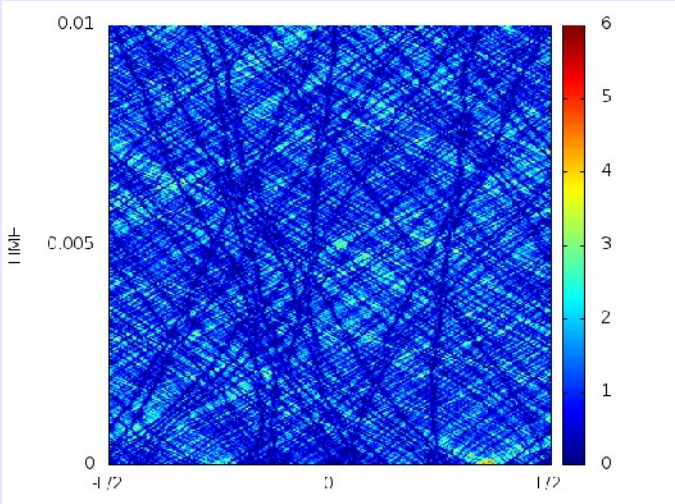
FINITE TEMPERATURES: naïve approach



$T = 10^4$ box u.



$T = 5 * 10^4$ box u.



$T = 10^5$ box u.

N = 1000

$n\, a_{dd} = 2$

$\lambda = 0.01$

Summary

Shapes of solitons

Shalow Inter-solitonic potential

Non-elastic collision → acceleration of solitons

In progress:
The solitons are present in thermal samples

STABILITY OF SOLITONS (radiation)

Solitons vs rotons