Adiabatic Preparation Of A Heisenberg Antiferromagnet Using An Optical Superlattice
(arXiv:1106.1628)

Michael Lubasch\textsuperscript{1}, Valentin Murg\textsuperscript{2}, Ulrich Schneider\textsuperscript{3}, J. Ignacio Cirac\textsuperscript{1}, Mari-Carmen Bañuls\textsuperscript{1}

\textsuperscript{1}Max Planck Institute of Quantum Optics, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany
\textsuperscript{2}University of Vienna, Faculty of Physics, Boltzmanngasse 5, 1090 Vienna, Austria
\textsuperscript{3}Ludwig-Maximilians-University Munich, Faculty of Physics, Schellingstrasse 4, 80799 Munich, Germany

31 August 2011
Outline

Motivation
High-$T_c$ Superconductivity
From Hubbard To $t - J$ Model

Our Experimental Proposal
The Adiabatic Protocol
No Holes & No Trap
Holes & Trap

Summary
Conclusions & Perspectives
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High-$T_c$ Superconductivity

Historical Remarks

- 1986: Georg Bednorz and Karl Mueller discover that LaBaCuO has $T_c = 30$ K, LaSrCuO discovered as another high-$T_c$ superconductor
- 1987: YBCO found to have $T_c = 90$ K
- 1988: BSCCO found having $T_c = 108$ K, TBCCO with $T_c = 127$ K
- 1993: $T_c = 135$ K realized with HgBaCaCuO, having $T_c = 164$ K under high pressure
High-$T_c$ Superconductivity

Phase Diagram Of The Cuprates

- parent compound $\text{La}_2\text{CuO}_4$ is a Mott insulator featuring long-range antiferromagnetic order below Néel temperature $T_N \approx 300$ K
- by substituting trivalent La by divalent Sr, $x$ holes are added to the CuO planes in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$
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High-$T_c$ Superconductivity

Effective Description

high-$T_c$ superconducting cuprate = doped Mott insulator ?

[Lee, Nagaosa & Wen, RMP'06]
From Hubbard To $t-J$ Model

Hubbard Model

$$\hat{H} = -t \sum_{\langle l,m \rangle, \sigma} (c^\dagger_{l,\sigma} c_{m,\sigma} + c^\dagger_{m,\sigma} c_{l,\sigma}) + U \sum_l \hat{n}_{l,\uparrow} \hat{n}_{l,\downarrow}$$

- $t$: hopping; $U$: on-site interaction
- effective model for electrons in a solid
- describes ultracold atoms in optical lattices
From Hubbard To $t - J$ Model

$t - J$ Model

- $t - J$ model is obtained from Hubbard model below half filling in the strong interaction limit $U \gg t$:

$$\hat{H} = -t \sum_{\langle l,m \rangle, \sigma} (\tilde{c}^{\dagger}_{l,\sigma} \tilde{c}_{m,\sigma} + \tilde{c}^{\dagger}_{m,\sigma} \tilde{c}_{l,\sigma}) + J \sum_{\langle l,m \rangle} (\vec{S}_l \vec{S}_m - \frac{\hat{n}_l \hat{n}_m}{4})$$

- $\tilde{c}_{l,\sigma} := (1 - \hat{n}_{l,\sigma})c_{l,\sigma}$ excludes double occupancy
- superexchange interaction $J := 4t^2/U$
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The Adiabatic Protocol

Experimental Status Quo

- recent experimental realization of the fermionic Hubbard model in an optical lattice [U. Schneider et al., Science 322, 1520 (2008)]:

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1. band insulating ground state $|BI\rangle$ of deep optical lattice:

2. dimerized ground state of double well lattice [S. Trotzky et al., PRL 105, 265303 (2010)]:
   $|\psi(\tau = 0)\rangle = |\phi\rangle_{L/2}$ with
   $|\phi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

3. quantum Heisenberg antiferromagnet:
   $|\psi(\tau = T)\rangle = |AFM\rangle$

Advantage compared to direct generation of Mott insulator:

- Band insulator can be created with less entropy!
- Band insulator has a larger gap, namely the bandgap, than Mott insulator, which has a gap $U$
- Band insulator can be realized with non-interacting fermions which are more mobile.
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Experimental Observables

• squared staggered magnetization [G. M. Bruun et al., PRA 80, 033622 (2009)]:

\[ \langle M_{stag}^2 \rangle = \frac{1}{N^2} \sum_{i,j=1}^{N} (-1)^{i+j} \left\langle \vec{S}_i \cdot \vec{S}_j \right\rangle \]

• 2D value in the thermodynamic limit: \( M_{stag}^2 \approx 0.095 \) [A. W. Sandvik and H. G. Evertz, PRB 82, 024407 (2010)]

• singlet fraction [S. Trotzky et al., PRL 105, 265303 (2010)]:

\[ \langle P_0 \rangle = \frac{1}{4} - \left\langle \vec{S}_l \cdot \vec{S}_{l+1} \right\rangle \]

