Laguerre-Gaussian laser modes for atomic physics
Atom channeling and information storage

QTC 2015

Laurence PRUVOST
Laboratoire Aimé Cotton, CNRS,
Univ. Paris-Sud,
ENS Cachan
Orsay, France
Twisted light: Atom channeling and information storage

**Introduction**: twisted light and orbital angular momentum

**Laguerre-Gaussian modes**
- properties.
- preparation, methods to detect

**Use of the ring shape**
- cold atoms: LG-Channelled-2D MOT

**Use of the quantum number of the phase**
- generalities
- OAM storage/retrieval in cold atoms by Four Wave Mixing and Coherent Population Oscillation

**Conclusion**
Twisted light /vortex beam

**Def:** EM wave imprinted/carrying an helical phase

![Twisted light/vortex beam diagram]

**Properties**

- Fresnel diffraction and symmetry dark center for $\ell \neq 0$
- Helical phase Orbital angular momentum (OAM), $L=\hbar \ell$
  - $\ell$ quantum number of the beam
- Classed in families of solutions of the Helmholtz (paraxial) equation
  - ex. Laguerre-Gaussian modes
Orbital Angular Momentum

- The associated OAM is \( \ell \).
- \( \ell \) characterizes how the phase turns.
- OAM quantized with the signed integer \( \ell \), varying from \(-\infty\) to \(+\infty\).
  \( \ell \) also called mode order, or azimuthal number.

OAM differs from the linear momentum \( k \) and the polarization (SAM)
Laguerre-Gaussian modes
Laguerre-Gauss Modes

Eigen solutions of the paraxial wave equation

$$\nabla_\perp^2 A - 2ik \frac{\partial A}{\partial z} = 0$$

Hermite-Gauss TEM$_{mn}$
Cartesian coordinates

Laguerre-Gauss LG$_{\ell p}^\ell$
cylindrical coordinates
Eigen solutions of the paraxial wave equation

\[ \nabla^2 A - 2i k \frac{\partial A}{\partial z} = 0 \]

- Propagates keeping the shape
- LG modes constitute a basis

\[ |\Psi_{\text{light}}\rangle = \sum_{\ell, p} a_{\ell, p} |LG^{\ell}_p\rangle \]

- \(LG^{\ell}_p\)
  - \(\ell\): OAM, azimuthal number
  - \(p\): radial number
Inside the Gaussian envelope, the intensity has $p+1$ rings.

- The center of the $p=0$ varies $\rho^{2\ell}$
  - power-law

- $\ell = 1$ harmonic
- $\ell \gg 1$ squared
\[ E = \frac{C_{p, l}^{LG}}{w(z)} \left( \frac{\rho \sqrt{2}}{w(z)} \right)^{|l|} L_p^{|l|} \left[ \frac{2\rho^2}{w(z)^2} \right] \exp \left[ -\frac{\rho^2}{w(z)^2} \right] \exp \left[ \frac{ik\rho^2}{2R(z)} - i\phi_{2p,|l|}(z) + il\theta \right] \]

- **Amplitude factor**
- **Gaussian envelope**
- **Wavefront curvature**
- **Helical phase**
- **Gouy phase**

- **Phase** \( \exp[il\theta] \). The electric field is angular dependent and changes of sign \( \ell \) times (figure for \( \ell=4, p=0,1,2,3 \))

**Rings of helical phases**
LG preparation and detection
LG generation

- Imprint a **helical phase** to an input beam

Use of a spatial light modulator (**SLM**) as phase mask
Spatial Light Modulator. Phase-only SLM

• SLM = Programmable 2D optical component able to modify the amplitude and/or the phase of the light at each point of its surface.
  • Micro mirror devices, deformed mirrors, liquid crystal devices

• In liquid-crystal SLM, the LC molecules are oriented by an electric field map, **changing locally the birefringence** (the index) and thus the phase of a beam going through.

SLM Hamamatsu, active area: 2cm x2 cm
Our setup

Detected on the CCD

$\ell = 10$

$\ell = 100$

Rem: very dark center
- LG $\ell=10$ $p=0$?
- Very dark center, close to $\rho^{2\ell}$ shape

- Thin principal ring
- extra rings due $p \neq 0$ components
Other methods of fabrication

- dark spot (absorbent) in a laser cavity
- conversion of HG to LG using a set of cylindrical lenses

- amplitude mask, being a fork pattern
  
  *picture: mask used for electron vortex beams.*

- wavefront imprinting by a vortex phase,
  by a shaped plate
  by an holographic plate
  by SLM \( \ell \) is easily changed
Methods for OAM detection

Ring shape not enough. Phase detection needs interferences

1. Phase analyser

Phase analyzer, Shark-Hartmann or micro lenses

2. Interference with a reference beam

Twisted pattern to analyzed

3. Diffraction by an aperture (double slit, triangle, wheel-hole… )
OAM detection

4. Self-interferences with a astigmatic system

- As the lens is turned:
- OAM determined by the fringe number:

LG modes use

Ring shape

- STED microscopy
cf Hell, Moermer
- Particule traps=optical tweezers
cf Grier,
- Dipole potential (squared)
cf Haebbebics

\[ U > 0 \]

Atoms in the LG dark center

OAM (phase)

- Transfer to objects i.e. rotation
cf Grier,
- Transfer to atomic waves
cf Philips
- Optics / Non-linear optics
cf Padgett, Zeilinger
- Information encoding
cf Wang, Tamburini, Bo Thidé

\[ \ell = 10, 7, 13 \]
superposition
Use of ring shape
LG-2D-MOT

V Carrat, C Cabrera, M Jacquey, J R W Tabosa, B Viaris de Lesegno, LP

Experiment done in Orsay (France)

cf C. Cabrera talk, Thursday 10:40
“Light tube” to channel the atoms

- Atoms exiting a 2D-MOT used to load a 3D-MOT
- Channel the atoms, reduces the divergence and increases the density, reduce the capture zone and efficient loading
Use of the helical phase
Storage/retrieval of OAMs in cold atoms


Experiments done in Recife (Brazil)
Context

- Orbital angular momentum of light as a variable for encoding
- Using an atomic system— as simple as possible - many groups explore the writing, storing, reading OAM.
- Atoms are a simple model to experience and explain the involved processes.

See Also: E Giacobino & J Laurat, Paris, France, Nat, Phot, 8, 234, 2014
G-C Guo, Hefei 230026, China, Nat, Com, 4, 252, 2013
S Franke-Arnold, Glasgow, UK,
Principle of the OAM storage/retrieval

- The information is encoded on $W$ (or $W'$) beam (Laguerre-Gaussian mode).
- The memory is a cold atom sample.

- The information is retrieved via **four wave mixing** (FWM) process.

\[
\begin{align*}
\text{Writing} & \quad \text{Reading}
\end{align*}
\]
Interaction with the atom: $\Lambda$ system. Dark state

- Atom submitted to 2 lasers $W$, $W'$ each realizing a transition. The Rabi frequencies are $\Omega$ and $\Omega'$.
- Interaction matrix in the initial natural basis $g_1$, $g_2$, $e$ is $V_{(g_1,g_2,e)}$.
- In a new basis (dark, bright, $e$)

\[
V_{(g_1,g_2,e)} = \begin{pmatrix}
0 & 0 & \Omega \\
0 & 0 & \Omega' \\
\Omega & \Omega' & 0
\end{pmatrix}
\]

\[
|\text{dark}\rangle = \frac{\Omega'}{\sqrt{\Omega^2 + \Omega'^2}} |g_1\rangle - \frac{\Omega}{\sqrt{\Omega^2 + \Omega'^2}} |g_2\rangle
\]

\[
|\text{bright}\rangle = \frac{\Omega}{\sqrt{\Omega^2 + \Omega'^2}} |g_1\rangle + \frac{\Omega'}{\sqrt{\Omega^2 + \Omega'^2}} |g_2\rangle
\]

the dark state (DS) is not coupled to $e$ state. Any atom falling into the DS, remains in it and becomes insensitive to the light.
Phase sensitive interaction of the dark state

The DS combines 2 ground states, having a \textbf{long lifetime}. In principle, it lives for a long time.

- the DS combines the Rabi frequencies $\Omega$, $\Omega'$. So it contains information carried by $\Omega$ and $\Omega'$.
- It contains the relative phase of $W$ and $W'$, so the OAM.

$$|\text{dark}\rangle = \frac{\Omega'}{\sqrt{\Omega^2 + \Omega'^2}} |g1\rangle - \frac{\Omega}{\sqrt{\Omega^2 + \Omega'^2}} |g2\rangle$$

$$\Omega = \mu E_w$$

$$\Omega' = \mu E_{w'}$$
Another point of view:

- The interference of $W$ and $W'$ making an angle $\theta$, creates a fork coherence pattern, imprinted in the cold atom sample.

- The reading beam $R$ diffracts on the fork pattern.

- The emitted beam (C) acquires the OAM and propagates with an angle $\theta$ with $R$. 

Coherence phase pattern:

$$\Lambda = \frac{2\pi}{|k_F - k_P|} = \frac{\lambda}{2\sin(\theta/2)}$$
The diffracted beam C is emitted in a direction different from the initial input direction.

Conservation of the OAM is observed without or with time delay.

The angle $\theta$ is small ($2^\circ$) & $\ell$ is small.
Delayed FWM realizes OAM storage/retrieval in cold atoms

- Storage of angular momentum of light OAM and collinear and Off-axis retrieval.
- Revieval using Larmor oscillations

OAM storage/ retrieval with CPO

Polarisation configurations

EIT : $\sigma^+ / \sigma^-$

CPO : lin $\perp$ lin (Coherent Population Oscillations)

$$\text{lin} = \sigma^+ + \sigma^-$$

$$\perp\text{lin} = \sigma^+ - \sigma^-$$

Eq. to 2 two-levels systems
OAM storage/ retrieval with CPO

Polarisation configurations

EIT : $\sigma^+ - \sigma^-$

CPO : $\text{lin} \perp \text{lin}$

CPO efficient and robust against magnetic field
EIT/ CPO memories

- CPO robust against magnetic field
- CPO storage longer than EIT ones
- Also observed in hot vapour
  Cs: Tabosa, 2014
  He: Laupretre, Goldfarb 2014
OAM storage / retrieval with CPO

- One beam carrying OAM
- Collinear and off-axis case
OAM storage / retrieval with CPO

- If two beams carry OAM
  \[ \mathcal{E}_D \propto \chi^{(3)} \mathcal{E}_W \mathcal{E}_W^* \mathcal{E}_R \]
- Phase matching: \( \ell_{\text{out}} = \ell_{w'} - \ell_w \)

Conclusion

- Ring shape for dipole potentials and atom manipulation.
- OAM for encoding information
  Cold atoms for storage/retrieval.
  FWM, CPO processes available
  CPO robust against magnetic field
- Use both $\ell$, $p$ variables?
- Next? Other transitions?
  OAM in two--photon transition
M Jacquey

J Tabosa (Recife)

B. Viaris
C Cabrera
V Carrat
LP