• 1D value in the thermodynamic limit: \( \langle P_0 \rangle \approx 0.693 \)

• 2D value in the thermodynamic limit: \( \langle P_0 \rangle \approx 0.585 \)
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No Holes & No Trap

\[ E = E_{\text{spin}} = J \sum_{\langle l,m \rangle} \vec{S}_l \cdot \vec{S}_m \]

Adiabaticity Of The Total Lattice Of Size \( N \) In 1D

- experimental observable: squared staggered magnetization
  \[ \langle M^2_{\text{stag}} \rangle = \frac{1}{N^2} \sum_{i,j=1}^{N} (-1)^{i+j} \langle \vec{S}_i \cdot \vec{S}_j \rangle: m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}} \]

- 1D: gap closes at the end of the adiabatic protocol [M. Matsumoto et al., PRB 65, 014407 (2001)]
- Landau-Zener formula: \( T \propto 1/\Delta^2 \)
- 1D: \( \Delta \propto 1/N \rightarrow T \propto N^2 \)
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- nevertheless a high magnetization value is obtained in a short ramping time
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Adiabaticity Of A Sublattice Of Size \( L \) In 1D

- experimental observable: squared staggered magnetization
  \[ \langle M_{\text{stag}}^2 \rangle = \frac{1}{N^2} \sum_{i,j=1}^{L} (-1)^{i+j} \langle \vec{S}_i \cdot \vec{S}_j \rangle: m^2(T) := M_{\text{stag}}^2(T)/M_{\text{stag,AFM}}^2 \]

- adiabaticity of a sublattice of length \( L \) is governed by an effective local gap: \( \Delta \propto 1/L \quad \rightarrow \quad T \propto L^2 \)

- given a large sample, high magnetization values can be obtained in short ramping times on small parts of the system \( L \propto \sqrt{T} \)
No Holes & No Trap

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\[ E = E_{\text{kin}} + E_{\text{spin}} = -t \sum_{\langle l,m \rangle, \sigma} (\tilde{c}_{l,\sigma} \tilde{c}_{m,\sigma} + \tilde{c}_{m,\sigma} \tilde{c}_{l,\sigma}) + E_{\text{spin}} \]

- 1D: hole spreads over antiferromagnetic background like free particle with velocity \( v = 2t \)
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Holes & No Trap

- energy is increased by $\Delta E_{\text{spin}} \approx |\langle \vec{S}_l \cdot \vec{S}_{l+1} \rangle|$
- hole reduces staggered magnetization substantially
- hole propagation like in $t - J_z$ model:

$$E_{\text{spin}} = J_z \sum_{\langle l,m \rangle} (S_l^z \otimes S_m^z) + \frac{J_\perp}{2} \sum_{\langle l,m \rangle} (S_l^+ \otimes S_m^- + S_l^- \otimes S_m^+)$$

a) b)

- effect of holes during adiabatic ramping:
Holes & No Trap

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- hole propagation like in $t - J_z$ model:

$$E_{\text{spin}} = J_z \sum_{\langle l,m \rangle} (S^z_l \otimes S^z_m) + \frac{J_\perp}{2} \sum_{\langle l,m \rangle} (S^+_l \otimes S^-_m + S^-_l \otimes S^+_m)$$

- effect of holes during adiabatic ramping:

$$m^2(L=42)$$

- 2 holes on 84 sites
- 4 holes on 86 sites
Holes & No Trap

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• effect of holes during adiabatic ramping:

a) b)
Holes & Trap

\[ E = E_{\text{kin}} + E_{\text{spin}} + E_{\text{trap}} = E_{\text{kin}} + E_{\text{spin}} + V_t \sum_l (l - l_0)^2 \hat{n}_l \]

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\[ \begin{array}{cccc}
\text{m}^2 (L=82) & \text{T} & \text{V=0.004J} & \text{V=0.006J} & \text{V=0.02J} \\
0 & 10 & 20 & 30 & 40 & 50
\end{array} \]

\[ \begin{array}{cccc}
\text{T(m}^2 = 0.85) & \text{sublattice size L} & \text{V=0.004J} & \text{V=0.006J} & \text{V=0.02J} \\
0 & 10 & 20 & 30 & 40 & 50
\end{array} \]

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Conclusions & Perspectives

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- the proposed adiabatic protocol reveals realistic timescales
- adiabaticity of a sublattice is governed only by its size
- the highly destructive effect of holes on the AFM can be controlled by the trap
- the initial band insulating ground state has particularly low entropy
- already the initial ground state features SU(2) symmetry
- the only crucial gap is the one of the final Heisenberg antiferromagnet

Perspectives

- controlled hole doping: phase diagram of $t - J$ model
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Appendix

Experimental Entropy Distribution

- the harmonic trap allows to push the entropy to the outside of the sample [U. Schneider et al., Science 322, 1520 (2008)]
- experimental entropies per particle are a factor 2 above the necessary \( \frac{\ln(2)}{2} \) for the Heisenberg antiferromagnet at Néel temperature [R. Jördens et al., PRL 104, 180401 (2010)]
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